DESIGN OF CONTROL SYSTEM WITH INCREASED POTENTIAL OF ROBUST STABILITY

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Abstract: Basing on results of catastrophe theory, a conception of the limit robust stability is discussed. One of the approaches to design extremely stable control system for linear plants is presented. Groups of real simple, multiple and complex conjugate eigenvalues with the nonlinear control action, given in the region of canonical coordinates of a system in the form of structurally stable function are considered. Widened ranges of robust stability are obtained to the control system with all possible structures.

Keywords: nonlinear control, robust stability, structurally stable functions (mappings)

I. INTRODUCTION

At present it is conventional that the most of real control systems function in conditions of small or considerable degree of uncertainty. The uncertainty can be conditioned by an ignorance of true values of controlled plant's parameters and unpredictable changes them in time. Therefore a robust stability plays the most important role in control theory of dynamic system. In common statement, the robust stability consists in the indication of restrictions on changes of parameters of a control system, at which a stability is saved.

The large number of works is devoted to the problem of the robust stability of control system [Siliak, Vidyasagar]. This article is devoted to a construction of the robust stable control system for linear plants, with nonlinear control actions, given in the form one-parametrical, structurally stable mappings [Poston, Gilmor], adding a limit robust stability to the designed control system.

The conception of the construction of the extremely robust stable control system of dynamic plants is based on results of the theory of catastrophes, where the main result is basic structurally stable mappings. They are limited and defined by number of controlling parameters immediately.

II. ANALYSIS OF STABILITY OF CONTROL SYSTEM WITH ONE OUTPUT VARIABLE

Let the control system is described by a state equation

$$\dot{x} = Ax + Bu \ , \qquad (1)$$

where x is a n-dimensional state vector; u is a m-dimensional control vector-function; A and B are n-column and n-rowed and n-column matrixes as

$$A = \begin{Vmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ . & . & . & \ldots & . \\ 0 & 0 & 0 & \ldots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \ldots & -a_1 \end{Vmatrix},$$

$$B = \begin{Vmatrix} b_{11} & 0 & \ldots & 0 \\ 0 & b_{22} & \ldots & 0 \\ . & . & \ldots & 0 \\ 0 & 0 & \ldots & b_{nn} \end{Vmatrix}$$

From the control conditions it follows that a number of non-zero diagonal element corresponds to the uncontrolled coordinate. In this case it is supposed that all variable states of the system (1) are controlled, and x_1 is an unique output coordinate. By the introduction of a regulator in the control circuit with the control action on each coordinate, given in the form one-parametrical, structurally stable function

$$u = \begin{Vmatrix} \frac{1}{b_{11}}(-x_1^3 + kx_1) \\ \frac{1}{b_{22}}(-x_1^3 + kx_1) \\ \vdots \\ \frac{1}{b_{nn}}(-x_1^3 + kx_1 + a_n x_1) \end{Vmatrix}, \qquad (2)$$

the control plant by the matrix A can be shifted in anyone beforehand given position.

We shall show that problem (1), (2) provides the limit robust stability to the designed system independently on changes of plant's parameters.

In real systems it is usually difficult to obtain exact values of the first-order derivatives of the output coordinate, practically it is impossible to measure the second-order and higher-order derivatives. But it is necessary, that a speed of the change of the output coordinate and the derivatives of the higher-order of the output value would be equal to zero in the system's steady state.

Taking in account (2) the system (1) can be described in the detailed form

$$\begin{cases} \dot{x}_1 = -x_1^3 + kx_1 + x_2 \\ \dot{x}_2 = -x_1^3 + kx_1 + x_3 \\ \dots\dots\dots \\ \dot{x}_n = -x_1^3 + kx_1 - a_{n-1}x_2 - \dots - a_1 x_n \end{cases}, \qquad (3)$$

Stationary states of the system (3) are defined

$$x_{1s}^1 = x_{2s} = \dots = x_{ns} = 0 \qquad (4)$$

and

$$x_{1s}^2 = \sqrt{k}\,,\ x_{1s}^3 = -\sqrt{k}\,,\ x_{2s} = x_{3s} = \dots = x_{ns} = 0 \qquad (5)$$

States (5) merge with (4) at the control parameter $k = 0$ and branch off from it at the $k>0$.

