Power-Law Entropy-Corrected New Agegraphic Dark Energy in Hořava-Lifshitz Cosmology

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Abstract

We investigate the new agegraphic dark energy (NADE) model with power-law corrected entropy in the framework of Hořava-Lifshitz cosmology. For a non-flat universe containing the interacting power-law entropy-corrected NADE (PLECNADE) with dark matter, we obtain the differential equation of the evolution of density parameter as well as the deceleration parameter. To study parametric behavior, we use an interesting form of state parameter as function of redshift $\omega_\Lambda(z) = \omega_0 + \omega_1 z$. We find that phantom crossing occurs for the state parameter for a non-zero coupling parameter, thus supporting interacting dark energy model.

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1 Introduction

Observational data of type Ia supernovae (SNeIa) collected by Riess et al. [1] in the High-redshift Supernova Search Team and by Perlmutter et al. [2] in the Supernova Cosmology Project Team independently reported that the present observable universe is undergoing an accelerated expansion phase. The exotic source for this cosmic acceleration is generally dubbed "dark energy" (DE) which is distinguished from ordinary matter (such as baryons and radiation), in the sense that it has negative pressure. This negative pressure leads to the accelerated expansion of the universe by counteracting the gravitational force. The astrophysical observations show that about 70% of the present energy of the universe is contained in DE. Although the nature and cosmological origin of DE is still enigmatic at the present, a great variety of models has been proposed to describe the DE (see e.g., the reviews [3, 4]). Two promising candidates are the holographic DE (HDE) [5] and the agegraphic DE (ADE) [6] models which are originated from some considerations of the features of the quantum theory of gravity.

It is curious to note that the HDE model has its origin (i.e. definition and derivation) depends on the Bekenstein-Hawking (BH) entropy-area relationship $S_{\rm BH} = A/4$ of black hole thermodynamics, where A is the area of the horizon [7]. However, this definition of HDE can be modified (or corrected) due to the various correction procedures applied to gravity theories [8]. For instance, the corrections to the entropy which appear in dealing with the entanglement of quantum fields in and out the horizon [9] generate a power-corrected area term in the entropy expression. The power-law corrected entropy has the form [10]

$$S = \frac{A}{4}\left[1 - K_\alpha A^{1-\frac{\alpha}{2}}\right], \tag{1}$$

where α is a dimensionless constant whose value is currently under debate and determining its unique and precise value requires separate investigation, and

$$K_\alpha = \frac{\alpha(4\pi)^{\frac{\alpha}{2}-1}}{(4-\alpha)r_c^{2-\alpha}}, \tag{2}$$

where r_c is the crossover scale. The second term in Eq. (1) can be regarded as a power-law correction to the entropy-area law, resulting from entanglement i.e. the wave function of the field is taken to be a superposition/entanglement of ground and exited states [9]. The entanglement entropy of the ground state satisfies the BH entropy-area relationship. Only the excited state contributes to the correction, and more excitations produce more deviation from the BH entropy-area law [11] (also see [12] for a review on the origin of black hole entropy through entanglement). This lends further credence to entanglement as a possible source of black hole entropy. The correction term is also more significant for higher excitations [9]. It is important to note that the correction term falls off rapidly with increasing A. So for large black holes the correction term falls off rapidly and the BH entropy-area law is recovered, whereas for the small black holes the correction is significant.

The ADE model is originated from the uncertainty relation of quantum mechanics together with the gravitational effect in general relativity (GR). The ADE model assumes that the observed DE comes from the spacetime and matter field fluctuations in the universe [6]. Following the line of quantum fluctuations of spacetime, Karolyhazy [13] proposed that the distance in Minkowski spacetime cannot be known to a better accuracy than $\delta t = \varepsilon t_P^{2/3} t^{1/3}$, where ε is a dimensionless constant of order unity and t_P is the reduced Planck time. Based on Karolyhazy relation, Maziashvili proposed that the energy density of metric fluctuations of Minkowski spacetime is given by [14]

