Eur. Phys. J. C (2013) 73:2267
DOI 10.1140/epjc/s10052-012-2267-8

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Wormholes in a viable $f(T)$ gravity

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Received: 24 September 2012 / Published online: 15 January 2013

Abstract In this paper, we derive some new exact solutions of static wormholes in $f(T)$ gravity. We discuss independent cases of the pressure components including isotropic and anisotropic pressure. Lastly we consider radial pressure satisfying a barotropic equation of state. We also check the behavior of null energy condition (NEC) for each case and observe that it is violated for the anisotropic case, while it is satisfied for isotropic and barotropic cases.

1 Introduction

Teleparallelism was proposed by Einstein as an attempt to unify electromagnetism and gravity. But as time rolled on scientists lost interest in this concept of unification and as a result, today teleparallelism is just considered as a theory of gravity. Teleparallel gravity corresponds to a gauge theory for the translation group [1, 2]. The crucial new idea, for Einstein, was the introduction of a tetrad field in this theory. Here the space-time is characterized by a curvature-free linear connection, called the Weitzenbock connection, presenting torsion but no curvature. In the framework of general relativity (GR), curvature is used to geometrize space-time, thus successfully describing the gravitational interaction. Teleparallelism, on the other hand, attributes gravitation to torsion. But in this case torsion accounts for gravitation not by geometrizing the interaction, but by acting as a force. This implies that, in the teleparallel equivalent of general relativity, instead of geodesics, there are force equations, which are quite analogous to the Lorentz force equation of electrodynamics [1, 2]. Therefore it seems to be just a matter of convention whether gravity requires a curved or a torsioned space-time.

A typical stationary spherically symmetric wormhole is a two-mouthed tunnel (also called a tube, throat, or handle) in a multiply connected space-time joining two remote asymptotically flat regions of the same space-time or two different space-times altogether (see [3] for a review). The concept of a "traversable wormhole" was proposed by Morris and Thorne [4, 5] with the aim of using a wormhole for time travel. Wormholes can be enlarged through a mechanism similar to cosmological inflation [6]. Recent interest has centered on matter that satisfies the equation of state (EoS) $p = w\rho$ with $w < 0$. Astrophysical observations of supernovae of type Ia and cosmic microwave background data suggest that the observable Universe is pervaded by a dynamic dark energy whose EoS parameter $\omega < -1$ [7]. Moreover, it comprises more than 70 percent of the critical density required for a flat Universe [8–12], suggesting that wormholes could form and stabilize in a dark-energy dominated universe [13]. A wormhole could decay into a black hole if the exotic matter is removed from the wormhole's throat [14]. In literature, wormholes have been constructed via different forms of energy [15–20]. It has been shown that the wormhole region that requires exotic matter can be made arbitrarily small by introducing a special shape function [21]. Later attempts were made to devise wormhole solutions that do not require a violation of energy conditions by modifying general relativity; examples are Gauss–Bonnet gravity, Kalb–Ramond background, braneworld scenarios, and Brans–Dicke gravity theory [22–29]. The modified gravity has gained enormous popularity in the cosmological community since it passes several solar system and astrophysical tests successfully [30–32]. In addition to this, modified gravity models can successfully account for the recent cosmic acceleration without resorting to any form of DE. In the context of modified gravity theory it is worth stating that loop quantum gravity [33–37], extra dimensional braneworlds [38, 39], $f(R)$ [40–53], $f(T)$ [54–61] are some of the popular modified gravity models.

In this paper, we derive some new exact solutions of static wormholes in $f(T)$ gravity. We discuss independent

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cases of the pressure components including isotropic and anisotropic pressure. Lastly we consider radial pressure satisfying a barotropic equation of state. We also check the behavior of null energy condition (NEC) for each case and observe that it is violated for anisotropic while it is satisfied for isotropic and barotropic cases. The paper is organized as follows. In Sect. 2, the basic equations for the $f(T)$ model is discussed. In Sect. 3, the field equations are evaluated. In Sect. 4, wormhole solutions in $f(T)$ gravity are discussed. In Sect. 5, we propose our viable $f(T)$ model. In Sect. 6, an exact anisotropic solution is derived. In Sect. 7, the isotropic solutions are derived. In Sect. 8, the barotropic solutions are investigated. Finally we end with some concluding remarks in Sect. 9.

2 Field equations with an off-diagonal tetrad

Consider the static spherically symmetric metric

$$ds^2 = e^{a(r)}\,dt^2 - e^{b(r)}\,dr^2 - r^2\big(d\theta^2 + \sin^2\theta\,d\varphi^2\big), \tag{1}$$

where $a(r)$ and $b(r)$ are two unknown metric functions. Following the above discussion of tetrads, we introduce the off-diagonal tetrad field, to avoid the TEGR with $f_{TT}=0$, given by

$$e^i{}_\mu = \begin{pmatrix} e^{a/2} & 0 & 0 & 0 \\ 0 & e^{b/2}\sin\theta\cos\phi & r\cos\theta\cos\phi & -r\sin\theta\sin\phi \\ 0 & e^{b/2}\sin\theta\sin\phi & r\cos\theta\sin\phi & r\sin\theta\cos\phi \\ 0 & e^{b/2}\cos\theta & -r\sin\theta & 0 \end{pmatrix}.$$

The off-diagonal basis tetrad is related to its diagonal form by doing a rotation.

The determinant of $e^i{}_\mu$ is $e=\exp((a+b)/2)r^2\sin\theta$. The torsion scalar is

$$T(r) = \frac{2e^{-b}(e^{b/2}-1)(e^{b/2}-1-ra')}{r^2}. \tag{2}$$

The equations of motion are given by [62–73]

$$4\pi\rho(r) = \frac{e^{-b/2}}{r}\big(1-e^{-b/2}\big)T' f_{TT} - \left(\frac{T}{4} - \frac{1}{2r^2}\right) f_T + \frac{e^{-b}}{2r^2}\big(rb'-1\big) f_T - \frac{f}{4}, \tag{3}$$

$$4\pi p_r(r) = \left[-\frac{1}{2r^2} + \frac{T}{4} + \frac{e^{-b}}{2r^2}\big(1+ra'\big)\right] f_T - \frac{f}{4}, \tag{4}$$

$$4\pi p_t(r) = \frac{e^{-b}}{2}\left(\frac{a'}{2} + \frac{1}{r} - \frac{e^{b/2}}{r}\right) T' f_{TT} + f_T\left\{\frac{T}{4} + \frac{e^{-b}}{2r}\left[\left(\frac{1}{2} + \frac{ra'}{4}\right)\big(a'-b'\big) + \frac{ra''}{2}\right]\right\} - \frac{f}{4}, \tag{5}$$

where $\rho(r)$, $p_r(r)$, $p_t(r)$ are the energy density, the radial pressure and the pressure measured in the tangential directions and orthogonal to the radial direction, respectively. The above field equations (3)–(5) give three independent equations for our six unknown quantities, i.e., $\rho(r)$, $p_r(r)$, $p_t(r)$, $a(r)$, $b(r)$, and $f(T)$. This system of equations is not closed, so we need to reduce the number of unknown functions by assuming some physically acceptable suitable reduction conditions.