2. A stability of stationary states (4) and (5) of the system (3) is researched on the base of the principle of the linear stability.

The linearized system has a characteristic equation

$$\lambda^n + q_1\lambda^{n-1} + q_2\lambda^{n-2} + \dots + q_n = 0, \qquad (6)$$

where

$$q_i = a_1 - d,\ q_2 = a_2 - (a_1 + 1)d, \dots,$$
$$q_{n-1} = a_{n-1} - (a_{n-2} + a_{n-3} + \dots + a_1 + 1)d,$$
$$q_n = -(a_{n-1} + a_{n-2} + \dots + a_1 + 1)d$$
$$d = -3(x_{1s})^2 + k$$

Using Hurwitz's criterion it is not difficult to be convinced that for a stability of the stationary state (4) it is necessary that at the negative amplification factor k, additional conditions are carried out

$$\frac{a_i}{a_{i-1} + \dots a_1 + 1} > k,\ i = \overline{1,n}\ , \qquad (7)$$

and for a stability of the stationary state (5) it is necessary that at the positive amplification factor k, additional conditions are carried out

$$\frac{a_i}{2(a_{i-1} + \dots + a_1 +)} < k\,,\ i = \overline{1,n}, \qquad (8)$$

Thus, it is shown, that the system (1) gains properties of the limiting robust stability because of introduction of the nonlinear control action, given in the form of the structurally stable function. At the fulfillment of conditions (7), states (4) are globally asymptotically stable if $k<0$, and they are not stable if $k>0$; at the fulfillment of conditions (8), states (5) are asymptotically, but not globally stable. At the $k=0$, a branching off happens, and new stable branches of stationary states appear. Practically there is a possibility to construct stable control system at any admissible changes of system's parameters.

III. THE CONSTRUCTION OF EXTREMELY STABLE CONTROL SYSTEM

One of the approaches to for linear plants by groups of real simple, multiple and complex conjugate eigenvalues with the nonlinear control action, given in the region of canonical coordinates of a system in the form of one-parametrical structurally stable function, adding the limit robust stability to the control system among all possible structures, is stated.

Let a states of the nominal control system are described by the equation

$$\dot{x} = Ax + Bu\,, \qquad (9)$$

where A is a n column and n rowed square matrix of coefficients; B is a m column and n rowed control matrix; x is a n-dimensional state vector; u is unknown m-dimensional control vector-function.

The matrix of control plant A can be leaded to the block-diagonal form using an unsingular matrix P, columns of which are eigenfunctions of a matrix A

$$A = P^{-1}AP = diag\{\Lambda, J_1, \dots, J_m J_1^i, \dots, J_k^i\} \qquad (10)$$

It has diagonal square blocks such as

$$\Lambda = diag\{\lambda_1, \dots, \lambda_l\}, \qquad (11)$$

$$J_j = \begin{Vmatrix} \lambda_j & 1 & . & . & . & 0 & 0 \\ o & \lambda_j & . & . & . & 0 & 0 \\ . & . & . & . & . & . & . \\ 0 & 0 & . & . & . & \lambda_j & 1 \\ 0 & 0 & . & . & . & 0 & \lambda_j \end{Vmatrix}, N_j \times N_j,\ j = \overline{1,m} \qquad (12)$$

$$J_j^i = \begin{Vmatrix} \alpha_j & -\beta_j \\ \beta_j & \alpha_j \end{Vmatrix},\ j = \overline{1,k} \qquad (13)$$

where $\lambda_1, \dots, \lambda_l$ are real simple eigenvalues of the matrix A, λ_j are real eigenvalues of the matrix A, N_j are multiple eigenvalues of the matrix A, $\lambda_j = \alpha_j \pm j\beta_j$ are completely conjugate eigenvalues of the matrix A, and it is obvious $l + N_1 + \dots + N_m + 2k = n$.