$$\rho_\Lambda \sim \frac{1}{t_P^2 t^2} \sim \frac{M_P^2}{t^2}, \tag{3}$$

where M_P is the reduced Planck mass $M_P^{-2} = 8\pi G$. Since in the original ADE model the age of the universe is chosen as the length measure, instead of the horizon distance, the causality problem in the HDE is avoided [6]. The original ADE model had some difficulties. In particular, it cannot justify the matter-dominated era [6]. This motivated Wei and Cai [15] to propose the new ADE (NADE) model, while the time scale is chosen to be the conformal time instead of the age of the universe. The NADE density is given by [15]

$$\rho_\Lambda = \frac{3n^2 M_P^2}{\eta^2}, \tag{4}$$

where $3n^2$ is the numerical factor and η is the conformal time and defined as

$$\eta = \int \frac{\mathrm{d}t}{a} = \int_0^a \frac{\mathrm{d}a}{Ha^2}. \tag{5}$$

The ADE models have been examined and constrained by various astronomical observations [16, 17, 18, 19, 20, 21]. Inspired by the power-law corrected entropy relation (1), and following the derivation of HDE [22] and entropy-corrected HDE (ECHDE) [23], we can easily obtain the so-called "power-law entropy-corrected" NADE (PLECNADE) whose the scale is chosen to be the conformal time η. Therefore, we write down the energy density of PLECNADE as [24, 25]

$$\rho_\Lambda = \frac{3n^2 M_P^2}{\eta^2} - \frac{\beta M_P^2}{\eta^\alpha}, \tag{6}$$

where β is a dimensional constant whose the precise value needs to be determined. In this paper, our aim is to investigate the PLECNADE model in Hořava-Lifshitz cosmology.

The plane of the paper as follows: In section 2, we give a brief review of Hořava-Lifshitz cosmology in detailed balance case. In section 3, we construct a model of interaction between DE and DM. In section 4, we discuss some cosmological implications of this model. We obtain the evolution of dimensionless energy density, deceleration parameter and equation of state parameter of PLECNADE model. In section 5 we give the conclusion.

2 Basics of Hořava-Lifshitz Cosmology: Detailed Balance Case

Recently a power-counting renormalizable UV complete theory of gravity was proposed by Hořava [26]. Quantum gravity models based on an anisotropic scaling of the space and time dimensions have recently attracted significant attention [27]. In particular, Hořava-Lifshitz point gravity might be has desirable features, but in its original incarnation one is forced to accept a non-zero cosmological constant of the wrong sign to be compatible with observations [28]. At a first look it seems that this non-relativistic model for quantum gravity has a well defined IR limit and it reduces to GR. But as it was first indicated by Mukohyama [29], Hořava-Lifshitz theory mimics GR plus dark matter (DM). This theory has a scale invariant power spectrum which describes inflation [30]. Moreover, some new integrable and nonintegrable cosmological models of the Hořava-Lifshitz gravity have been discussed in [31]. Phenomenologically, in Hořava-Lifshitz gravity the radiation energy density decreases proportional to a^{-6} [32]. Hence the resultant baryon asymmetry as well as the stochastic gravity waves can be enhanced. Some cosmological solutions to in Hořava-Lifshitz gravity are obtained previously [33]. Saridakis formulated Horava-Lifshitz cosmology with an additional scalar field and showed that Horava-Lifshitz dark energy naturally presents very interesting behaviors, possessing a varying equation-of-state parameter, exhibiting phantom behavior and allowing for a realization of the phantom divide crossing. In addition, Horava-Lifshitz dark energy guarantees for a bounce at small scale factors and it may trigger the turnaround at large scale factors, leading naturally to cyclic cosmology [34]. The large scale evolution and curvature perturbations in HL gravity are explored in [35], while the origin of primordial large-scale magnetic fields in the Horava's non-relativistic gravity have been discussed in [36]. The generalized second law of thermodynamics in Horava-Lifshitz cosmology is studied in [37]. For reviews on the scenario where the cosmological evolution is governed by Hořava-Lifshitz gravity see [29, 38].

