

Bianchi type I cosmology in generalized Saez–Ballester theory via Noether gauge symmetry

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Received: 10 February 2012 / Revised: 23 March 2012 / Published online: 28 April 2012
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Abstract In this paper, we investigate the generalized Saez–Ballester scalar–tensor theory of gravity via Noether gauge symmetry (NGS) in the background of Bianchi type I cosmological spacetime. We start with the Lagrangian of our model and calculate its gauge symmetries and corresponding invariant quantities. We obtain the potential function for the scalar field in the exponential form. For all the symmetries obtained, we determine the gauge functions corresponding to each gauge symmetry which include constant and dynamic gauge. We discuss cosmological implications of our model and show that it is compatible with the observational data.

1 Introduction

Several cosmological observations indicate that the observable universe is undergoing a phase of accelerated expansion [1–5]. There are two major approaches to address the problem of cosmic acceleration: either introducing a ‘dark energy’ component in the universe and study its dynamics [6] or interpreting it as a failure of general relativity (GR) and consider modifying GR theory, termed as the ‘modified gravity’ approach [7–9]. Although both approaches have novel features with some deep theoretical problems, we here focus only on the modified gravity approach.

One of the earlier modifications to Einstein’s general relativity was termed Brans–Dicke gravity, in which besides a gravitational part, a dynamical scalar field was introduced

to account for a variable gravitational constant [10]. This modification was introduced due to lack of compatibility of Einstein’s theory with the Mach principle. Another cosmologically viable model is the covariant scalar–tensor–vector theory which allows the gravitational constant G [11, 12]. The theory can explain successfully galaxy rotation curves and cluster data without non-baryonic dark matter. Moreover the theory is consistent with solar system observational tests. Later Saez & Ballester [13] introduced a scalar–tensor theory of gravity in which metric is coupled to a scalar field. Here the strength of the coupling between gravity and the field was governed by a parameter ω . With this modification, they were able to solve a ‘missing-mass problem’. Several aspects of Saez & Ballester theory in relation to Bianchi cosmological models have been explored in literature [14–17].

Noether symmetries are the symmetries of the Lagrangian. In literature, the approach of Noether symmetry is used to obtain exact forms of gravitational theories including $f(T)$ gravity, where T is torsion scalar [18, 19], $f(R)$ gravity, R being Ricci scalar [20–26] and scalar–tensor theories [27]. Spherically symmetric solutions in $f(R)$ gravity via Noether symmetry were discussed in [28]. This approach gives a power-law evolutionary form of scale factor which is consistent with the astrophysical observations. Moreover the evolution of state-parameter obtained in such an approach also gives a phantom crossing behavior of dark energy [29–31]. The Noether symmetry approach has been applied to Bianchi cosmological models in literature: Capozziello et al. [32] investigated the Bianchi universes via Noether symmetries. Camci & Kucukakca [33] studied the Noether symmetries of Bianchi type I, III and Kantowski–Sachs spacetimes in scalar coupled theories. They obtained the exact solutions for potential functions, scalar field and the scale factors, see also [34] which is a similar work

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as [33]. Scalar–tensor theories have been investigated via Noether symmetry [35] but via NGS approach, the analysis is not reported earlier. The application of Noether theorem in higher-order theory of gravity turned out to be a powerful tool to find the solution of the field equations and physically reasonable solutions like power-law inflation have been discussed in [36]. The NS approach has been applied to pure gravity model with variable cosmological constant Λ and gravitational constant G in [37].

In this paper, we consider a Bianchi type I spacetime in the framework of Saez–Ballester theory of gravity. We set up a Lagrangian in which the metric variables and scalar potential play the role of dynamical variables. The Lagrangian is so constructed that its variation with respect to the metric components and the scalar potential leads to the correct equations of motion. We explicitly calculate the form of scalar potential by demanding the Lagrangian admits the desired Noether symmetry. Unlike the usual approach to obtain Noether symmetries $\mathcal{L}_X L = 0$ as followed in [20–26], we employ the full Noether Gauge Symmetry Condition $(X^{[1]}L + (D_t \xi)L = D_t G(t, \phi, A, B, C))$ introduced earlier in [38, 39]. The advantage of this later scheme is that it yields extra symmetries than the former one, hence full depth of the theory is realized in this manner.