1. We shall show, that the accepted structure (10) allows a separate control by canonical coordinates (harmonics) of the system (9) corresponding to any diagonal block of matrix $\tilde{A}$. Similarly to (9) we note

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u = \begin{Vmatrix} \Lambda & & 0 \\ & J & \\ 0 & & J^1 \end{Vmatrix}\tilde{x} + \begin{Vmatrix} \tilde{B}_1 \\ \tilde{B}_2 \\ \tilde{B}_3 \end{Vmatrix} u \qquad (14)$$

where $\tilde{x} = P^{-1}x,\ \tilde{A} = P^{-1}AP,\ \tilde{B} = P^{-1}B$

The dimensions of matrixes, $\tilde{B}_1, \tilde{B}_2, \tilde{B}_3$ and the control vector-function u correspond to dimensions of square matrixes Λ, J, J^i. Accepting $\tilde{B}_2 = 0$, $\tilde{B}_3 = 0$, at the base of (14) it is not difficult to be convinced, that we can control by canonical coordinates of the system (9), corresponding to the matrix Λ, saving canonical coordinates of the system (9), determined by matrixes J and J^i, are unchanged . The similar results can be obtained concerning matrixes J or

J^i, accepting $\widetilde{B}_1 = 0$, $\widetilde{B}_3 = 0$ or $\widetilde{B}_1 = 0$, $\widetilde{B}_2 = 0$. Thus, the further problem is in a sequential construction of extremely robust stable control systems for canonical plant

$$\dot{\widetilde{x}}_1 = \Lambda\widetilde{x}_1 + B_1 u, \quad (15)$$

$$\dot{\widetilde{x}}_2 = J\widetilde{x}_2 + B_2 u, \quad (16)$$

$$\dot{\widetilde{x}}_3 = J^i\widetilde{x}_3 + B_3 u, \quad (17)$$

where $\widetilde{x}_1 = \begin{Vmatrix} \widetilde{x}_1 \\ \widetilde{x}_2 \\ . \\ . \\ . \\ \widetilde{x}_l \end{Vmatrix}$, $\widetilde{x}_2 = \begin{Vmatrix} \widetilde{x}_{l+1} \\ \widetilde{x}_{l+2} \\ . \\ . \\ . \\ \widetilde{x}_{l+L} \end{Vmatrix}$, $\widetilde{x}_3 = \begin{Vmatrix} \widetilde{x}_{l+L+1} \\ \widetilde{x}_{l+L+2} \\ . \\ . \\ . \\ \widetilde{x}_n \end{Vmatrix}$,

$L = N_1 + \ldots + N_m$,

with matrixes (11) - (13).

Let's consider problems (15), (16) and (17) by turns.

2. Suppose, that a transformed matrix of control *B*, and $\widetilde{B}_1$, $\widetilde{B}_2$ and $\widetilde{B}_3$ in (15), (16) and (17) correspondingly are diagonal. Then for a full controllability of canonical plant (15) it was necessary and sufficiently, that all diagonal elements of the matrix $\widetilde{B}_1$ would be non-zero. The availability of zero elements $\widetilde{b}_{ii}$ = 0 means, that corresponding canonical coordinates $\widetilde{x}_i$ are uncontrolled.

Components of the control vector-function *u* for $i = \overline{1,l}$ are chosen as one-parametrical, structurally stable functions which are described by equation

$$u_i = \gamma_i(-\widetilde{x}_i^3 + k_i\widetilde{x}_i),\ \gamma_i = 1/b_{ii},\ i = \overline{1,l} \quad (18)$$

The system (15) can be represented in the expanded form

$$\dot{\widetilde{x}}_i = -\widetilde{x}_i^3 + (\lambda_i + k_i)\widetilde{x}_i,\ i = \overline{1,l} \quad (19)$$

The stationary (steady) states of the system (11) will be described by equations

$$-\widetilde{x}_{is}^3 + (\lambda_i + k_i)\widetilde{x}_{is} = 0,\ i = \overline{1,l} \quad (20)$$