3 Wormhole solutions in $f(T)$ gravity

Consider a typical static spherically symmetric wormhole is given by the metric (1), with the following metric function:

$$e^{-b(r)} = 1 - \frac{\beta(r)}{r}. \tag{6}$$

In the context of wormhole physics $a(r)$ and $\beta(r)$ are arbitrary functions of the radial coordinate r. $a(r)$ is the redshift function; it is related to the gravitational redshift g_{00}, and $\beta(r)$ denotes the shape function, as shown by embedding diagrams, it determines the shape of the wormhole [74, 75]. The coordinate r is bounded by $r_0 < r < +\infty$, r_0 representing the location of the wormhole throat, where $b(r_0)=r_0$, and then it increases from r_0 to $+\infty$. To ensure that we have a typical solution of a wormhole, one needs to impose the flaring out of the throat, which is given by the condition $(\beta-\beta' r)/2\beta^2 > 0$ [74, 75]. At the throat we verify that the shape function satisfies the condition $\beta'(r_0) < 1$.

The flaring out condition of the throat is a fundamental property in wormholes, and through the Einstein field equations it was found that some of these solutions possess a peculiar property, namely exotic matter, involving a stress-energy tensor that violates the null energy condition. One good example for such fluids is dark energy with EoS $p \approx -\rho$. In fact, they violate all known point-wise energy conditions and averaged energy conditions. Note that the weak energy condition (WEC) assumes that the local *positive definite* energy density

$$\rho = T_{\mu\nu}U^\mu U^\nu \geq 0,$$

for all timelike vectors U^μ, where $T_{\mu\nu}$ is the usual energy-momentum (EM) tensor. In the limit of the quantum effects, this expression of the EM tensor can be replaced by the semi classical quantum expectation value $\langle T_{\mu\nu}^{\text{vaccua}}\rangle$, where it can be replaced as a source term in the right hand side of the gravitational field equation. For example, this kind of EM tensors with conformal anomaly can produce a rich family of the exact (A)dS black holes in the context of the $f(R)$ gravity [76].

Further we can show that by replacing the general expression of the EM tensor for a typical Lorentz invariance Lagrangian $\mathcal{L}$, by the expression

$$T_{\mu\nu} = g_{\mu\nu}\mathcal{L} - 2\frac{\delta\mathcal{L}}{\delta g^{\mu\nu}},$$

and by adopting a timelike velocity vector stream by $U^\mu = \phi^{;\mu}$, again this new expression remains positive definite. In the locally Lorentzian frame of the matter, for example in the co moving frame in perfect fluids, this amounts to $\rho > 0$ and $\rho + p_i \geq 0$, where $i = r, t$. By continuity, the WEC implies the null energy condition (NEC), $T_{\mu\nu}k^\mu k^\nu \geq 0$, where k^μ is a null vector [77]. Our goal is to find some exact solutions for wormholes and check whether they satisfy or violate the above energy conditions. We briefly review the energy conditions in the next section.

4 Energy conditions

The energy conditions are used in different contexts to derive a variety of general results which hold for different situations. Under these conditions, one allows not just gravity to be attractive but also the energy density to be positive and flows not to be faster than light [78]. The notion of energy conditions arises from the Raychaudhuri equation, given by

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}u^\mu u^\nu, \tag{7}$$

where u^μ is a vector field representing the congruence of "timelike geodesics". Also $R_{\mu\nu}$, θ, $\sigma_{\mu\nu}$ and $\omega_{\mu\nu}$ represent Ricci tensor, the expansion parameter, the shear and the rotation associated with the congruence, respectively. Similarly, in the case of a congruence of "null geodesics" defined by the vector field k^μ, the Raychaudhuri equation becomes

$$\frac{d\theta}{d\tau} = -\frac{1}{2}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}k^\mu k^\nu. \tag{8}$$

From both Raychaudhuri equations, it is apparent that these are purely geometric and independent of any gravity theory. The origin of these energy conditions is independent of any gravity theory and these are purely geometrical (for a review on the energy conditions, see the classic text [79]). Using the modified (effective) gravitational field equations the null energy condition (NEC), weak energy condition (WEC), strong energy condition (SEC) and the dominant energy condition (DEC) are given by [62–73, 80]

$$\text{NEC} \iff \rho_{\text{eff}} + p_{\text{eff}} \geq 0, \tag{9}$$

$$\text{WEC} \iff \rho_{\text{eff}} \geq 0 \quad \text{and} \quad \rho_{\text{eff}} + p_{\text{eff}} \geq 0, \tag{10}$$

$$\text{SEC} \iff \rho_{\text{eff}} + 3p_{\text{eff}} \geq 0 \quad \text{and} \quad \rho_{\text{eff}} + p_{\text{eff}} \geq 0, \tag{11}$$

$$\text{DEC} \iff \rho_{\text{eff}} \geq 0 \quad \text{and} \quad \rho_{\text{eff}} \pm p_{\text{eff}} \geq 0. \tag{12}$$

The matter supporting the wormhole geometry is termed "exotic" since it violates the standard energy conditions (null and weak) that are generally obeyed by classical matter. In other words, its energy density takes on negative values in some suitable reference system, as does the pressure. It turns out that injecting this exotic matter into the wormhole's throat can significantly widen the radius of the throat [81, 82].

5 Equations of motion

We choose a suitable expression for $f(T)$ which contains a constant, linear and a non-linear form of torsion, specifically [83]

$$f(T) = 2c_1\sqrt{-T} + \alpha T + c_2, \tag{13}$$

where α, c_1 and c_2 are arbitrary constants. Note that choosing $c_1 = 0$, $\alpha = \frac{1}{16\pi}$ in (13) leads to teleparallel gravity plus one cosmological constant term $\Lambda = c_2$. In this model the combination of the first and the third term corresponds to the equation of state of the cosmological constant in the framework of $f(T)$ gravity [84]. In this way by shuffling various terms or by the introduction of new terms, cosmologists have succeeded in establishing different models. In fact many of them have been reconstructed from various dark-energy models. The model in (13) that we are currently dealing with may have been inspired from the proposed model of the Veneziano ghost [85].