The plan of the paper is as follows: In Sect. 2, we model our system by writing the Lagrangian and then deriving the dynamical equations of motion for Bianchi type I spacetime. In Sect. 3, we consider pure vacuum solution and construct a Lagrangian. Using it, we solve system of coupled differential equations to obtain Noether gauge symmetries and corresponding invariant quantities. We discuss some cosmological implications of our model in Sect. 4. We conclude in Sect. 5.

2 The model

The metric of Bianchi model of type I in coordinates $x^\mu = (t, x, y, z)$ is [40]

$$g_{\mu\nu} = \text{diag}(1, -A^2(t), -B^2(t), -C^2(t)). \tag{1}$$

The exact solutions of Einstein field equations based on metric (1) have been investigated in detail in the literature [40–42]. The geometrical quantities of (1) are the average scale factor $a = \sqrt[3]{ABC}$; the volume of the space-like hypersurface, defined by $V = ABC = a^3$; the generalized (or mean) Hubble parameter: $H = \frac{1}{3} \sum_i H_i$ where $H_i = \partial_t \log(A_i)$, $A_i = \{A, B, C\}$. For the isotropic case, $A = B = C$, the mean Hubble parameter converts to the Friedmann–Robertson–Walker form $H = \partial_t \log(a)$ where a is the scale factor. In this paper we are interesting to investigate the anisotropic models in which the cosmology described by metric (1) with $A \neq B \neq C$.

We consider the case of a homogeneous but anisotropic Bianchi type-I model with a scalar field ϕ based on a non-standard scalar–tensor theory. The action of this model reads [13]

$$S = \frac{1}{16\pi} \int \sqrt{-g} d^4x \left(R + \frac{\omega}{2} \phi^k \phi_{,\mu} \phi^{,\mu} - V(\phi) \right), \tag{2}$$

where k and ω are arbitrary dimensionless constants. Choosing $k = 0$ reduces our model to the minimally coupled massless scalar field coupled to Einstein gravity. Different aspects of this model have been explored in the literature [43–45]. Varying (2) w.r.t. the metric $g_{\mu\nu}$ lead to a generalized Einstein equation

$$G_{\mu\nu} = \omega \phi^k \left(\phi_{,\mu} \phi_{,\nu} - \frac{1}{2} g_{\mu\nu} \phi^{,\sigma} \phi_{,\sigma} \right) - V(\phi) g_{\mu\nu}. \tag{3}$$

The generalized Klein–Gordon equation for scalar field is

$$\begin{aligned} \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} (\sqrt{-g} g^{\mu\nu} \omega \phi^k \phi_{,\nu}) \\ = -V'(\phi) + \frac{\omega k}{2} \phi^{k-1} \phi_{,\sigma} \phi^{,\sigma}. \end{aligned} \tag{4}$$

Using metric (1) in the field equations (3) and (4), we obtain

$$\sum_{i,j}^3 \left(\frac{\ddot{A}_i}{A_i} + \frac{\ddot{A}_j}{A_j} + \frac{\dot{A}_i \dot{A}_j}{A_i A_j} \right) = \frac{1}{2} \omega \phi^k \dot{\phi}^2, \tag{5}$$

$$\sum_{i,j}^3 \frac{\dot{A}_i \dot{A}_j}{A_i A_j} = -\frac{1}{2} \omega \phi^k \dot{\phi}^2 + V(\phi). \tag{6}$$

The Klein–Gordon equation is

$$\frac{1}{a^3} \frac{d}{dt} (a^3 \omega \phi^k \dot{\phi}) = -V'(\phi) + \frac{\omega k}{2} \phi^{k-1} \dot{\phi}^2. \tag{7}$$