From (20) stationary states of canonical coordinates of the system (19) are defined

$$\widetilde{x}_{1is} = 0,\ i = \overline{1,l} \quad (21)$$

and

$$\widetilde{x}_{is}^{2,3} = \pm\sqrt{\lambda_i + k_i},\ i = \overline{1,l}, \quad (22)$$

A stability of stationary states (21) and (22) of the system (19) can be reached by the linear principle of the stability

$$\dot{\widetilde{x}}_i = \left[-3(\widetilde{x}_{is})^2 + \lambda_i + k_i\right]\widetilde{x}_i\ \ i = \overline{1,l}\quad, \quad (23)$$

Hence, stationary states of canonical coordinates (21) of the system (20) are globally asymptotically stable, if $\lambda_i + k_i < 0$, $i = \overline{1,l}$, and additional stationary states (14), appearing at $\lambda_i + k_i > 0$, $i = \overline{1,n}$ also are not globally but asymptotically stable.

3. For a full controllability of the canonical plant (16) it was necessary and sufficiently, that at least all last diagonal elements of the matrix $\widetilde{B}_2$, corresponding N_j - multiple eigenvalues of the matrix *J* at $j = \overline{1,m}$, would be different from zero. Proceeding from a practical expediency, further we shall assume, that all diagonal elements of a matrix $\widetilde{B}_2$ are different from zero, that is all canonical coordinates $\widetilde{x}_i$, $i = \overline{l+1,l+L}$ are controlled immediately.

Components of the control vector-function *u* for $i = \overline{l+1,l+L}$ are presented as

$$u_i = \gamma_i(-\widetilde{x}_i^3 + k_i\widetilde{x}_i - \widetilde{x}_{i+1}),$$
$$\gamma_i = 1/b_{ii},\ i = \overline{l+1,l+L}, \quad (24)$$

Taking in account (24) the system (16) is represented as

$$\dot{\widetilde{x}}_i = -\widetilde{x}_i^3 + (\lambda_j + k_i)\widetilde{x}_i,\ j = \overline{1,m},\ i = \overline{l+1,l+L}, \quad (25)$$

Stationary states of the system (17) are determined from equations

$$-\widetilde{x}_{is}^3 + (\lambda_j + k_i)\widetilde{x}_{is} = 0,\ j = \overline{1,m},\ i = \overline{l+1,l+L} \quad (26)$$

The system (26) has solutions

$$\widetilde{x}_{is}^1 = 0,\ i = \overline{l+1,l+L} \quad (27)$$

and

$$\widetilde{x}_{is}^{2,3} = \pm\sqrt{\lambda_j + k_i}\ \ j = \overline{1,m}\ i = \overline{l+1,l+L} \quad (28)$$

A stability of stationary states of canonical coordinates (27) and (28) of the system (25) is determined on the base of the linear principle of the stability by analogy with section II.2.

Stationary states of canonical coordinates (27) of the system (25) are globally asymptotically stable, at $\lambda_j + k_i < 0$, $j = \overline{1,n}$, $i = \overline{l+1,l+L}$, and additional stationary states (20), appearing at $\lambda_j + k_i > 0$, $j = \overline{1,m}$, $i = \overline{j+1,l+L}$ are asymptotically stable too, but not globally.

4. For a full controllability of the canonical plant (17) with the matrix J^i in (13) it was necessary and sufficiently, that at least one from complex conjugate diagonal elements b_{ii}, $b_{i+1,i+1,}$, $i = \overline{1,2k}$ of the matrix $\widetilde{B}_3$ corresponding to complex conjugate eigenvalues of the matrix J^i would be different from zero. The validity of this statement is obvious, but for a generality we assume, that all canonical variables of the system (17) are controlled immediately, that is all diagonal elements of the matrix $\widetilde{B}_3$ are different from zero.

