

Statefinder Parameters for Different Dark Energy Models with Variable G Correction in Kaluza-Klein Cosmology

Shuvendu Chakraborty · Ujjal Debnath ·
Mubasher Jamil · Ratbay Myrzakulov

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Abstract In this work, we have calculated the deceleration parameter, statefinder parameters and EoS parameters for different dark energy models with variable G correction in homogeneous, isotropic and non-flat universe for Kaluza-Klein Cosmology. The statefinder parameters have been obtained in terms of some observable parameters like dimensionless density parameter, EoS parameter and Hubble parameter for holographic dark energy, new agegraphic dark energy and generalized Chaplygin gas models.

Keywords Kaluza-Klein gravity · Dark energy

S. Chakraborty

Department of Mathematics, Seacom Engineering College, Howrah, 711 302, India
e-mail: shuvendu.chakraborty@gmail.com

U. Debnath

Department of Mathematics, Bengal Engineering and Science University, Shibpur, Howrah 711 103, India
e-mail: ujjaldebnath@yahoo.com

U. Debnath

e-mail: ujjal@iucaa.ernet.in

M. Jamil (✉)

Center for Advanced Mathematics and Physics (CAMP), National University of Sciences and Technology (NUST), H-12, Islamabad, Pakistan
e-mail: jamil.camp@gmail.com

M. Jamil · R. Myrzakulov

Eurasian International Center for Theoretical Physics, Eurasian National University, Astana 010008, Kazakhstan

R. Myrzakulov

e-mail: rmyrzakulov@gmail.com

R. Myrzakulov

Department of Physics, California State University, Fresno, CA 93740, USA
e-mail: rmyrzakulov@csufresno.edu

1 Introduction

Recent cosmological observations obtained by SNe Ia [1, 2], WMAP [3], SDSS [4] and X-ray [5, 6] indicate that the observable universe experiences an accelerated expansion. To explain this phenomena the notion known as dark energy (DE) with large negative pressure is proposed. At present there are a lot of theoretical models of DE. But the most suitable models of DE is the cosmological constant. According of the modern observational cosmology, the present value of cosmological constant is 10^{-55} cm^{-2} . At the same time, the particle physics tells us that its value must be 10^{120} times greater than this factor. It is one main problem modern cosmology and known as the cosmological constant problem. In order to solve this problem, some authors considered the cosmological constant as a varying parameter (see e.g. [7–11]). Here we can mention that Dirac showed that some fundamental constants do not remain constant forever rather they vary with time due to some causal connection between micro and macro physics [12–15] that is known as Large Number Hypothesis (LNH). The field equations of General Relativity (GR) involve two physical constants, namely, the gravitational constant G (couples the geometry and matter) and cosmological constant Λ (vacuum energy in space). According to the LNH, the gravitational constant should also vary with time. In [16] LNH was extended by taking cosmological constant as $\Lambda = \frac{8\pi G^2 m_p^2}{h^4}$, where m_p^2 is the mass of proton and h is the Planck's constant. It was showed that Λ produces the same gravitational effects in vacuum as that produced by matter [16]. As result, this cosmological term must be included in the physical part of the field equations. In [16] also defined gravitational energy of the vacuum as the interactions of virtual particles separated by a distance $\frac{h}{m_p c}$, where c is the speed of light. It is also interesting to note that a time varying gravitational constant also appears in the entropic interpretations of gravity [17, 18].

In the literature, many modifications of cosmological constant have been proposed for the better description and understanding of DE (see e.g. [19]). For example, in [20–26] was studied the field equations by using three different forms of the cosmological constant, i.e., $\Lambda \sim (\frac{\dot{a}}{a})^2$, $\Lambda \sim (\frac{\ddot{a}}{a})$ and $\Lambda \sim \rho$ and shown that these models yield equivalent results to the FRW spacetime. From these investigations follow that an investigation about the scale factor and other cosmological parameters with varying G and Λ may be interesting especially for description the accelerated expansion of the universe.