3 NGS analysis

We eliminate the terms \ddot{A}_i and obtain the following Lagrangian which is suitable for calculating the gauge symmetries:

$$\begin{aligned} L(t, \phi, A, B, C, \dot{\phi}, \dot{A}, \dot{B}, \dot{C}) \\ = \left(\frac{\omega}{2} \phi^k \dot{\phi}^2 - V(\phi) \right) ABC \\ - 2(\dot{A}\dot{B}\dot{C} + A\dot{B}\dot{C} + \dot{A}B\dot{C}), \quad \omega \neq 0 \end{aligned} \tag{8}$$

Varying the Lagrangian (8) w.r.t. ϕ, A, B and C , we get a system of Euler–Lagrange equations (or field equations):

$$\ddot{\phi} = -\frac{2\phi ABCV_\phi + 2\omega\phi^k\dot{\phi}(kABC\dot{\phi} + \phi(\dot{A}BC + A\dot{B}C + AB\dot{C}))}{2\omega\phi^{k+1}ABC}, \tag{9}$$

$$\ddot{A} = \frac{4(A\dot{B}\dot{C} - \dot{A}B\dot{C} - \dot{A}\dot{B}C) + 2ABCV - w\phi^k\dot{\phi}^2ABC}{8BC}, \tag{10}$$

$$\ddot{B} = \frac{4(\dot{A}B\dot{C} - A\dot{B}\dot{C} - \dot{A}\dot{B}C) + 2ABCV - w\phi^k\dot{\phi}^2ABC}{8AC}, \tag{11}$$

$$\ddot{C} = \frac{4(\dot{A}\dot{B}C - \dot{A}B\dot{C} - A\dot{B}\dot{C}) + 2ABCV - w\phi^k\dot{\phi}^2ABC}{8AB}. \tag{12}$$

The Noether symmetry is given by

$$X = \xi \frac{\partial}{\partial t} + \eta_1 \frac{\partial}{\partial T} + \eta_2 \frac{\partial}{\partial A} + \eta_3 \frac{\partial}{\partial B} + \eta_4 \frac{\partial}{\partial C} \tag{13}$$

where the coefficients ξ, η_i ($i = 1, 2, 3, 4$) are determined from the Noether symmetry conditions. The first-order prolongation of the above symmetry to the first-order jet space comprising of all derivatives is

$$X^{[1]} = X + \dot{\eta}_1 \frac{\partial}{\partial \dot{T}} + \dot{\eta}_2 \frac{\partial}{\partial \dot{A}} + \dot{\eta}_3 \frac{\partial}{\partial \dot{B}} + \dot{\eta}_4 \frac{\partial}{\partial \dot{C}}.$$

The Noether gauge symmetry condition is [38, 39]

$$X^{[1]}L + (D_t\xi)L = D_tG(t, \phi, A, B, C), \tag{14}$$

where G is the gauge function. We emphasize here the difference between the Noether and Noether gauge symmetries: In fact the NS is a very special case of NGS i.e. ignoring the gauge function and first prolongation, we find the restricted (or a special form of) Noether symmetry. The set of Noether symmetries is always a subset of Noether gauge symmetries.

The condition (14) yields the following system of linear partial differential equations:

$$\begin{aligned} \xi_\phi &= 0, \\ \xi_A &= 0, \\ \xi_B &= 0, \\ \xi_C &= 0, \\ C\eta_{3,A} + B\eta_{4,A} &= 0, \\ C\eta_{2,B} + A\eta_{4,B} &= 0, \\ B\eta_{2,C} + A\eta_{3,C} &= 0, \\ G_A + 2B\eta_{4,t} + 2C\eta_{3,t} + VABC\xi_A &= 0, \\ G_B + 2C\eta_{2,t} + 2A\eta_{4,t} + VABC\xi_B &= 0, \\ G_C + 2A\eta_{3,t} + 2B\eta_{2,t} + VABC\xi_C &= 0, \\ 2C\eta_{3,\phi} + 2B\eta_{4,\phi} - \eta_{1,A}w\phi^kABC &= 0, \end{aligned}$$