Choosing the control action for the canonical plant by the earlier indicated way, state equations and stationary states of canonical coordinates are defined as:

$u_i = \gamma_i(-\tilde{x}_i^3 + k_i\tilde{x}_i + \beta_j\tilde{x}_{i+1})$,
$j = \overline{1,k}$, $i = \overline{j+L+1,n}$, if *j* is odd,
or $u_i = \gamma_i(-\tilde{x}_i^3 + k_i\tilde{x}_i - \beta_j\tilde{x}_{i-1})$,
$j = 1,k$, $i = \overline{l+L+1,n}$, if *j* is even.

$$\dot{\tilde{x}}_i = -\tilde{x}_i^3 + (\alpha_j + k_i)\tilde{x}_i,\ j = 1,k\ \ i = \overline{l+L+1,n} \quad (29)$$

$$-\tilde{x}_{is}^3 + (\alpha_j + k_i)\tilde{x}_{is} = 0,\ j = 1,k\ , i = \overline{l+L+1,n}$$

Hence,

$$\tilde{x}_{is}^1 = 0,\ i = \overline{l+L+1,n} \quad (30)$$

and

$$\tilde{x}_{is}^{2,3} = \pm\sqrt{\alpha_j + k_i},\ j = 1,k\ , i = \overline{l+L+1,n} \quad (31)$$

The stationary state (30) of the system (29) is globally asymptotically stable, if $\alpha_j + k_i < 0$, $j = \overline{1,k}$; $i = \overline{l+L+1,n}$, and additional stationary states (30) appearing at $\alpha_j + k_i > 0$, $j = \overline{1,k}$; $i = \overline{l+L+1,n}$ are also asymptotically stable, but not globally.

Thus, choosing the control action in the region of canonical variables in dependence on eigenvalues of control plant's matrix *A*, we can add properties of the limit robust stability to the initial system (9), that is a system becomes stable at any change of parameters of the control plant and regulator.

IV EXAMPLE

Consider the automatic control system described by the system of equations

$$\begin{cases} \dot{x}_1 = \dfrac{1}{T_1}x_2 \\ \dot{x}_2 = \dfrac{1}{T_2}kx_1 \end{cases}, \quad (31)$$

where x_1 is deflection and x_2 is the first-order derivative

It is obviously, that the system (31) by linear control action $u = -kx_1$ (*k* = const) is either at limit of stability (k<0) or unstable (k>0).

Therefore we study the stability of the control system with nonlinear element $f(x_1) = x_1^3$.

This control system is described by the system of equations

$$\begin{cases} \dot{x}_1 = \dfrac{1}{T_1}(-x_1^3 + kx_1 + x_2) \\ \dot{x}_2 = \dfrac{1}{T_2}(-x_1^3 + k_1x_1) \end{cases} \quad (32).$$

The stationary states of the system will be described by equations

$$\begin{cases} -x_{1s}^3 + kx_{1s} + x_{2s} = 0 \\ -x_{1s}^3 + kx_{1s} = 0 \end{cases}$$

Hence, the stationary states of the system (32) are defined, as

$$x_{1s}^1 = x_{2s}^1 = 0 \quad (33)$$

and

$$x_{1s}^{2,3} = \pm\sqrt{k},\quad x_{2s}^{2,3} = 0\,. \quad (34)$$

The stability of stationary states is researched on the base of the principles of the linear stability.

$$\begin{cases} \dot{x}_1 = \dfrac{1}{T_1}\left[(-3(x_{1s})^2 + k)x_1 + x_2\right] \\ \dot{x}_2 = \dfrac{1}{T_2}\left(-3(x_{1s})^2 + k\right)x_1 \end{cases}$$

For the stability of stationary states (33)-(34) and accordingly the system (32) it is necessary and sufficiently, that the condition is carried out

$$-3(x_{1s} + k) < 0\,.$$

Then for the stability states (34) it is necessary and sufficiently, that the value of the coefficient *k* is less 0, but for the stability states ()it is necessary and sufficiently, that the value of the coefficient k is more 0.

Thus we see, if the control function is chosen as one-parametrical structurally stable function, the unstable linear system of the second order will be become stable by any values of parameters *k*, T_1, T_2.

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