According modern point of views, multidimensional gravity theories may play important role to explain main problems of cosmology and astrophysics in particular DE. One of classical examples of such theories is the theory of Kaluza-Klein (KK) [27, 28]. It is a 5 dimensional GR in which extra dimension is used to couple the gravity and electromagnetism (see e.g., the review [29–31] and references therein). In the context of our interest—DE, recently it was studied [32] that the non-compact, non-Ricci KK theory and coupled the flat universe with non-vacuum states of the scalar field. For the suitable choose of the equation of state (EoS), the reduced field equations describe the early inflation and late time acceleration. Moreover, the role played by the scalar field along the 5th coordinate in the 5D metric is in general very impressed by the role of scale factor over the 4D universe.

In recent years, the holographic dark energy (HDE) has been studied as a possible candidate for DE. It is motivated from the holographic principle which might lead to the quantum gravity to explain the events involving high energy scale. Another interesting models of DE are the so-called new-agegraphic dark energy which is originated from the uncertainty relation of quantum mechanics together with the gravitational effect of GR. In general, the agegraphic DE model assumes that the observed DE effect comes from spacetime and matter field fluctuations in the universe.

In the interesting paper [33] it was introduced a new cosmological diagnostic pair $\{r, s\}$ called statefinder which allows one to explore the properties of DE independent of model. This pair depends on the third derivative of the scale factor, $a(t)$, just like the dependence of Hubble and deceleration parameter on first and second derivative of respectively. It is used to distinguish flat models of the DE and this pair has been evaluated for different models [34–50]. In [42–50] it was solved the field equations of the FRW universe with variable G and Λ (see also [51–53] where was considered the flat KK universe with variable Λ but keeping G fixed). There are many works on higher dimensional space-time also [54–59].

In this work, we have calculated the statefinder parameters for different dark energy models with variable G correction in Kaluza-Klein cosmology. We evaluate different cosmological parameters with the assumption that our universe is filled with different types of matter. The scheme of the paper is as follows. In the next section, the KK model and its field equations are presented. In Sect. 3, solution of the field equations for the HDE are presented and Sect. 4 the new-agegraphic dark energy case is considered. Generalized Chaplygin Gas model is studied in the Sect. 5. In Sect. 6, we summarize the results.

2 Kaluza-Klein Model

The metric of a homogeneous and isotropic universe in the Kaluza-Klein model is

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1 - kr^2)d\psi^2 \right] \tag{1}$$

where $a(t)$ is the scale factor, $k = -1, 0, 1$ is the curvature parameter for spatially closed, flat and open universe respectively.

We assume that the universe is filled with dark energy and matter whose energy-momentum tensor is given by

$$T_{\mu\nu} = (\rho_m + \rho_x + p_x)u_\mu u_\nu - p_x g_{\mu\nu} \tag{2}$$

where u_μ is the five velocities satisfying $u^\mu u_\mu = 1$. ρ_m and ρ_x are the energy densities of matter and dark energy respectively and p_x is the pressure of the dark energy. We consider here the pressure of the matter as zero.

The Einstein’s field equations are given by

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(t)T_{\mu\nu} \tag{3}$$

where $R_{\mu\nu}$, $g_{\mu\nu}$ and R are Ricci tensor, metric tensor and Ricci scalar respectively. Here we consider gravitational constant G as a function of cosmic time t . Now from (1), (2) and (3) we have the Einstein’s field equations for the isotropic Kaluza-Klein space time (1) are

$$H^2 + \frac{k}{a^2} = \frac{4\pi G(t)}{3}(\rho_m + \rho_x) \tag{4}$$

$$\dot{H} + 2H^2 + \frac{k}{a^2} = -\frac{8\pi G(t)}{3}p_x \tag{5}$$

Let the dark energy obeying the equation of state $p_x = \omega\rho_x$. Equation (4) gives

$$\Omega = \Omega_m + \Omega_x - \Omega_k \tag{6}$$

where Ω_m, Ω_x and Ω_k are dimensionless density parameters representing the contribution in the total energy density. The deceleration parameter q in terms of these parameters are given by

$$q = \Omega_m + (1 + 2\omega)\Omega_x \quad \text{where } \omega = \frac{q - \Omega - \Omega_k}{2\Omega_x} \tag{7}$$