$$2C\eta_{2,\phi} + 2A\eta_{4,\phi} - \eta_{1,B}w\phi^kABC = 0, \tag{15}$$

$$2B\eta_{2,\phi} + 2A\eta_{3,\phi} - \eta_{1,C}w\phi^kABC = 0,$$

$$A\eta_{4,A} + B\eta_{4,B} + C\xi_t + C(\eta_{3,B} - \xi_t)$$

$$+ C(\eta_{2,A} - \xi_t) + \eta_4 = 0,$$

$$B\eta_{2,B} + C\eta_{2,C} + A\xi_t + A(\eta_{3,B} - \xi_t)$$

$$+ A(\eta_{4,C} - \xi_t) + \eta_2 = 0,$$

$$A\eta_{3,A} + C\eta_{3,C} + B\xi_t + B(\eta_{2,A} - \xi_t)$$

$$+ B(\eta_{4,C} - \xi_t) + \eta_3 = 0,$$

$$kABC\eta_1 + \phi(\eta_4AB + \eta_2BC + \eta_3AC + \xi_tABC$$

$$+ 2(\eta_{1,\phi} - \xi_t)wABC) = 0,$$

$$G_t + \eta_1ABCV_\phi + V(ABC\xi_t + \eta_3AC + \eta_4AB$$

$$+ \eta_2BC) = 0,$$

$$G_\phi + ABC(V\xi_\phi - \eta_{1,t}w\phi^k) = 0,$$

corresponding to the gauge functions $G(t, \phi, A, B, C)$. We numerically solve above system of linear partial differential equations. The potential function $V(\phi)$ is an arbitrary function whose form will be specified by the determining equations. We have the following cases.

3.1 $V(\phi) = 0$

The above system (15) altogether yields nine Noether symmetries comprising of translation, scalings and other symmetries:

$$X_1 = \frac{\partial}{\partial t}, \quad G = \text{const.}$$

$$X_2 = \phi^{-k/2} \frac{\partial}{\partial \phi}, \quad G = \text{const.}$$

$$X_3 = t \frac{\partial}{\partial t} + C \frac{\partial}{\partial C}, \quad G = \text{const.}$$

$$X_4 = A \frac{\partial}{\partial A} - C \frac{\partial}{\partial C}, \quad G = \text{const.}$$

$$X_5 = B \frac{\partial}{\partial B} - C \frac{\partial}{\partial C}, \quad G = \text{const.}$$

$$X_6 = \frac{t^2}{2} \frac{\partial}{\partial t} + \frac{tA}{3} \frac{\partial}{\partial A} + \frac{tB}{3} \frac{\partial}{\partial B} + \frac{tC}{3} \frac{\partial}{\partial C}, \quad G = -\frac{4}{3} ABC$$

$$X_7 = \phi^{-k/2} \ln\left(\frac{B}{C}\right) \frac{\partial}{\partial \phi} - \frac{\omega B \phi^{1+k/2}}{k+2} \frac{\partial}{\partial B} + \frac{\omega C \phi^{1+k/2}}{k+2} \frac{\partial}{\partial C},$$

$$G = \text{const.}$$

$$X_8 = \phi^{-k/2} \ln\left(\frac{A}{C}\right) \frac{\partial}{\partial \phi} - \frac{\omega A \phi^{1+k/2}}{k+2} \frac{\partial}{\partial A} + \frac{\omega C \phi^{1+k/2}}{k+2} \frac{\partial}{\partial C},$$

$$G = \text{const.}$$

$$X_9 = A \ln\left(\frac{B}{C}\right) \frac{\partial}{\partial A} + B \ln\left(\frac{C}{A}\right) \frac{\partial}{\partial B} + C \ln\left(\frac{A}{B}\right) \frac{\partial}{\partial C},$$

$$G = \text{const.}$$

The corresponding invariants are as follows:

