MODELLING OF TRANSITION PROCESSES IN ECONOMIC SYSTEM

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Development of methods of analysis of the economic system reforming processe realized in many post-soviet countries is actual scientific problem. Pricing, taxation, state budget control, different aspects of monetary policy are very important issues. Scientific analysis of them should be based on precise mathematical description of economic system functioning process as a whole. Therefore construction of mathematical models and development of information system of economic system imitative modelling by the model imitative experiment on computer has a great interest.

At present great efforts to develop this problem are undertaken .

In the article microdynamic model of transitional economic system with macroeconomic regulation presented by complex of systems of differential and algebraic equations describing main economic functions and relations of industries, population, bank, government, labor market, good market and money market is developed.

Economic system is closed. In system *n* goods and services are produced, they are used as production factors and consumer goods. Production factors are divided into fixed assets (machines, equipment, etc.), which are depreciated in each time partially, and inputs (raw materials, fuel, etc.), which are spent completely in each time.

This article is based on Leontiev model of production, that is good with an index $l = 1,2$, is produced by one and only one *l*-th industry. Each industry is divided into three subindustries in dependence on the form of ownership: state, joint-stock, and private. Each subindustry $i = 1,2,3$ uses a unique technology. *t* is time ($t \geq 0$), *i* is an index subindustry which has *i*-th form of ownership ($i = 1,2,3$), *l* is an index of industry or produced good ($l = 1,2, ..., n$).

Cobb-Douglass production function determines production of subindustry of *l*-th industry and *i*-th ownership form in the moment *t*

$$y_l^i(t) = F(\chi, x_l^i(t), L_l^i(t))$$

where χ is a vector of production function parameters; $x_l^i(t)$ is a volume of fixed assets; $L_l^i(t)$ is a number of workers.

Dynamics of fixed assets is described by equation

$$\frac{dx_l^i(t)}{dt} = \Phi_l^{Ii}(t) - \mu_l^i x_l^i(t)$$

where μ_l^i is a coefficient of leaving of fixed assets; $\Phi_l^{Ii}(t)$ is investments on production expansion.

Dynamics of loan capital of subindustry of *l*-th industry and *i*-th ownership form is

$$\frac{d\Phi_l^{Ki}(t)}{dt} = (1-\pi)U_l^i(t) - r(t)\Phi_l^{Ki}(t),$$

where π is a share of returned loan; $r(t)$ is rate of return; $U_l^i(t) = D_l^i(t)(1-\eta)/(1+r(t))$ is demand for credit, $D_l^i(t) = P_l(k)y_l^i(t)$ is revenues from sales; P_l (t) is price of *l*-th good (service); η is income tax rate.

Demand for *l*-th good of *m*-th population group (*m* = 1,2,3) is

$$W_i^m(t) = \frac{C_l^m}{\sum_{l=1}^{n} C_l^m P_l(t)} \Phi^m(t)$$

where $\boldsymbol{C^b} = (\boldsymbol{C_1^b},\ldots,\boldsymbol{C_n^m})$ is a vector of *m*-th population group's demand structure; (respectively workers of productive, non-productive and not sufficiently provided stratums); $\Phi^m(t)$ - consumer spending.

Dynamics for bank deposits of *m*-th population group is

$$\frac{dH^m(t)}{dt} = (1-d^m)\Phi^m(t), \qquad m = 1,2,3$$

where d^m is a coefficient of propensity to consume of *m*-th population group.

Flows of payments (buyers' expenditures)

$$\Phi(t) = \Phi^{\mathrm{A}}(t) + \Phi^I(t) + \sum_{m=1}^{3} \Phi^m(t) + \Phi^B(t) + G^3(t)$$

where $\Phi^{\mathrm{A}}(t)$ is total expenditures of enterprises on purchase of raw materials; $\Phi^I(t)$ is total investments in production; $G^3(t)$ is government expenditures; $\Phi^B(t)$ is expenditures of bank on purchase of goods (services).

Demand of banking system for *l*-th good

$$W_l^{\mathrm{B}}(t) = \frac{C_l^{\mathrm{B}}}{\sum_{l=1}^{n} C_l^{\mathrm{B}} P_l(t)} \Phi^{\mathrm{B}}(t)$$

where $C^{\mathrm{B}} = (C_1^{\mathrm{B}},\ldots,C_n^{\mathrm{B}})$ is a vector of bank's demand structure.

Dynamics of money in circulation is

$$\frac{dM(t)}{dt} = \varepsilon H(t) + \theta\Phi(t) + K\sum_{l=1}^{n}\sum_{i=1}^{3} P_l(t)y_l^i(t) - M(t)$$

where ε is rate of reservation of bank accounts, $0 < \varepsilon < 1$; K is an rate of currency emission; θ - velocity of money in circulation, $H(t)$ - savings of population.