Under the detailed balance and the projectability conditions, the modified Friedmann equations in the framework of Hořava-Lifshitz (HL) gravity are given by [26]

$$H^2 = \frac{\kappa^2}{6(3\lambda-1)}\rho_{\rm m} + \frac{\kappa^2}{6(3\lambda-1)}\left[\frac{3\kappa^2\mu^2k^2}{8(3\lambda-1)a^4} + \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)}\right] - \frac{\kappa^4\mu^2\Lambda k}{8(3\lambda-1)^2a^2}, \tag{7}$$

$$\dot{H} + \frac{3}{2}H^2 = -\frac{\kappa^2}{4(3\lambda-1)}p_{\rm m} - \frac{\kappa^2}{4(3\lambda-1)}\left[\frac{\kappa^2\mu^2k^2}{8(3\lambda-1)a^4} - \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)}\right] - \frac{\kappa^4\mu^2\Lambda k}{16(3\lambda-1)^2a^2}, \tag{8}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, λ is a dimensionless constant and Λ is a positive constant which as usual is related to the cosmological constant in the IR limit. The parameters κ and μ are constants. Also k denotes the curvature of space $k = 0, 1, -1$ for a flat, closed an open universe, respectively. Furthermore, $\rho_{\rm m}$ and $p_{\rm m}$ are the energy density and pressure of the matter.

Noticing the form of the above Friedmann equations, we can define the energy density ρ_Λ and pressure p_Λ for DE as

$$\rho_\Lambda \equiv \frac{3\kappa^2\mu^2k^2}{8(3\lambda-1)a^4} + \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)}, \tag{9}$$

$$p_\Lambda \equiv \frac{\kappa^2\mu^2k^2}{8(3\lambda-1)a^4} - \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)}. \tag{10}$$

The first term on the right hand side proportional to a^{-4} is effectively the "dark radiation term", present in HL cosmology [39], while the second term is referred as an explicit cosmological constant.

Finally in order for these expressions to match with the standard Friedmann equations we set [39, 40]

$$G_{\rm c} = \frac{\kappa^2}{16\pi(3\lambda-1)}, \tag{11}$$

$$\frac{\kappa^4\mu^2\Lambda}{8(3\lambda-1)^2} = 1, \tag{12}$$

where $G_{\rm c}$ is the "cosmological" Newton's constant. Note that in gravitational theories with the violation of Lorentz invariance (like HL gravity) the "gravitational" Newton's constant $G_{\rm g}$, which is present in the gravitational action, differs from the "cosmological" Newton's constant

$G_{\rm c}$, which is present in the Friedmann equations, unless Lorentz invariance is restored [41]. For the sake of completeness we write

$$G_{\rm g} = \frac{\kappa^2}{32\pi}. \qquad (13)$$

Note that in the IR limit ($\lambda = 1$), where Lorentz invariance is restored, $G_{\rm c}$ and $G_{\rm g}$ are the same.

Further we can rewrite the modified Friedmann Eqs. (7) and (8) in the usual forms as

$$H^2 + \frac{k}{a^2} = \frac{8\pi G_{\rm c}}{3}(\rho_{\rm m} + \rho_\Lambda), \qquad (14)$$

$$\dot{H} + \frac{3}{2}H^2 + \frac{k}{2a^2} = -4\pi G_{\rm c}(p_{\rm m} + p_\Lambda). \qquad (15)$$