The trajectories in the $\{r, s\}$ plane [60] corresponding to different cosmological models depict qualitatively different behaviour. The statefinder diagnostic along with future SNAP observations may perhaps be used to discriminate between different dark energy models. The above statefinder diagnostic pair for cosmology are constructed from the scale factor a . The statefinder parameters are given by

$$r = \frac{\ddot{a}}{aH^2}, \quad s = \frac{r - 1}{3(q - 1/2)}$$

From the expression of one of the statefinder parameter r , we have a relation between r and q is given by

$$r = q + 2q^2 - \frac{\dot{q}}{H} \tag{8}$$

From (7) we have

$$\dot{q} = \dot{\Omega}_m + (1 + 2\omega)\dot{\Omega}_x + 2\dot{\omega}\Omega_x \tag{9}$$

Also we have

$$\Omega = \frac{\rho}{\rho_{cr}} - \frac{k}{a^2H^2} \quad \text{which gives } \dot{\Omega} = \frac{\dot{\rho}}{\rho_{cr}} - \frac{2kq}{a^2H} - \frac{\rho\dot{\rho}_{cr}}{(\rho_{cr})^2} \tag{10}$$

where

$$\rho_{cr} = \frac{3H^2}{4\pi G(t)} \quad \text{which gives after differentiation } \dot{\rho}_{cr} = \rho_{cr} \left(2\frac{\dot{H}}{H} - \frac{\dot{G}}{G} \right) \tag{11}$$

which implies

$$\dot{\rho}_{cr} = -H\rho_{cr}(2(1 + q) + \Delta G) \tag{12}$$

where, $\Delta G \equiv \frac{G'}{G}, \dot{G} = HG'$. Now from (10) we have

$$\dot{\Omega} = \frac{\dot{\rho}}{\rho_{cr}} + \Omega_k H(2 + \Delta G) + \Omega H(2(1 + q) + \Delta G) \tag{13}$$

We assume that matter and dark energy are separately conserved. For matter, $\dot{\rho}_m + 4H\rho_m = 0$. So from (13)

$$\dot{\Omega}_m = \Omega_m H(-2 + 2q + \Delta G) + \Omega_k H(2 + \Delta G) \tag{14}$$

For dark energy, $\dot{\rho}_x + 4H(1 + \omega)\rho_x = 0$. So from (13)

$$\dot{\Omega}_x = \Omega_x H(-2 - 4\omega + 2q + \Delta G) + \Omega_k H(2 + \Delta G) \tag{15}$$

From (8), (9), (14), (15) we have expression for r and s given by

$$r = 3\Omega_m + (3 + 10\omega + 8\omega^2)\Omega_x - 4(1 + \omega)\Omega_k - \Delta G(\Omega_m + (1 + 2\omega)\Omega_x + 2(1 + \omega)\Omega_k) - \frac{2\dot{\omega}\Omega_x}{H} \tag{16}$$

$$s = \frac{3\Omega_m + (3 + 10\omega + 8\omega^2)\Omega_x - 4(1 + \omega)\Omega_k - \Delta G(\Omega_m + (1 + 2\omega)\Omega_x + 2(1 + \omega)\Omega_k) - \frac{2\dot{\omega}\Omega_x}{H} - 1}{3(-1/2 + \Omega_m + \Omega_x + 2\omega\Omega_x)} \tag{17}$$