$$I_1 = \frac{\omega}{2} \phi^k \dot{\phi}^2 ABC - 2(\dot{A}\dot{B}C + A\dot{B}\dot{C} + \dot{A}B\dot{C}),$$

$$I_2 = \phi^{k/2} \omega \dot{\phi} ABC,$$

$$I_3 = 2t(\dot{A}\dot{B}\dot{C} + \dot{A}\dot{B}\dot{C} + \dot{A}\dot{B}\dot{C}) - 2C(\dot{A}\dot{B} + \dot{A}\dot{B}) - \frac{1}{2} t \dot{\phi}^2 \omega \phi^k ABC,$$

$$I_4 = B(\dot{A}C - \dot{A}C),$$

$$I_5 = A(\dot{B}C - \dot{B}C),$$

$$I_6 = t^2(\dot{A}\dot{B}\dot{C} + \dot{A}\dot{B}\dot{C} + \dot{A}\dot{B}\dot{C}) + \frac{4}{3} ABC - \frac{4t}{3}(\dot{A}BC + A\dot{B}C + AB\dot{C}) - \frac{t^2}{4} \dot{\phi}^2 \omega \phi^k ABC,$$

$$I_7 = \frac{2\omega A(\dot{B}C - \dot{B}C)\phi^{\frac{k+2}{2}}}{k+2} + \omega \phi^{k/2} \dot{\phi} ABC \ln\left(\frac{B}{C}\right),$$

$$I_8 = \frac{2\omega B(\dot{A}C - \dot{A}C)\phi^{\frac{k+2}{2}}}{k+2} + \omega \phi^{k/2} \dot{\phi} ABC \ln\left(\frac{A}{C}\right),$$

$$I_9 = 2(\dot{B}C - \dot{B}C)A \ln A - 2(\dot{A}C - \dot{A}C)B \ln B + 2(A\dot{B} - \dot{A}B)C \ln C.$$

The first invariant is the Hamiltonian of the system.

$$3.2 \quad V(\phi) = \frac{2}{3} \alpha^2 \neq 0$$

In this case the system (15) again yields nine Noether symmetries of which six NSs $X_1, X_4, X_5, X_7, X_8, X_9$ are the same as above. The additional two NSs are

$$\tilde{X}_3 = e^{\alpha t} \frac{\partial}{\partial t} + \frac{\alpha e^{\alpha t}}{3} \left(A \frac{\partial}{\partial A} + B \frac{\partial}{\partial B} + C \frac{\partial}{\partial C} \right),$$

$$G = -\frac{4}{3} ABC \alpha^2 e^{\alpha t},$$

$$\tilde{X}_6 = e^{-\alpha t} \frac{\partial}{\partial t} - \frac{\alpha e^{-\alpha t}}{3} \left(A \frac{\partial}{\partial A} + B \frac{\partial}{\partial B} + C \frac{\partial}{\partial C} \right),$$

$$G = -\frac{4}{3} ABC \alpha^2 e^{-\alpha t}.$$

The corresponding invariants are as follows:

$$\tilde{I}_3 = e^{\alpha t} (4\alpha^2 ABC + 12(\dot{A}\dot{B}C + A\dot{B}\dot{C} + \dot{A}B\dot{C}) - 8\alpha(\dot{A}\dot{B}C + A\dot{B}\dot{C} + B\dot{A}C) - 3\omega \phi^k \dot{\phi}^2 ABC),$$

$$\tilde{I}_6 = e^{-\alpha t} (4\alpha^2 ABC + 12(\dot{A}\dot{B}C + A\dot{B}\dot{C} + \dot{A}B\dot{C}) + 8\alpha(\dot{A}\dot{B}C + A\dot{B}\dot{C} + B\dot{A}C) - 3\omega \phi^k \dot{\phi}^2 ABC).$$

$$3.3 \quad V(\phi) = \alpha \exp(\beta \phi^{\frac{k+2}{2}}), \quad k \neq -2$$

The system (15) yields five Noether symmetries of which four X_1, X_4, X_5, X_9 are the same as above. An additional NSs is

$$\tilde{X}_2 = t \frac{\partial}{\partial t} - \frac{4\phi^{-k/2}}{\beta(k+2)} \frac{\partial}{\partial \phi} + C \frac{\partial}{\partial C}, \quad G = \text{const.}$$