As a matter of fact this model was preferentially chosen over other alternatives, purely because of its simplicity as far as numerical computation is concerned. Another advantage of this model is that the obtained results are easier to compare or differentiate from the corresponding results in GR. In order to facilitate this, the linear middle term is included in the model. In connection with this model, it is worth stating that Capozziello et al. [86] made an attempt to investigate the cosmography of $f(T)$ cosmology by using data from BAO, supernovae Ia and WMAP. The analysis performed by Capozziello and his colleagues unveiled the fact that by choosing $c_2 = 0$, $\alpha = \Omega_{m0}$, and $c_1 = \sqrt{6}H_0(\Omega_{m0} - 1)$, it is possible to estimate the parameters of our proposed $f(T)$ model as functions of Hubble constant H_0, the cosmographic parameters and the value of matter density parameter.

Now field equations (3)–(5) and (13) give us the following system of equation of motion:

$$4\pi\rho(r) = \frac{1}{8}e^{-b}\left(-1 + e^{-\frac{1}{2}b}\right)\sqrt{2}e^{-\frac{1}{2}b}\left(-4e^b + \left(-2r^2a'' + \left(2r + b'r^2\right)a' + 8 + 2rb'\right)e^{\frac{1}{2}b} + 2r^2a''\right.$$

$$
\begin{aligned}
&+\left(-2b'r^2-2r\right)a'-2rb'-4\Big) \\
&\times c_1\left(\frac{\mathrm{e}^{-b}(\mathrm{e}^{\frac{1}{2}b}-1)(\mathrm{e}^{\frac{1}{2}b}-1-ra')}{r^2}\right)^{-\frac{3}{2}} r^{-3} \\
&-\left(\frac{1}{2}c_1\sqrt{2}\frac{1}{\sqrt{\frac{\mathrm{e}^{-b}(\mathrm{e}^{\frac{1}{2}b}-1)(\mathrm{e}^{\frac{1}{2}b}-1-ra')}{r^2}}}+\alpha\right) \\
&\times\left(\frac{1}{2}\frac{\mathrm{e}^{-b}(\mathrm{e}^{\frac{1}{2}b}-1)(\mathrm{e}^{\frac{1}{2}b}-1-ra')}{r^2}-\frac{1}{2}r^{-2}\right) \\
&-\frac{1}{2}c_1\sqrt{2}\sqrt{\frac{\mathrm{e}^{-b}(\mathrm{e}^{\frac{1}{2}b}-1)(\mathrm{e}^{\frac{1}{2}b}-1-ra')}{r^2}} \\
&-\frac{1}{2}\frac{\alpha\mathrm{e}^{-b}(\mathrm{e}^{\frac{1}{2}b}-1)(\mathrm{e}^{\frac{1}{2}b}-1-ra')}{r^2} \\
&-\frac{1}{4}c_2+\frac{1}{4}\mathrm{e}^{-b}\left(c_1\sqrt{2}\frac{1}{\sqrt{\frac{\mathrm{e}^{-b}(\mathrm{e}^{\frac{1}{2}b}-1)(\mathrm{e}^{\frac{1}{2}b}-1-ra')}{r^2}}}+2\alpha\right) \\
&\times\left(rb'-1\right)r^{-2}, \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
&4\pi p_r(r) \\
&=-\frac{1}{4}\Bigg(\left(\left(-2\alpha-2\alpha ra'\right)\mathrm{e}^{-b}+r^2c_2+2\alpha\right) \\
&\quad\times\sqrt{\frac{\mathrm{e}^{-b}(\mathrm{e}^{\frac{1}{2}b}-1)(\mathrm{e}^{\frac{1}{2}b}-1-ra')}{r^2}} \\
&\quad-\left(-1-\left(\mathrm{e}^{\frac{1}{2}b}\right)^2\mathrm{e}^{-b}+\mathrm{e}^{-b}\left(2+ra'\right)\mathrm{e}^{\frac{1}{2}b}\right)\sqrt{2}c_1\Bigg) \\
&\quad\times\frac{1}{\sqrt{\frac{\mathrm{e}^{-b}(\mathrm{e}^{\frac{1}{2}b}-1)(\mathrm{e}^{\frac{1}{2}b}-1-ra')}{r^2}}}r^{-2}, \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
&4\pi p_t(r) \\
&=\frac{1}{8}\frac{1}{\sqrt{\frac{\mathrm{e}^{-b}(\mathrm{e}^{\frac{1}{2}b}-1)(\mathrm{e}^{\frac{1}{2}b}-1-ra')}{r^2}}}r^{-2} \\
&\quad\times\Big(r\left(\alpha\left(2ra''+\left(a'-b'\right)\left(2+ra'\right)\right)\mathrm{e}^{-b}-2rc_2\right) \\
&\quad\times\sqrt{\frac{\mathrm{e}^{-b}(\mathrm{e}^{1/2b}-1)(\mathrm{e}^{\frac{1}{2}b}-1-ra')}{r^2}} \\
&\quad+\frac{1}{2}\big(-4\mathrm{e}^{b}+\left(4ra'+8\right)\mathrm{e}^{\frac{1}{2}b}+2r^2a''+r^2a'^2 \\
&\quad+\left(-2r-b'r^2\right)a'-2rb'-4\big)\mathrm{e}^{-b}\sqrt{2}c_1\Big). \qquad (16)
\end{aligned}
$$

Now we will discuss the possible physical solutions in the following three cases.

1. Anisotropic fluid: $p_r(r)\neq p_t(r)$.
2. Isotropic fluid: $p_r(r)=p_t(r)$.
3. Barotropic EoS: $p_r(r)=k\rho(r)$.

6 Anisotropic solution

Consider the specific redshift function and shape function given by [62–73]

$$
a(r)=c_1,\qquad b(r)=-\log\left(1-\left(\frac{r_0}{r}\right)^{n+1}\right), \qquad (17)
$$

where c_1, n, and r_0 are positive constants. Inserting these functions (17) into the EM tensor system, Eqs. (3)–(5), provide the following solutions:

$$
\begin{aligned}
&4\pi\rho(r) \\
&=\frac{1}{4}\Bigg(c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1}nr+3c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1} \\
&\quad-2\alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2r^{-2}}\left(\frac{r_0}{r}\right)^{n+1}n \\
&\quad+4\alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2r^{-2}}\left(\frac{r_0}{r}\right)^{n+1} \\
&\quad+3c_1\sqrt{2}r\left(\frac{r_0}{r}\right)^{n+1}-c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1}n \\
&\quad-8\alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2r^{-2}} \\
&\quad-6c_1\sqrt{2}+8\alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2r^{-2}} \\
&\quad\times\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}-2c_1\sqrt{2}r \\
&\quad+6\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2} \\
&\quad-c_2r^2\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2r^{-2}} \\
&\quad+2\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2}r\Bigg)r^{-2} \\
&\quad\times\frac{1}{\sqrt{(-1+\sqrt{1-(\frac{r_0}{r})^{n+1}})^2r^{-2}}}, \qquad (18)
\end{aligned}
$$