3 Model with Interaction

Here we would like to investigate the PLECNADE in HL theory. To do this we consider a spatially non-flat Friedmann-Robertson-Walker (FRW) universe containing the PLECNADE and DM. Let us define the dimensionless energy densities as

$$\Omega_{\rm m} = \frac{\rho_{\rm m}}{\rho_{\rm cr}} = \frac{8\pi {\rm G_c}}{3H^2}\rho_{\rm m}, \qquad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{\rm cr}} = \frac{8\pi {\rm G_c}}{3H^2}\rho_\Lambda, \qquad \Omega_k = -\frac{k}{a^2H^2}, \qquad (16)$$

thus the Friedmann Eq. (7) can be rewritten as

$$1 - \Omega_k = \Omega_\Lambda + \Omega_{\rm m}. \qquad (17)$$

Taking time derivative of Eq. (6) and using relation $\dot{\eta} = 1/a$, we get

$$\dot{\rho}_\Lambda = \left(\frac{1}{a\eta}\right)\left[-2\rho_\Lambda + \frac{\beta M_P^2}{\eta^\alpha}(\alpha - 2)\right]. \qquad (18)$$

Also, if we take the time derivative of the second relation in Eq. (16) after using (18), as well as relations $\dot{\eta} = 1/a$ and $\dot{\Omega}_\Lambda = H\Omega'_\Lambda$, we obtain the equation of motion for Ω_Λ as

$$\Omega'_\Lambda = \left[-2\Omega_\Lambda\frac{\dot{H}}{H^2} - \frac{2\Omega_\Lambda}{aH\eta} + \frac{G_{\rm c}}{G_{\rm g}}\frac{\beta(\alpha-2)}{3aH^3\eta^{\alpha+1}}\right]. \qquad (19)$$

Here, prime denotes the derivative with respect to $x = \ln a$. Taking derivative of $\Omega_k = -k/(a^2H^2)$ with respect to $x = \ln a$, one gets

$$\Omega'_k = -2\Omega_k\left(1 + \frac{\dot{H}}{H^2}\right). \qquad (20)$$

To be more general we consider an interaction between DM and PLECNADE. The recent observational evidence provided by the galaxy clusters supports the interaction between DE and DM [42]. In this case, the energy densities of DE and DM no longer satisfy independent conservation laws. They obey instead

$$\dot{\rho}_\Lambda + 3H(1 + \omega_\Lambda)\rho_\Lambda = -Q, \qquad (21)$$

$$\dot{\rho}_{\rm m} + 3H\rho_{\rm m} = Q, \qquad (22)$$

where $Q = 3b^2 H\rho_\Lambda$ stands for the interaction term with coupling constant b^2. Note that the form of Q is chosen purely phenomenologically i.e. to obtain certain desirable cosmological findings including phantom crossing and accelerated expansion. In literature, one can find numerous forms of $Q(H\rho)$ while we chose a simpler form sufficient for our purpose. A more general form of Q was proposed in [43]. Also the three interacting fluids has been studied before to investigate the triple coincidence problem [44]. Differentiating the Friedmann Eq. (7) with respect to time and using Eqs. (16), (17), (18), (21) and (22) we find

$$\frac{\dot{H}}{H^2} = \frac{1}{2}\Big[\Omega_k - 3(1-\Omega_\Lambda) + 3b^2\Omega_\Lambda\Big] - \frac{\Omega_\Lambda}{aH\eta} + \frac{G_{\rm c}}{G_{\rm g}}\frac{\beta(\alpha-2)}{6aH^3\eta^{\alpha+1}}. \tag{23}$$

Inserting this result into Eq. (19) one gets

$$\Omega'_\Lambda = \Omega_\Lambda\left[3(1-\Omega_\Lambda) - 3b^2\Omega_\Lambda - \Omega_k\right] + (1-\Omega_\Lambda)\left[\frac{-2\Omega_\Lambda}{aH\eta} + \frac{G_{\rm c}}{G_{\rm g}}\frac{\beta(\alpha-2)}{3aH^3\eta^{\alpha+1}}\right]. \tag{24}$$

Combining Eq. (23) with (20) we have

$$\Omega'_k = \Omega_k\left[(1-\Omega_k) - 3\Omega_\Lambda - 3b^2\Omega_\Lambda + \frac{2\Omega_\Lambda}{aH\eta} - \frac{G_{\rm c}}{G_{\rm g}}\frac{\beta(\alpha-2)}{3aH^3\eta^{\alpha+1}}\right]. \tag{25}$$