In each moment *t* government revenue $D(t)$ consists of income taxes imposed on industries $N(t)$, population groups $N_H(t)$, banks $N_o(t)$, dividends on shares $D_G^{\mathrm{A}}(t)$,

revenues from sales of bonds $D\Pi_G^{\text{A}}(t)$, revenues from sales of a state property as a result of privatisation $D^G(t)$, loans and other revenues ÇÏ (t)

$$D(t) = N(t) + N_H(t) + N_0(t) + D_G^{\text{A}}(t) + D\Pi_G^{\text{A}}(t) + D^G(t) + 3\Pi(t)$$

In moment *t* government expenditures $G^P(t)$ consist of transfers to population groups $V_0^m(t)$ (m = 1,2,3), expenditures on government purchases $G^3(t)$, expenditures on education, culture and science $G^C(t)$, dotations $G^D(t)$ and other expenditures $G^{\Pi}(t)$

$$G^P(t) = \sum_{m=1}^{4} V_0^m(t) + G^3(t) + G^C(t) + G^D(t) + G^{\Pi}(t)$$

Demand for *l*-th good of government in moment *t*

$$W_l^G(t) = \frac{C_l^G}{\sum_{l=1}^{n} C_l^G P_l(t)} G^3(t)$$

where $C^G = (C_1^G,\ldots,C_n^G)$ is government demand structure.

Dynamics of population structure is presented by the differential equation

$$\frac{dN(t)}{dt} = AN(t)$$

where $N(t) = (N_1(t),$ is a vector of population by age groups.

$$\text{À} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots & a_{1,n-1} & a_n \\ a_{21} & a_{22} & 0 & \ldots & 0 & 0 \\ 0 & a_{32} & a_{33} & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & a_{n,n-1} & a_{nn} \end{bmatrix}$$

N is matrix of intensity of the birth rate, death rate, immigration and emigration in age groups; $a_{12,}a_{13},\ldots,a_{1n}$ is an intensity of the birth rate in age groups; $a_{jj}(j = 1,\ldots,n)$ is an intensity of immigration; $a_{j+1,j} = 1 - \mu_i - m_j;$ μ_j and m_j are intensities of death rate and emigration in *j*-th age group.

There is an equilibrium in the labour market when labour demand L_D (*t*) is equal to labour supply N_S (*t*). Number of the employed in each subindustry is determined from nonlinear model of labour market in dependence on wage rate $W_l^i(t)$

$$\frac{dL_l^i(t)}{dt} = L_l^i(t)\left\{1 - \frac{L_l^i(t)\sum_{i=1}^{3}\sum_{l=1}^{n} W_l^i(t)}{L_D(t)W_l^i(t)}\right\}, \quad L_l^i(t) \le L_{Dl}^i(t) \quad i = 1,2,3; \quad l = \overline{1,n}$$

Number of the unemployed is

$$L_0(t) = N_S(t) - \sum_{l=1}^{n}\sum_{i=1}^{3} L_l^i(t)$$

It is assumed that the money market is in equilibrium; the interest rate r (t) is determined by equality between demand for credit and supply of credit.

Money market is described by the differential equation

$$\frac{dr(t)}{dt} = \left[\mathrm{M}(t) - \varepsilon H(t) - \Phi^{\mathrm{A}}(t) - \sum_{m=1}^{3} \Phi^m(t) - \Phi^{\mathrm{B}}(t) - G^3(t) - \frac{1-\eta}{1+r(t)} \sum_{l=1}^{n}\sum_{i=1}^{3} P_l(t) y_l^i(t) \right] \times \quad \times \frac{1}{\sum_{l=1}^{n} P_l(t)\tilde{y}_l(t)}$$

where $\tilde{y}_l(t) = \sum_{i=1}^{3} y_l^i(t) + y_l^u(t) - y_l^э(t)$ is supply of l-th good in good market, $y_l^э(t)$ and $y_l^u(t)$ are total export and import of l-th good respectively in moment t.

Interindustry relations are determined by equation (Leontiev model)

$$\tilde{y}_l(t) = \sum_{i=1}^{3}\sum_{j=1}^{n} a_{lj}^i y_j^i(t) + \sum_{i=1}^{3}\sum_{j=1}^{n} b_{lj}^i \vartheta_l^i(t) + \sum_{m=1}^{3} w_l^m(t) + w_l^{\mathrm{B}}(t) + w_l^G(t)], \quad l = \overline{1,n},$$

where a_{jl}^i and b_{jl}^i are material and capital coefficients respectively, $\vartheta_l^i(t)$ is expected increase of production.

Price level is a main indicator in the good market $P_l(t)$.Dynamics of changes in price of l-th good is

$$\frac{dP_l(t)}{dt} = \frac{P_l(t)}{\tilde{y}_l(t)} \left[\sum_{i=1}^{3}\sum_{j=1}^{n} a_{lj}^i y_j^i(t) + \sum_{i=1}^{3}\sum_{j=1}^{n} b_{lj}^i \vartheta_l^i(t) + \sum_{m=1}^{3} w_l^m(t) + w_l^B(t) + w_l^G(t)] - P_l(t); \qquad l = \overline{1,n}.$$

Information system is developed on the basis of constructed mathematical models. It is intended for a imitative modeling of transitional economy. It allows to estimate different scenarios of economic system development at various combinations and values of government regulation mechanisms. Results of numerical tests on the basis of economy consisting of two industries uncover main conformities to natural laws of economic processes development and confirm capacity and usefulness of information system.

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