3 Holographic Dark Energy

To study the dark energy models from the holographic principle it is important to mention that the number of degrees of freedom is directly related to the entropy scale with the enclosing area of the system, not with the volume [61, 62]. Where as Cohen et al. [63] suggest a relation between infrared (IR) and the ultraviolet (UV) cutoff in such a way that the total energy of the system with size L must not exceed the mass of the same size black hole. The density of holographic dark energy is

$$\rho_x = \frac{3c^2}{8\pi G} \frac{1}{L^2} \tag{18}$$

Here c is the holographic parameter of order unity. Considering $L = H_0^{-1}$ one can found the energy density compatible with the current observational data. However, if one takes the Hubble scale as the IR cutoff, the holographic dark energy may not capable to support an accelerating universe [64]. The first viable version of holographic dark energy model was proposed by Li [65], where the IR length scale is taken as the event horizon of the universe. The holographic dark energy has been explored in various gravitational frameworks [66–76].

The time evolution is

$$\dot{\rho}_x = -\rho_x H \left(2 - \frac{2\sqrt{2\Omega_x}}{c} \cos y + \Delta G \right) \tag{19}$$

where L is defined as $L = ar(t)$ with a is the scale factor. Also $r(t)$ can be obtained from the relation

$$R_H = a \int_t^\infty \frac{dt}{a} = \int_0^{r(t)} \frac{dr}{\sqrt{1 - kr^2}}$$

where R_H is the event horizon. When R_H is the radial size of the event horizon measured in the r direction, L is the radius of the event horizon measured on the sphere of the horizon.

For closed (or open) universe we have $r(t) = \frac{1}{\sqrt{k}} \sin y$, where $y = \frac{\sqrt{k}R_H}{a}$. using the definition $\Omega_x = \frac{\rho_x}{\rho_{cr}}$ and $\rho_{cr} = \frac{3H^2}{4\pi G(t)}$ we have $HL = \frac{c}{\sqrt{2\Omega_x}}$.

And using all these we ultimately obtain the relation $\dot{L} = HL + a\dot{r}(t) = \frac{c}{\sqrt{2\Omega_x}} - \cos y$, by which we find (19).

From the energy conservation equation and (19) we have the holographic energy equation of state given by

$$\omega = \frac{1}{4} \left(-2 - \frac{2\sqrt{2\Omega_x}}{c} \cos y + \Delta G \right) \tag{20}$$

where, $\Omega_k = \frac{k}{a^2 H^2}$, $\Omega_x = \frac{c^2}{2L^2 H^2}$ are usual fractional densities in KK model.

From the ration of the fractional densities we have, $\sin^2 y = \frac{c^2 \Omega_k}{2\Omega_x}$ and naturally, $\cos y = \sqrt{\frac{2\Omega_x - c^2 \Omega_k}{2\Omega_x}}$.

Now differentiating (20) and using (15) we have

$$\frac{\dot{\omega}}{H} = \frac{16\Omega_x^2(-1 + \Omega_x) + c^2\Omega_x(3\Delta'G + \Omega_k(2 - 8\Omega_x)) - 4c\sqrt{-c^2\Omega_k + 2\Omega_x}((2 + \Delta G)\Omega_k + \Omega_x(2\Omega_m + \Delta G\Omega_x))}{12c^2\Omega_x} \tag{21}$$

Now putting (21) in (16) and (17), we have

$$r = \frac{1}{6c^2} [8(5 - 2\Omega_x)\Omega_x^2 - c^2(3(2(-3 + \Delta G)\Omega_m + (-\Delta G + \Delta'G)\Omega_x) + \Omega_k(3(2 + \Delta G)^2 + 14\Omega_x - 8\Omega_x^2)) + 2c\sqrt{-c^2\Omega_k + 2\Omega_x}(5(2 + \Delta G)\Omega_k + \Omega_x(-3 + 4\Omega_m + \Delta G(-3 + 2\Omega_x)))] \tag{22}$$