Adding Eqs. (24) and (25) yields

$$\Omega'_\Lambda + \Omega'_k = (1-\Omega_k-\Omega_\Lambda)\left[\Omega_k + 3\Omega_\Lambda - \frac{3b^2\Omega_\Lambda(\Omega_k+\Omega_\Lambda)}{(1-\Omega_k-\Omega_\Lambda)} + \frac{G_{\rm c}}{G_{\rm g}}\frac{\beta(\alpha-2)}{3aH^3\eta^{\alpha+1}} - \frac{2\Omega_\Lambda}{aH\eta}\right]. \tag{26}$$

For completeness we give the deceleration parameter which is defined as

$$q = -\left(1 + \frac{\dot{H}}{H^2}\right). \tag{27}$$

After combining Eq. (23) with (27) we get

$$q = \frac{1}{2}[1 - \Omega_k - 3(1+b^2)\Omega_\Lambda] + \frac{\Omega_\Lambda}{aH\eta} - \frac{G_{\rm c}}{G_{\rm g}}\frac{\beta(\alpha-2)}{3aH^3\eta^{\alpha+1}}. \tag{28}$$

Using definitions (16) as well as Eq. (17) we have

$$\rho_\Lambda = \frac{\rho_{\rm m}}{\Omega_{\rm m}}\Omega_\Lambda = \frac{\rho_{\rm m}}{(1-\Omega_k-\Omega_\Lambda)}\Omega_\Lambda, \tag{29}$$

which from it we can obtain

$$\frac{{\rm d}\ln\rho_\Lambda}{{\rm d}\ln a} = \frac{\rho'_{\rm m}}{\rho_{\rm m}} - \frac{\Omega'_{\rm m}}{\Omega_{\rm m}} + \frac{\Omega'_\Lambda}{\Omega_\Lambda}. \tag{30}$$

4 Cosmological Implications

In this section, we study some cosmological consequences of a phenomenologically time-dependent parameterization for the PLECNADE equation of state as

$$\omega_\Lambda(z) = \omega_0 + \omega_1 z. \tag{31}$$

It was shown in [45] that this parameterization allows to divide the parametric plane (ω_0, ω_1) in defined regions associated to distinct classes of DE models that can be confirmed or excluded from a confrontation with current observational data.

After using Eq. (21), the evolution of the DE density is obtained as [3, 46]

$$\frac{\rho_\Lambda}{\rho_{\Lambda_0}} = a^{-3(1+\omega_0-\omega_1+b^2)} e^{3\omega_1 z}. \tag{32}$$

The Taylor expansion of the DE density around $a_0 = 1$ at the present time yields

$$\ln \rho_\Lambda = \ln \rho_{\Lambda_0} + \frac{\mathrm{d}\ln \rho_\Lambda}{\mathrm{d}\ln a}\Big|_0 \ln a + \frac{1}{2}\frac{\mathrm{d}^2 \ln \rho_\Lambda}{\mathrm{d}(\ln a)^2}\Big|_0 (\ln a)^2 + \cdots. \tag{33}$$

Using the fact that for small redshifts, $\ln a = -\ln(1+z) \simeq -z + \frac{z^2}{2}$, Eqs. (32) and (33), respectively, reduce to

$$\frac{\ln(\rho_\Lambda/\rho_{\Lambda_0})}{\ln a} = -3(1+\omega_0+b^2) - \frac{3}{2}\omega_1 z, \tag{34}$$

$$\frac{\ln(\rho_\Lambda/\rho_{\Lambda_0})}{\ln a} = \frac{\mathrm{d}\ln \rho_\Lambda}{\mathrm{d}\ln a}\Big|_0 - \frac{1}{2}\frac{\mathrm{d}^2 \ln \rho_\Lambda}{\mathrm{d}(\ln a)^2}\Big|_0 z. \tag{35}$$