$4\pi p_r(r)$

$$
\begin{aligned}
&= -\frac{1}{4}\Bigg(2\alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2 r^{-2}} \\
&\quad \times \left(\frac{r_0}{r}\right)^{n+1}\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}} \\
&\quad + \sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2 r^{-2}}\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}} \\
&\quad \times c_2 r^2 + 2\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2} - 2c_1\sqrt{2} \\
&\quad + 2c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1}\Bigg)\frac{1}{\sqrt{(-1+\sqrt{1-(\frac{r_0}{r})^{n+1}})^2 r^{-2}}} \\
&\quad \times \frac{1}{\sqrt{1-(\frac{r_0}{r})^{n+1}}} r^{-2},
\end{aligned}
\tag{19}
$$

$4\pi p_t(r)$

$$
\begin{aligned}
&= \frac{1}{8}\Bigg(2\alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2 r^{-2}} \\
&\quad \times \left(\frac{r_0}{r}\right)^{n+1}\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\, n \\
&\quad + 2\alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2 r^{-2}} \\
&\quad \times \left(\frac{r_0}{r}\right)^{n+1}\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}} \\
&\quad - 2\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2 r^{-2}}\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}} \\
&\quad \times c_2 r^2 - 4\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2} + 4c_1\sqrt{2} \\
&\quad - 4c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1} + c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1} n\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}} \\
&\quad + 3\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1}\Bigg) \\
&\quad \times \frac{1}{\sqrt{(-1+\sqrt{1-(\frac{r_0}{r})^{n+1}})^2 r^{-2}}}\frac{1}{\sqrt{1-(\frac{r_0}{r})^{n+1}}} r^{-2}.
\end{aligned}
\tag{20}
$$

The energy conditions read

$\rho + p_r$

$$
\begin{aligned}
&= \frac{1}{16\pi}\frac{1}{\sqrt{(-1+\sqrt{1-(\frac{r_0}{r})^{n+1}})^2 r^{-2}}}\frac{1}{\sqrt{1-(\frac{r_0}{r})^{n+1}}} \\
&\quad \times r^{-2}\Bigg(\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1} nr \\
&\quad + 3\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1} \\
&\quad - 2\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}} \\
&\quad \times \alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{x}\right)^{n+1}}\right)^2 r^{-2}}\left(\frac{r_0}{r}\right)^{n+1} n \\
&\quad + 2\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2 r^{-2}} \\
&\quad \times \left(\frac{r_0}{r}\right)^{n+1} + 3\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2}r\left(\frac{r_0}{r}\right)^{n+1} \\
&\quad - \sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1} n \\
&\quad - 8\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}} \\
&\quad \times \alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2 r^{-2}} \\
&\quad - 8\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2} \\
&\quad + 8\alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2 r^{-2}} \\
&\quad - 8\alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2 r^{-2}}\left(\frac{r_0}{r}\right)^{n+1} \\
&\quad - 2\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2}r + 8c_1\sqrt{2} \\
&\quad - 8c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1} \\
&\quad - 2\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2 r^{-2}}
\end{aligned}
$$

$$\times\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_2r^2 + 2c_1\sqrt{2}r - 2c_1\sqrt{2}r\left(\frac{r_0}{r}\right)^{n+1}\Bigg). \tag{21}$$

$\rho + p_t$

$$= \frac{1}{32\pi}\frac{1}{\sqrt{(-1+\sqrt{1-(\frac{r_0}{r})^{n+1}})^2r^{-2}}}\frac{1}{\sqrt{1-(\frac{r_0}{r})^{n+1}}}$$

$$\times r^{-2}\Bigg(2\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1}nr + 6\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1} - 4\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}$$

$$\times\alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2r^{-2}}\left(\frac{r_0}{r}\right)^{n+1}n + 8\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2r^{-2}}$$

$$\times\left(\frac{r_0}{r}\right)^{n+1} + 6\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2}r\left(\frac{r_0}{r}\right)^{n+1} - 2\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1}n - 16\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}$$

$$\times\alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2r^{-2}} - 12\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2} + 16\alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2r^{-2}}$$

$$- 16\alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2r^{-2}}\left(\frac{r_0}{r}\right)^{n+1} - 4\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2}r + 12c_1\sqrt{2}$$

$$- 12c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1} - 2\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2r^{-2}}$$

$$\times\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_2r^2 + 4c_1\sqrt{2}r - 4c_1\sqrt{2}r\left(\frac{r_0}{r}\right)^{n+1} + 2\pi\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}$$

$$\times\alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2r^{-2}}\left(\frac{r_0}{r}\right)^{n+1}n + 2\pi\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}$$

$$\times\alpha\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2r^{-2}}\left(\frac{r_0}{r}\right)^{n+1} - 2\pi\sqrt{\left(-1+\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}\right)^2r^{-2}}$$

$$\times\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_2r^2 - 4\pi\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2} + 4\pi c_1\sqrt{2} - 4\pi c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1}$$

$$+ \pi\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1}n + 3\pi\sqrt{1-\left(\frac{r_0}{r}\right)^{n+1}}c_1\sqrt{2}\left(\frac{r_0}{r}\right)^{n+1}\Bigg). \tag{22}$$

The qualitative behavior of the energy density; the radial pressure, and the tangential pressure are plotted in Fig. 1, left panel. Note that the energy density is not positive throughout the space-time. So the anisotropic case violates the WEC also NEC. But just for a small region which is denoted by $r < 0.25$, the energy density is positive. So, there is just one possibility to have *micro* or *tiny* wormholes but in any case as we observe in the right panel, the NEC is violated. The NEC along the radial and the tangential directions are given by the positivity of the left-hand side of the following expressions of the (21), (22). The violation of the NEC is for both the case of the radial and of the tangential components of the pressure. Note that we adopted the parameters of our $f(T)$ to meet the observational data by the *cosmography* technique.