The corresponding invariant is

$$\tilde{I}_2 = 2t(\dot{A}\dot{B}\dot{C} + \dot{A}\dot{B}\dot{C} + \dot{A}\dot{B}\dot{C}) - 2C(\dot{A}\dot{B} + \dot{A}\dot{B}) - ABC \left(\alpha t e^{\beta \phi^{\frac{k+2}{2}}} + \frac{4\omega \dot{\phi} \phi^{k/2}}{\beta(k+2)} + \frac{\omega t \phi^k \dot{\phi}^2}{2} \right).$$

In this approach, we have obtained the exponential potential of the scalar field. Such exponential potentials are most favored in cosmology to study dark energy dynamics and fulfill many issues of dark energy approach, both from a theoretical point of view and in comparison with available observational data [46–48]. Therefore in the present model, we have obtained a solution representing accelerated expansion and is of immense cosmological interest. The case with $k = 0$ is interesting as it leads to $(V(\phi) = \alpha \exp(\beta\phi))$. Such potential forms have been used a lot in phenomenological models of dark energy such as quintessence, phantom and quintom [49–51]. For this potential, we have acceleration solution. Further in this case, when the kinetic term has much larger pressure than the potential term, then the potential domination epoch is an attractor solution as long as the potential is flat, i.e. the case $\beta = 0$.

3.4 Arbitrary $V(\phi)$

The system (15) gives four Noether symmetries which are the same as X_1, X_4, X_5, X_9 in Sect. 3.1. Therefore we do not get any new NGS and corresponding invariant in this case.

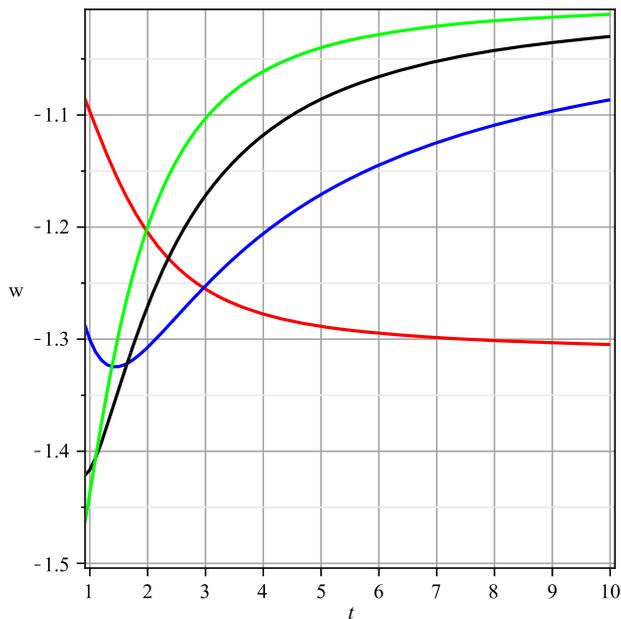


Fig. 1 Cosmological evolution of w vs. time t . The model parameters chosen as $\alpha = 1, \beta = -1, \omega = 1$. Curves in various colors correspond to (red, $k = 1$), (blue, $k = 2$), (black, $k = 3$), (green, $k = 4$)

4 Cosmological implications

In this section we investigate the general cosmic evolution of the model proposed in Sect. 2 with an exponential potential given by

$$V(\phi) = \alpha \exp(\beta\phi^{\frac{k+2}{2}}). \tag{16}$$

The EoS parameter w and the deceleration parameter q can be constructed analytically. Since w and q depend on metric functions and scalar field, their evolutionary behavior is obtained by numerically solving the Euler–Lagrange equations (9)–(12) for an appropriate set of the parameters and the initial conditions.