Comparing Eq. (34) with (35), we find that these two equations are consistent provided we have

$$\omega_0 = -\frac{1}{3}\frac{\mathrm{d}\ln \rho_\Lambda}{\mathrm{d}\ln a}\Big|_0 - 1 - b^2, \tag{36}$$

$$\omega_1 = \frac{1}{3}\frac{\mathrm{d}^2 \ln \rho_\Lambda}{\mathrm{d}(\ln a)^2}\Big|_0. \tag{37}$$

Inserting Eq. (30) in (36) and (37), after using (22), yields

$$\omega_0 = -\frac{1}{3}\left[\frac{\Omega'_\Lambda}{\Omega_\Lambda} + \frac{\Omega'_\Lambda + \Omega'_k}{(1-\Omega_k-\Omega_\Lambda)}\right]_0 - b^2\left(\frac{1-\Omega_k}{1-\Omega_k-\Omega_\Lambda}\right)_0, \tag{38}$$

$$\begin{aligned}\omega_1 = \frac{1}{3}\Bigg[&\frac{3b^2\Omega'_\Lambda}{(1-\Omega_k-\Omega_\Lambda)} + \frac{3b^2\Omega_\Lambda(\Omega'_\Lambda+\Omega'_k)}{(1-\Omega_k-\Omega_\Lambda)^2} + \frac{\Omega''_\Lambda}{\Omega_\Lambda} - \frac{\Omega'^2_\Lambda}{\Omega^2_\Lambda}\\ &+\frac{\Omega''_\Lambda+\Omega''_k}{(1-\Omega_k-\Omega_\Lambda)} + \frac{(\Omega'_\Lambda+\Omega'_k)^2}{(1-\Omega_k-\Omega_\Lambda)^2}\Bigg]_0.\end{aligned} \tag{39}$$

Substituting Eqs. (24) and (26) into (38) we reach

$$\omega_0 = \frac{-1}{3\Omega_{\Lambda_0}}\left(\frac{G_{\rm c}}{G_{\rm g}}\frac{\beta(\alpha-2)}{3H_0^3\eta_0^{\alpha+1}} - \frac{2\Omega_{\Lambda_0}}{H_0\eta_0}\right) - b^2 - 1. \tag{40}$$

Taking derivative of Eqs. (24) and (25) with respect to $x = \ln a$ and using (40), one gets

$$\begin{aligned}\Omega''_\Lambda &= -(\Omega_\Lambda\Omega'_k + \Omega'_\Lambda\Omega_k) - 3\Omega_\Lambda(\Omega_\Lambda - \Omega'_\Lambda - 1)(\omega_0+b^2+1) + (1-\Omega_\Lambda)A\\ &\quad -6\Omega'_\Lambda\Omega_\Lambda(b^2+1) + 3\Omega'_\Lambda,\end{aligned} \tag{41}$$

and

$$\Omega''_k = \Omega'_k(1-2\Omega_k) - 3\Omega_\Lambda(\Omega_k-\Omega'_k)(\omega_0+b^2+1) - A\Omega_k - 3(\Omega'_k\Omega_\Lambda + \Omega_k\Omega'_\Lambda)(b^2+1), \tag{42}$$

where A is given by

$$A = \frac{2\Omega_\Lambda}{aH\eta}\left(\frac{\dot{H}}{H^2} + \frac{\dot{\eta}}{H\eta}\right) - \frac{2\Omega'_\Lambda}{aH\eta} - \frac{G_\mathrm{c}}{G_\mathrm{g}}\frac{\beta(\alpha-2)}{3aH^3\eta^{\alpha+1}}\left(\frac{3\dot{H}}{H^2} + \frac{(\alpha+1)\dot{\eta}}{H\eta}\right). \tag{43}$$