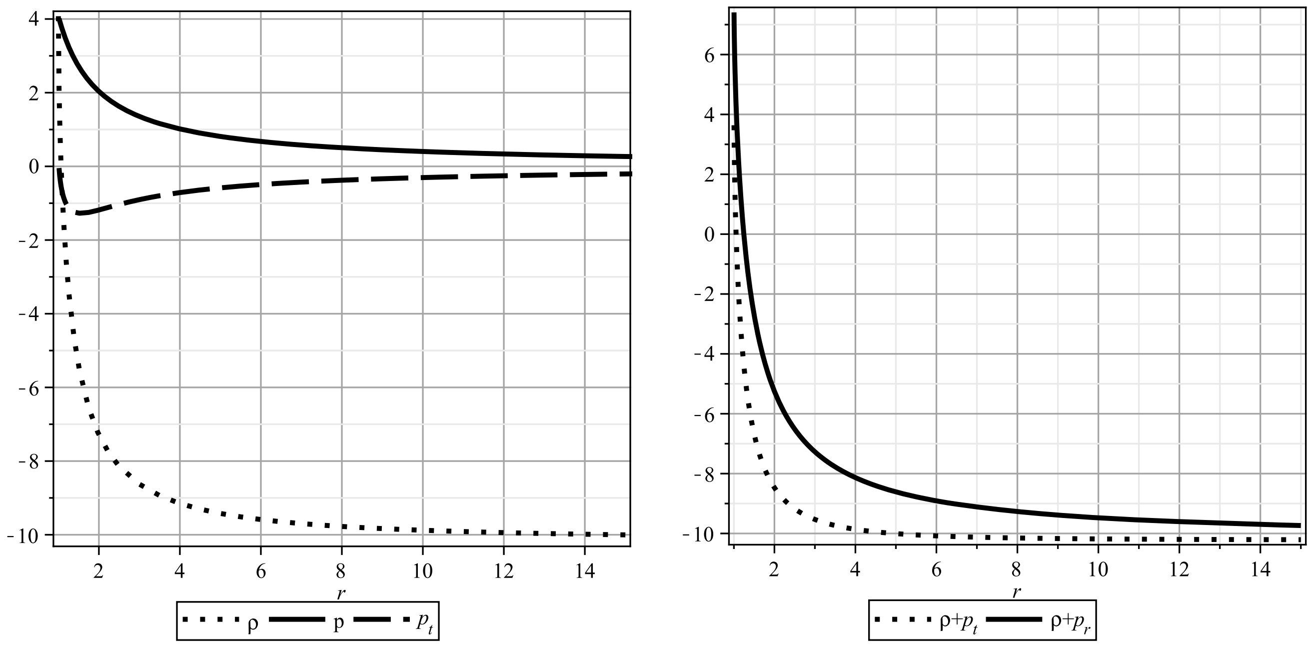


Fig. 1 (*Left*) The *dotted curve* depicts the energy density; the *solid* the radial pressure; and the *dashed curve* the tangential pressure. We have defined the following quantities: $r_0 = 1$, $n = 0.2$, $c_1 = 6^{1/2}/2H_0(\Omega_{m0} - 1)$, $c_2 = \Omega_{m0}$, $\alpha = 0$. (*Right*) Variation of the $\rho + p_r$ *solid*, and $\rho + p_t$ *dotted* for the following quantities: $r_0 = 1$, $n = 0.2$, $c_1 = 6^{1/2}/2H_0(\Omega_{m0} - 1)$, $c_2 = \Omega_{m0}$, $\alpha = 0$

7 Solution for isotropic pressure

Consider the specific redshift function and shape function given by

$$a(r) = c, \qquad p_r(r) = p_t(r) = p(r). \tag{23}$$

Inserting these functions into the stress-energy tensor profile, Eqs. (3)–(5), provides the following solutions:

$$b_{\pm}(r) = 2\log\left[\frac{-1 \pm \sqrt{-r^2C + 1}}{1 - r^2C \mp \sqrt{-r^2C + 1}}\right], \tag{24}$$

where C is a constant of integration. The asymptotic behavior of β is

$$\beta_{\pm} \simeq Cr^3 + O\left(\frac{1}{r^2}\right),$$

which diverges as $r \to \infty$. so the wormhole space-time is not asymptotically flat in both cases of $b_{\pm}$. Due to this reason, we cannot define the ADM mass of wormhole.

To find the radius of the wormhole's throat, we consider the following line element [87]:

$$ds^2 = e^{2\Phi(l)}\,dt^2 - dl^2 - r^2(l)\,d\Omega^2. \tag{25}$$

To obtain the radius of throat, we have the proper distance

$$l(r) = \pm\int_{r_0}^{r} \frac{dr'}{\sqrt{1 - \frac{\beta_{\pm}(r')}{r'}}}. \tag{26}$$

Solving the above integral, we obtain

$$l(r) = \mp\frac{\arcsin(\sqrt{C}r)}{\sqrt{C}}.$$

Now by taking inverse on both sides

$$r = \mp\frac{1}{\sqrt{C}}\sin(l\sqrt{C}).$$

Now the radius of the throat, $r_0 = \min\{r(l)\}$, is given by

$$|r_0| = \frac{1}{\sqrt{C}}.$$

Finally $C = \frac{1}{r_0^2}$. By this solution we have the following functions for the energy density ρ, and the pressure $p(r)$:

$$\begin{aligned}
&4\pi\rho_{+}(r)\\
&\quad = 5/2\biggl(-3/5{r_0}^2\biggl(r\biggl(\left(-16 - 16/3r\right){r_0}^6\\
&\qquad + \left(28r^2 + \frac{20}{3}r^3\right){r_0}^4\\
&\qquad + \left(-5/3r^5 - 13r^4\right){r_0}^2 + r^6\biggr)c_1\sqrt{2}\\
&\qquad + \left(-8/3c_2r^2 - \frac{128}{3}\alpha\right){r_0}^6
\end{aligned}$$

$$+10/3r^2\left(\frac{136}{5}\alpha+c_2r^2\right){r_0}^4$$
$$-5/6r^4\left(c_2r^2+\frac{344}{5}\alpha\right){r_0}^2+\frac{29}{3}\alpha r^6\right)$$
$$\times\sqrt{-\frac{r^2}{{r_0}^2}+1}+\left(-1/5rc_1\big((16r+48){r_0}^4\right.$$
$$+\left(-12r^3-60r^2\right){r_0}^2$$
$$+r^4(15+r)\big){r_0}^2\sqrt{2}$$
$$+\left(-8/5c_2r^2-\frac{128}{5}\alpha\right){r_0}^6$$
$$+6/5r^2\left(c_2r^2+\frac{104}{3}\alpha\right){r_0}^4$$
$$-1/10r^4\left(c_2r^2+168\alpha\right){r_0}^2+\alpha r^6\Big)(r+r_0)$$
$$\times(-r_0+r)\Big)\frac{1}{\sqrt{-\frac{r^2}{{r_0}^2}+1}}{r_0}^{-6}r^{-2}$$
$$\times\left(-2{r_0}^2-2\sqrt{-\frac{r^2}{{r_0}^2}+1}{r_0}^2+r^2\right)^{-1}$$
$$\times\left(1+\sqrt{-\frac{r^2}{{r_0}^2}+1}\right)^{-3}, \tag{27}$$