The EoS parameter is constructed using the expressions of total energy density and averaged pressure

$$\rho = \frac{1}{2}\omega\phi^k\dot{\phi}^2 + V(\phi), \tag{17}$$

$$p = \frac{1}{2}\omega\phi^k\dot{\phi}^2 - V(\phi). \tag{18}$$

When $k = 0$, (17) and (18) transform to the canonical scalar field model with a rescaling of the field. The EoS parameter is defined as

$$w \equiv \frac{p}{\rho}. \tag{19}$$

The numerical simulations of w is drawn in Fig. 1 which shows that w behaves like the phantom form of dark energy. This conclusion is exciting since there exists convincing astrophysical evidence that the observable universe is

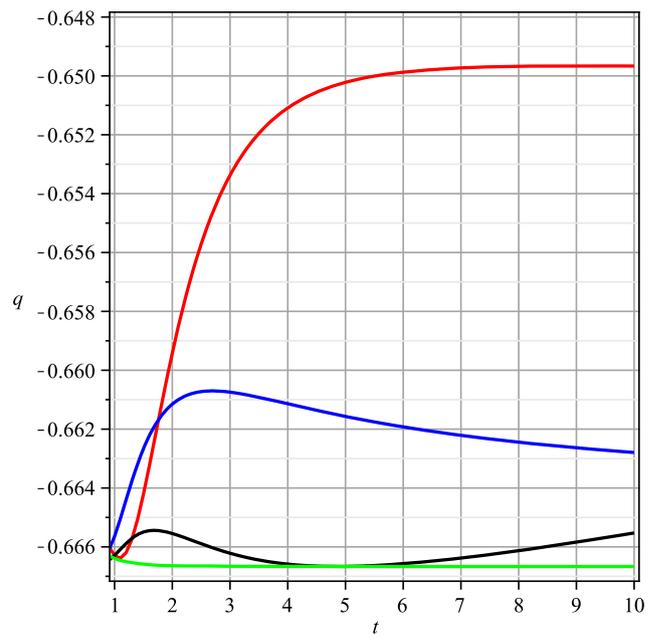


Fig. 2 Variation of q vs. time t . The model parameters chosen as $\alpha = 1, \beta = -1, \omega = 1$. Curves in various colors correspond to (red, $k = 1$), (blue, $k = 2$), (black, $k = 3$), (green, $k = 4$)

currently in the phantom phase [52, 53]. In Figs. 1 and 2, we chose the initial conditions $A(0) = 0.2, B(0) = 0.1, C(0) = 0.3, \phi(0) = 1, \dot{A}(0) = 1, \dot{B}(0) = 1, \dot{C}(0) = 1, \dot{\phi}(0) = 0.3$.

Further we calculate the deceleration parameter q using the average scale factor a

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = -1 - 3 \left[\frac{\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - (\frac{\dot{A}}{A})^2 - (\frac{\dot{B}}{B})^2 - (\frac{\dot{C}}{C})^2}{(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C})^2} \right]. \tag{20}$$

Again using the numerical calculation displayed in Fig. 2, we show that q remains always negative indicating the accelerated expansion of the universe. Note that our model predicts the present value $q_0 = -0.67$ which is in good agreement with the astrophysical data [54].

5 Conclusion

In this paper, we investigated the Noether gauge symmetries of a simple extension of an old model proposed by Saez–Ballester in a homogeneous but anisotropic Bianchi type I backgrounds. We solved the gauge equations and classified the models depending on potential function. One of the models is a generalization of the exponential families which have been used frequently in phenomenological models of dark energy such as quintessence, phantom and quintom. By performing numerical simulation of cosmological parameters w and q , we demonstrated that the universe lies in the

phantom energy dominated phase while the present value of deceleration parameter is compatible with the observations.

Acknowledgements M. Jamil and D. Momeni would like to thank the warm hospitality of Eurasian National University, Astana, Kazakhstan where this work was completed. All authors would thank the anonymous referee for the enlightening comments on our paper.

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