The above expression for A can also be rewritten as

$$\begin{aligned} A &= \frac{2}{aH\eta}\Big[\Omega_k\Omega_\Lambda - 3\Omega_\Lambda(1-\Omega_\Lambda - b^2\Omega_\Lambda)\Big] + 3\Omega_\Lambda(\omega_0 + b^2 + 1)\left[\frac{1}{aH\eta} - 2\Omega_\Lambda - q - 1\right] \\ &\quad - \frac{G_\mathrm{c}}{G_\mathrm{g}}\frac{\beta(\alpha-2)}{3aH^3\eta^{\alpha+1}}\left[\frac{\alpha}{aH\eta} - 2(1+q)\right], \end{aligned} \tag{44}$$

where we have used Eqs. (23), (24) as well as the relation $\dot{\eta} = 1/a$. Adding Eqs. (41) and (42) gives

$$\begin{aligned} \Omega''_\Lambda + \Omega''_k &= (1-\Omega_\Lambda-\Omega_k)\Big[\Omega'_k + 3\Omega_\Lambda(1+\omega_0+b^2) + A\Big] + (3\Omega_\Lambda\omega_0 - \Omega_k)(\Omega'_k + \Omega'_\Lambda) \\ &\quad + 3\Omega'_\Lambda\Big[1 - (\Omega_k + \Omega_\Lambda)(1+b^2)\Big]. \end{aligned} \tag{45}$$

Finally, by combining Eqs. (24), (26), (41), (44) and (45) with (39) we find

$$\omega_1 = (1+\omega_0+b^2)\Big[3\omega_0(\Omega_{\Lambda_0} - 1) - \Omega_{k_0} - 3b^2 + 1\Big] + \frac{A_0}{3\Omega_{\Lambda_0}}, \tag{46}$$

which more explicitly can be written as

$$\begin{aligned} \omega_1 &= (1+\omega_0+b^2)\Big[\Omega_{\Lambda_0}(3\omega_0 - 2) - 3(\omega_0 + b^2) - \Omega_{k_0} - q_0\Big] \\ &\quad + \frac{1}{H_0\eta_0}\left[\omega_0 + (1+b^2)(1+2\Omega_{\Lambda_0}) + 2\left(\frac{\Omega_{k_0}}{3} - 1\right)\right] \\ &\quad - \frac{G_\mathrm{c}}{G_\mathrm{g}}\frac{\beta(\alpha-2)}{9H_0^3\eta_0^{\alpha+1}\Omega_{\Lambda_0}}\left[\frac{\alpha}{H_0\eta_0} - 2(1+q_0)\right]. \end{aligned} \tag{47}$$

Therefore, with ω_0 and ω_1 at hand we can easily write down the explicit expression for $\omega_\Lambda(z)$ in Eq. (31) in terms of model parameters such as Ω_Λ, Ω_k, the running parameter λ of HL gravity, the parameter n of PLECNADE, the interaction coupling b^2, and the correction coefficients α and β. From Fig. 1 we notice that in the absence of interaction $b^2 = 0$, $\omega_0 = -0.99$ showing quintessence state. By introducing interaction term, the state parameter evolves to a phantom state and gradually goes to more super-phantom state. Figure 1 also shows that the phantom crossing for ω_0 happens for $b^2 = 0.01$ which is compatible with the observation [47]. This is also in agreement with the result obtained by [48]. Note that phantom crossing has sound empirical support: analysis of Gold SNe and other observational datasets suggests that $\omega(z) \leq -1$, for $0 \leq z \leq 0.5$ [52]. From Fig. 2, we notice that in the absence of interaction, the first order correction ω_1 to state parameter behaves like quintessence but when interaction is introduced, the parameter ω_1 evolves towards -1, cosmological constant. Here it is probable that ω_1 can cross the cosmological constant boundary if $b^2 > 0.20$. For reader's clarity, we did not use any initial conditions for plotting both figures since Eq. (40) and Eq. (47) are not differential equations. However the behavior of curves in figures is strongly sensitive to the values of free parameters.

5 Conclusions

It has been shown that the origin of black hole entropy may lie in the entanglement of quantum fields between inside and outside of the horizon [9]. Since the modes of gravitational fluctuations in a black hole background behave as scalar fields, one is able to compute the entanglement entropy of such a field, by tracing over its degrees of freedom inside a sphere. In this way the authors of [9] showed that the black hole entropy is proportional to the area of the sphere when the field is in its ground state, but a correction term proportional to a fractional power of area results when the field is in a superposition of ground and excited states. For large horizon areas, these corrections are relatively small and the BH entropy-area law is recovered.