$4\pi p_+(r)$

$$=-1/2\left(\left(20r^2{r_0}^4-5r^4{r_0}^2-16{r_0}^6\right)\sqrt{-\frac{r^2}{{r_0}^2}+1}\right.$$
$$\left.-13r^4{r_0}^2+r^6+28r^2{r_0}^4-16{r_0}^6\right)$$
$$\times\left(c_1\sqrt{2}{r_0}^2+r\left(1/2c_2{r_0}^2+\alpha\right)\right)\frac{1}{\sqrt{-\frac{r^2}{{r_0}^2}+1}}$$
$$\times{r_0}^{-6}r^{-1}\left(-2{r_0}^2-2\sqrt{-\frac{r^2}{{r_0}^2}+1}{r_0}^2+r^2\right)^{-1}$$
$$\times\left(1+\sqrt{-\frac{r^2}{{r_0}^2}+1}\right)^{-3}. \tag{28}$$

So the expression for the NEC reads by the following expression:

$\rho_+ + p_+$

$$=1/2\left(-3/4\left(c_1r\left(\left(-16/3r-\frac{64}{3}\right){r_0}^6\right.\right.\right.$$
$$+\frac{20}{3}r^2\left(r+\frac{26}{5}\right){r_0}^4$$
$$+\left(-5/3r^5-\frac{44}{3}r^4\right){r_0}^2+r^6\Big)\sqrt{2}$$
$$+\left(-\frac{128}{3}\alpha-16/3c_2r^2\right){r_0}^6$$
$$+\frac{20}{3}r^2\left(c_2r^2+\frac{64}{5}\alpha\right){r_0}^4$$
$$-5/3r^4\left(c_2r^2+\frac{152}{5}\alpha\right){r_0}^2+8\alpha r^6\Big){r_0}^2$$
$$\times\sqrt{-\frac{r^2}{{r_0}^2}+1}+(r+r_0)(-r_0+r)\big(-1/4c_1r$$
$$\times\left((16r+64){r_0}^4+\left(-72r^2-12r^3\right){r_0}^2\right.$$
$$+r^4(16+r)\big){r_0}^2\sqrt{2}+\left(-32\alpha-4c_2r^2\right){r_0}^6$$
$$+\left(48\alpha r^2+3c_2r^4\right){r_0}^4+\left(-18r^4\alpha-1/4r^6c_2\right){r_0}^2$$
$$+\alpha r^6\big)\Big)\pi^{-1}\frac{1}{\sqrt{-\frac{r^2}{{r_0}^2}+1}}{r_0}^{-6}r^{-2}\Big(-2{r_0}^2$$
$$-2\sqrt{-\frac{r^2}{{r_0}^2}+1}{r_0}^2+r^2\Big)^{-1}\left(1+\sqrt{-\frac{r^2}{{r_0}^2}+1}\right)^{-3}. \tag{29}$$

The qualitative behavior of the energy density and the pressure for positive branch is plotted in Fig. 2, left panel. Note that the energy density remains positive throughout the space-time. So the isotropic case satisfies the WEC. Further, as we observe in the right panel, also the NEC satisfies here the EM components. The NEC along both the radial and the tangential directions is given by the positivity of the left-hand side of the expressions of (29). The NEC is satisfied for both the case of the radial and of the tangential components of the pressure. So there is at least one possibility to have a physically reasonable wormhole solution with isotropic pressures in our viable $f(T)$ model. Finally in the isotropic case, it is clear that the NEC is satisfied throughout the space-time. But for the negative branch, from Fig. 3, both null and weak energy conditions are violated.

For the minus branch:

$4\pi\rho_-(r)$

$$=5/2\left(3/5{r_0}^2\left(c_1r\left(\left(-16/3r-16\right){r_0}^6\right.\right.\right.$$
$$+\left(\frac{20}{3}r^3+28r^2\right){r_0}^4$$
$$+\left(-13r^4-5/3r^5\right){r_0}^2+r^6\Big)\sqrt{2}$$
$$+\left(-8/3c_2r^2-\frac{128}{3}\alpha\right){r_0}^6$$

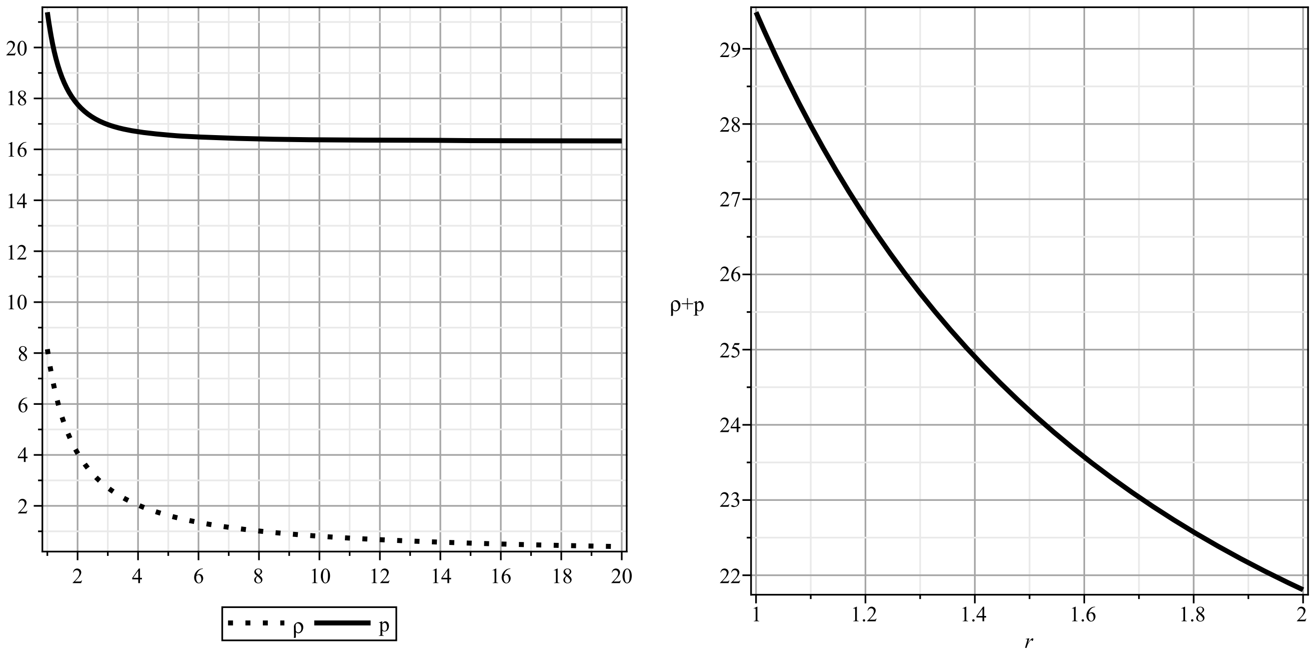


Fig. 2 (*Left*) Positive branch: The *dotted curve* depicts the energy density; the *solid* the radial pressure. We have defined the following quantities: $r_0 = 1$, $n = 0.2$, $c_1 = 6^{\frac{1}{2}}/2H_0(\Omega_{m0} - 1)$, $c_2 = \Omega_{m0}$, $\alpha = 0$. (*Right*) Variation of the $\rho + p_r$ for the following quantities: $r_0 = 1$, $n = 0.2$, $c_1 = 6^{\frac{1}{2}}/2H_0(\Omega_{m0} - 1)$, $c_2 = \Omega_{m0}$, $\alpha = 0$