$$s = \frac{1}{9c(-2\Omega_x\sqrt{-c^2\Omega_k + 2\Omega_x} + c(-1 + 2\Omega_m + \Delta G\Omega_x)) \times [8(5 - 2\Omega_x)\Omega_x^2 - c^2(3(2 + 2(-3 + \Delta G)\Omega_m + (-\Delta G + \Delta'G)\Omega_x) + \Omega_k(3(2 + \Delta G)^2 + 14\Omega_x - 8\Omega_x^2)) + 2c\sqrt{-c^2\Omega_k + 2\Omega_x}(5(2 + \Delta G)\Omega_k + \Omega_x(-3 + 4\Omega_m + \Delta G(-3 + 2\Omega_x)))]} \tag{23}$$

This is the expressions for $\{r, s\}$ parameters in terms of fractional densities of holographic dark energy model in Kaluza-Klein cosmology for closed (or open) universe.

4 New Agegraphic Dark Energy

There are another version of the holographic dark energy model called, the new agegraphic dark energy model [77–79], where the time scale is chosen to be the conformal time. The new agegraphic dark energy is more acceptable than the original agegraphic dark energy, where the time scale is chosen to be the age of the universe. The original ADE suffers from the difficulty to describe the matter-dominated epoch while the NADE resolved this issue. The density of new agegraphic dark energy is

$$\rho_x = \frac{3n^2}{8\pi G} \frac{1}{\eta^2} \tag{24}$$

where n is a constant of order unity. where the conformal time is given by $\eta = \int_0^a \frac{da}{Ha^2}$.

If we consider η to be a definite integral, the will be a integral constant and we have $\dot{\eta} = \frac{1}{a}$.

Considering KK cosmology and using the definition $\Omega_x = \frac{\rho_x}{\rho_{cr}}$ and $\rho_{cr} = \frac{3H^2}{4\pi G(t)}$ we have $H\eta = \frac{n}{\sqrt{2\Omega_x}}$.

After introducing the fractional energy densities we have the time evolution of NADE as

$$\dot{\rho}_x = -\rho_x H \left(\frac{2\sqrt{2\Omega_x}}{na} + \Delta G \right) \tag{25}$$

From the energy conservation equation and the equation (25) we have the new agegraphic energy equation of state given by

$$\omega = \frac{1}{4} \left(-4 + \frac{2\sqrt{2\Omega_x}}{na} + \Delta G \right) \tag{26}$$

where, $\Omega_k = \frac{k}{a^2 H^2}$, $\Omega_x = \frac{n^2}{2\eta^2 H^2}$ are usual fractional densities in KK model.

Differentiating (26) and using (15) we have

$$\frac{\dot{\omega}}{H} = \frac{a^2 \Delta' G n^2 \sqrt{x} + 4(-1 + \Omega_x)\Omega_x^{3/2} + \sqrt{2}an((2 + \Delta G)\Omega_k + \Omega_x(2\Omega_m + (-2 + \Delta G)\Omega_x))}{4a^2 n^2 \sqrt{\Omega_x}} \tag{27}$$

Now putting (27) in (16) and (17), we have the expression for r, s as

$$r = -\frac{1}{2a^2 n^2} \left[4(-3 + \Omega_x)\Omega_x^2 + \sqrt{2}an\sqrt{\Omega_x}(3(2 + \Delta G)\Omega_k + (2(3 + \Omega_m - \Omega_x) + \Delta G(-2 + \Omega_x))\Omega_x) + a^2 n^2 (\Delta G^2 \Omega_k - 6\Omega_m + (-2 + \Delta' G)\Omega_x + \Delta G(2(\Omega_k + \Omega_m) + \Omega_x)) \right] \tag{28}$$

$$s = -\frac{1}{3an(2\sqrt{2}\Omega_x^{3/2} + an(-1 + 2\Omega_m + (-2 + \Delta G)\Omega_x))} \left[4(-3 + \Omega_x)\Omega_x^2 + \sqrt{2}an\sqrt{\Omega_x}(3(2 + \Delta G)\Omega_k + (2(3 + \Omega_m - \Omega_x) + \Delta G(-2 + \Omega_x))\Omega_x) + a^2 n^2 (2 + \Delta G^2 \Omega_k - 6\Omega_m + (-2 + \Delta' G)\Omega_x + \Delta G(2(\Omega_k + \Omega_m) + \Omega_x)) \right] \tag{29}$$