Here, we investigated the PLECNADE scenario in the framework of HL gravity. We considered an arbitrary spatial local curvature for the background geometry and allowed for an interaction between the PLECNADE and DM. We obtained the deceleration parameter as well as the differential equation which determines the evolution of the PLECNADE density parameter. Using a low redshift expansion of the EoS parameter of PLECNADE as $\omega_\Lambda(z) = \omega_0 + \omega_1 z$, we calculated ω_0 and ω_1 as functions of the PLECNADE and curvature density parameters, Ω_Λ and Ω_k respectively, of the running parameter λ of HL gravity, of the parameter n of PLECNADE, of the interaction coupling b^2, and of the coefficients of correction terms α and β. It is quite interesting to note that phantom crossing for ω_0 happens for $b^2 = 0.01$ i.e. a small but non-zero interaction parameter. In literature, there are two well-studied ways for phantom crossing: via modified gravities including scalar-tensor or Gauss-Bonnet braneworld models [53] or by introducing an interaction between fluid dark energy and dark matter. In the later models, its a generic feature of interacting dark energy models to have phantom crossing, for instance, using different forms of dark energy including Chaplygin gas [54], quintessence [55], new-agegraphic and holographic dark energy [20]. In [56], the present authors studied the PLECNADE interacting with matter in Brans-Dicke gravity and obtained the state parameter behaving to cross the phantom divide for small values of coupling parameter b.

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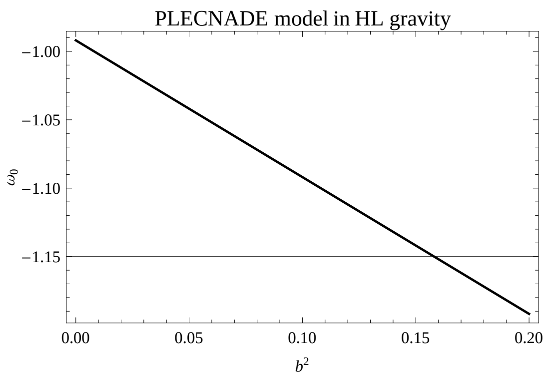

Figure 1: The EoS parameter ω_0 of the PLECNADE in HL gravity, Eq. (40), versus the interacting coupling parameter b^2. Auxiliary parameters are: $n = 2.716$ [16], $\alpha = -7.5$, $\beta = -14.8$, $\eta_0 = 1.1$ [49], $\Omega_{\Lambda_0} = 0.728$, $\Omega_{k_0} = -0.013$ [47], $\lambda = 1.02$ [50], $G_{\rm c}/G_{\rm g} = 2/(3\lambda - 1) = 0.97$, $H_0 = 74.2$ Km S^{-1} Mpc^{-1} [51] and $M_P^{-2} = 8\pi G_{\rm g} = 1$.

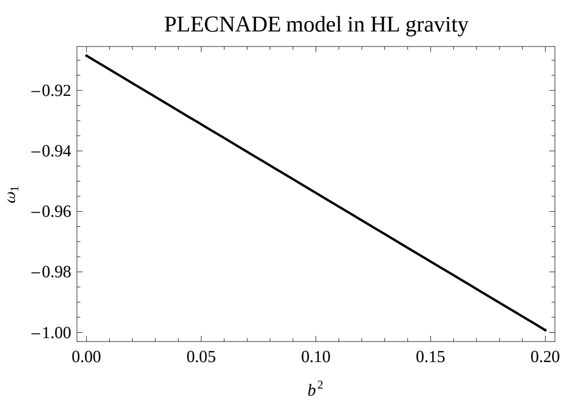

Figure 2: The EoS parameter ω_1 of the PLECNADE in HL gravity, Eq. (47), versus the interacting coupling parameter b^2. Auxiliary parameters as in Fig. 1.