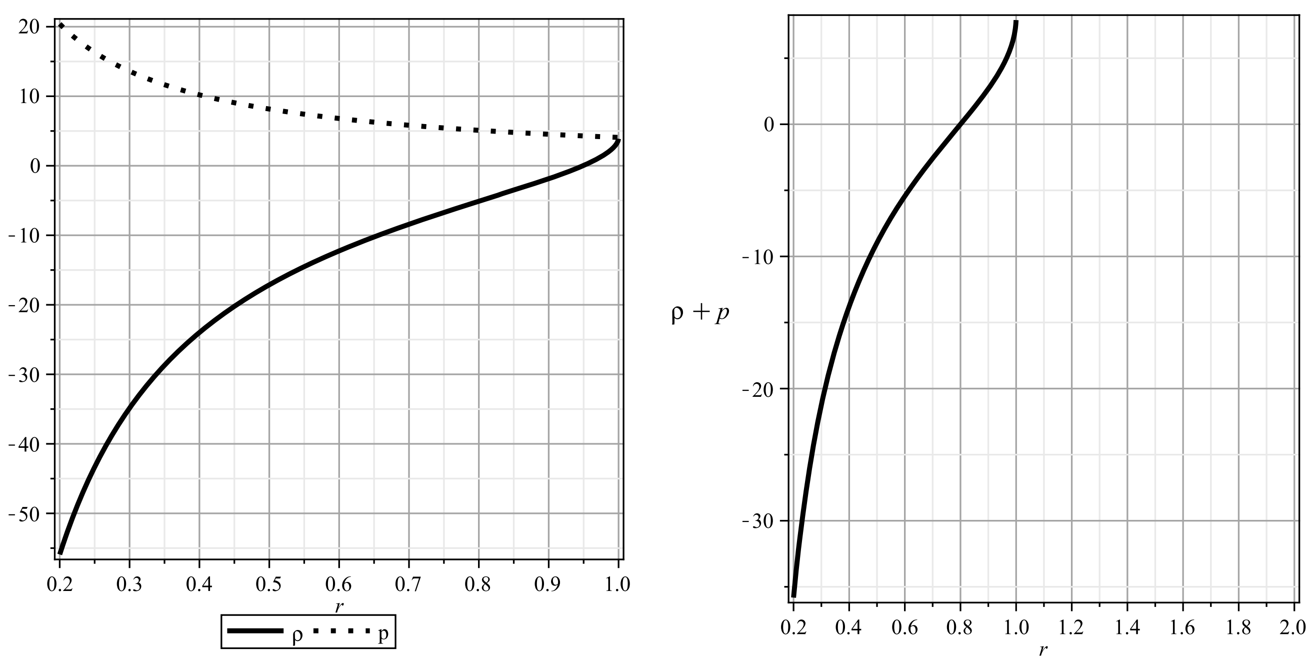


Fig. 3 (*Left*) Minus branch: The *solid curve* depicts the energy density; the *dotted* the radial pressure. We have defined the following quantities: $r_0 = 1$, $n = 0.2$, $c_1 = 6^{\frac{1}{2}}/2H_0(\Omega_{m0} - 1)$, $c_2 = \Omega_{m0}$, $\alpha = 0$. (*Right*) Variation of the $\rho + p_r$ for the following quantities: $r_0 = 1$, $n = 0.2$, $c_1 = 6^{\frac{1}{2}}/2H_0(\Omega_{m0} - 1)$, $c_2 = \Omega_{m0}$, $\alpha = 0$

$$
\begin{aligned}
&+10/3r^2\left(c_2r^2+\frac{136}{5}\alpha\right)r_0{}^4\\
&-5/6\left(c_2r^2+\frac{344}{5}\alpha\right)r^4r_0{}^2+\frac{29}{3}\alpha r^6\Big)\\
&\times\sqrt{-\frac{r^2}{r_0{}^2}+1}+(r+r_0)\Big(-1/5c_1r_0{}^2r\\
&\times\big((48+16r)r_0{}^4+\big(-60r^2-12r^3\big)r_0{}^2\\
&+r^4(15+r)\big)\sqrt{2}+\left(-\frac{128}{5}\alpha-8/5c_2r^2\right)r_0{}^6\\
&+6/5\left(c_2r^2+\frac{104}{3}\alpha\right)r^2r_0{}^4\\
&-1/10r^4\big(168\alpha+c_2r^2\big)r_0{}^2+\alpha r^6\Big)(-r_0+r)\Big)\\
&\times\frac{1}{\sqrt{-\frac{r^2}{r_0{}^2}+1}}r_0{}^{-6}r^{-2}\\
&\times\left(-2r_0{}^2+2\sqrt{-\frac{r^2}{r_0{}^2}+1}r_0{}^2+r^2\right)^{-1}\\
&\times\left(-1+\sqrt{-\frac{r^2}{r_0{}^2}+1}\right)^{-3},
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
&4\pi p_-(r)\\
&\quad=-1/2\big(c_1\sqrt{2}r_0{}^2+r\big(1/2c_2r_0{}^2+\alpha\big)\big)\\
&\qquad\times\Big(\big(16r_0{}^6-20r^2r_0{}^4+5r^4r_0{}^2\big)\sqrt{-\frac{r^2}{r_0{}^2}+1}\\
&\qquad-13r^4r_0{}^2\\
&\qquad+r^6+28r^2r_0{}^4-16r_0{}^6\Big)\frac{1}{\sqrt{-\frac{r^2}{r_0{}^2}+1}}r_0{}^{-6}r^{-1}\\
&\qquad\times\left(-2r_0{}^2+2\sqrt{-\frac{r^2}{r_0{}^2}+1}r_0{}^2+r^2\right)^{-1}\\
&\qquad\times\left(-1+\sqrt{-\frac{r^2}{r_0{}^2}+1}\right)^{-3},
\end{aligned}
\tag{31}
$$