This is the expressions for $\{r, s\}$ parameters in terms of fractional densities of new agegraphic dark energy model in Kaluza-Klein cosmology for closed (or open) universe.

5 Generalized Chaplygin Gas

It is well known to everyone that Chaplygin gas provides a different way of evolution of the universe and having behaviour at early time as pressureless dust and as cosmological constant at very late times, an advantage of GCG, that is it unifies dark energy and matter into a single equation of state. This model can be obtained from generalized version of the Born-Infeld action. The equation of state for generalized Chaplygin gas is [80–82]

$$p_x = -\frac{A}{\rho_x^\alpha} \tag{30}$$

where $0 < \alpha < 1$ and $A > 0$ are constants. Inserting the above equation of state (30) of the GCG into the energy conservation equation we have

$$\rho_x = \left[A + \frac{B}{a^{4(\alpha+1)}} \right]^{\frac{1}{\alpha+1}} \tag{31}$$

where B is an integrating constant.

$$\omega = -A \left(A + \frac{B}{a^{4(1+\alpha)}} \right)^{-1} \tag{32}$$

Differentiating (32) and using (15) we have

$$\frac{\dot{\omega}}{H} = -4AB(1 + \alpha) \frac{1}{a^{4(1+\alpha)}} \left(A + \frac{B}{a^{4(1+\alpha)}} \right)^{-2} \tag{33}$$

Now putting (33) in (16) and (17), we have

$$r = 3\Omega_m - \Delta G\Omega_m + \Omega_x + \Delta G\Omega_x - \frac{2B((2 + \Delta G)\Omega_k + \Omega_x(-1 + \Delta G - 4\alpha))}{(a^{4+4\alpha}A + B)} - \frac{8B^2\Omega_x\alpha}{(Aa^{4+4\alpha} + B)^2} \tag{34}$$

$$s = \frac{3\Omega_m - \Delta G\Omega_m + \Omega_x + \Delta G\Omega_x - \frac{2B((2+\Delta G)\Omega_k + \Omega_x(-1+\Delta G-4\alpha))}{(a^{4+4\alpha}A+B)} - \frac{8B^2\Omega_x\alpha}{(Aa^{4+4\alpha}+B)^2}}{3(-1/2 + \Omega_m + \Omega_x - \frac{2A\Omega_x}{A+a^{-4(1+\alpha)}B})} \tag{35}$$

This is the expressions for $\{r, s\}$ parameters in terms of fractional densities of generalized Chaplygin gas model in Kaluza-Klein cosmology for closed (or open) universe.

6 Conclusions

In this work, we have considered the homogeneous, isotropic and non-flat universe in 5D Kaluza-Klein Cosmology. We have calculated the corrections to statefinder parameters due to variable gravitational constant in Kaluza-Klein Cosmology. These corrections are relevant because several astronomical observations provide constraints on the variability of G . We have investigated three multipromising models of DE such as the Holographic dark energy, the new-agegraphic dark energy and generalized Chaplygin gas. These dark energies derive the accelerating phase of the Kaluza-Klein model of the universe. We have assumed that the dark energies do not interact with matter. In this case, the deceleration parameter and equation state parameter for dark energy candidates have been found. The statefinder parameters have been found in terms of the dimensionless density parameters as well as EoS parameter ω and the Hubble parameter. An important thing to note is that the G -corrected statefinder parameters are still geometrical since the parameter ΔG is a pure number and is independent of the geometry.

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