and the NEC reads

$$
\begin{aligned}
&\rho_-+p_-\\
&\quad=1/2\Big(3/4r_0{}^2\Big(c_1r\Big(\Big(\left(-\frac{64}{3}-16/3r\right)r_0{}^6\\
&\qquad+\frac{20}{3}\left(r+\frac{26}{5}\right)r^2r_0{}^4\\
&\qquad+\left(-\frac{44}{3}r^4-5/3r^5\right)r_0{}^2+r^6\Big)\sqrt{2}\\
&\qquad+\left(-\frac{128}{3}\alpha-16/3c_2r^2\right)r_0{}^6\\
&\qquad+\frac{20}{3}r^2\left(\frac{64}{5}\alpha+c_2r^2\right)r_0{}^4\\
&\qquad-5/3\left(c_2r^2+\frac{152}{5}\alpha\right)r^4r_0{}^2+8\alpha r^6\Big)\\
&\qquad\times\sqrt{-\frac{r^2}{r_0{}^2}+1}+(r+r_0)\big(-1/4r_0{}^2\big((16r+64)r_0{}^4\\
&\qquad+\big(-12r^3-72r^2\big)r_0{}^2\\
&\qquad+r^4(16+r)\big)c_1r\sqrt{2}+\big(-4c_2r^2-32\alpha\big)r_0{}^6\\
&\qquad+\big(3c_2r^4+48r^2\alpha\big)r_0{}^4\\
&\qquad+\big(-1/4r^6c_2-18r^4\alpha\big)r_0{}^2+\alpha r^6\big)(-r_0+r)\Big)\\
&\qquad\times\pi^{-1}\frac{1}{\sqrt{-\frac{r^2}{r_0{}^2}+1}}r_0{}^{-6}r^{-2}\\
&\qquad\times\left(-2r_0{}^2+2\sqrt{-\frac{r^2}{r_0{}^2}+1}r_0{}^2+r^2\right)^{-1}\\
&\qquad\times\left(-1+\sqrt{-\frac{r^2}{r_0{}^2}+1}\right)^{-3}.
\end{aligned}
\tag{32}
$$

8 Solution with barotropic equation of state

In this case the exact solution exists only for $\alpha=0$. Consider the specific redshift function and shape function given by

$$a(r)=c,\qquad p_r(r)=k\rho(r),\tag{33}$$

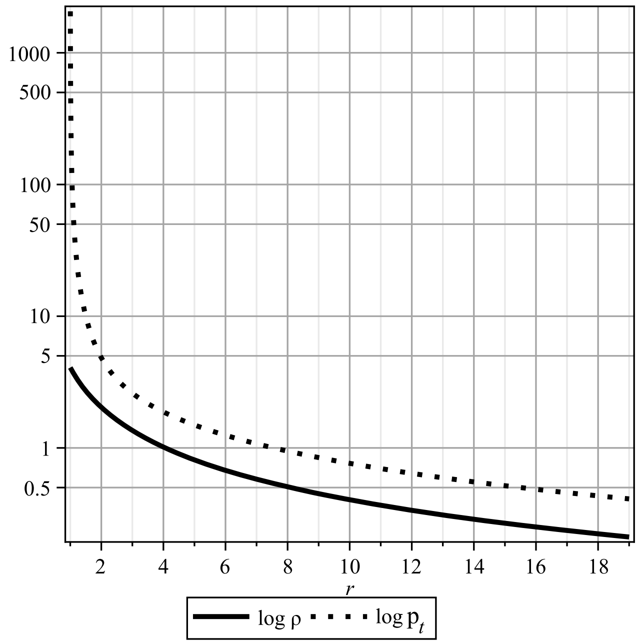
where k is an arbitrary but finite constant. Using (33) in the equations of motion, we obtain

$$b(r)=2\log\left(\frac{r^2}{r^2+C(1-3r+3r^2-r^3)}\right),\tag{34}$$

$$4\pi\rho(r)=-\frac{c_2r^6+2c_1\sqrt{2}r^5}{4r^6},\tag{35}$$

$$4\pi p_r(r)=-k\frac{c_2r^6+2c_1\sqrt{2}r^5}{4r^6},\tag{36}$$

$$
\begin{aligned}
4\pi p_t(r)=\frac{1}{4r^6(-1+r)}&\big(-c_2r^7+c_2r^6+3C\sqrt{2}c_1r^6\\
&-c_1\sqrt{2}r^6-9C\sqrt{2}c_1r^5\\
&-2c_1\sqrt{2}r^5+9C\sqrt{2}c_1r^4-3C\sqrt{2}c_1r^3\big).
\end{aligned}
\tag{37}
$$



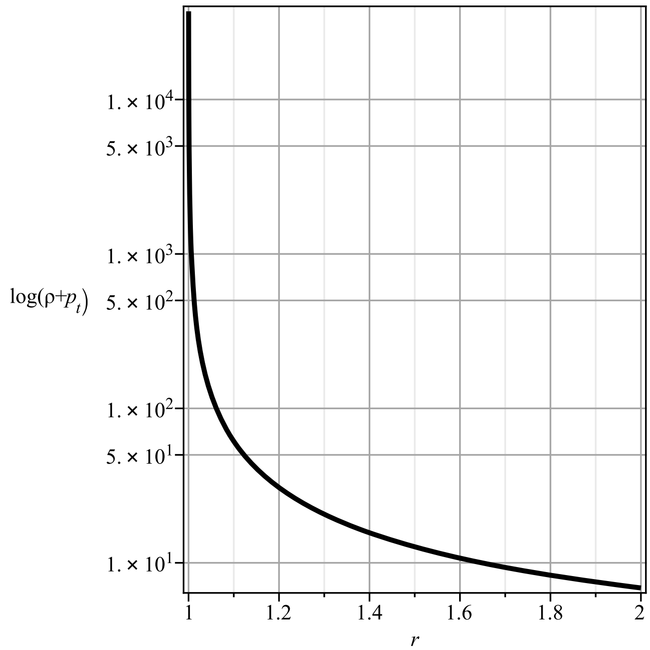


Fig. 4 (*Left*) We have defined the following quantities: $r_0 = 1$, $n = 0.2$, $c_1 = 6^{\frac{1}{2}}/2H_0(\Omega_{m0} - 1)$, $c_2 = \Omega_{m0}$, $\alpha = 0$. (*Right*) Variation of the $\log(\rho + p_t)$ for the following quantities: $r_0 = 1$, $n = 0.2$, $c_1 = 6^{\frac{1}{2}}/2H_0(\Omega_{m0} - 1)$, $c_2 = \Omega_{m0}$, $\alpha = 0$

So, the expression for the transverse null energy condition $\rho + p_t > 0$ reads

$$\begin{aligned}
&\rho + p_t \\
&\quad = \frac{1}{16\pi r^3(1-r)}\big(2c_2r^4 + \big(-2c_2 + 3c_1\sqrt{2} \\
&\qquad - 3C\sqrt{2}c_1\big)r^3 + 9C\sqrt{2}c_1r^2 \\
.&\qquad - 9C\sqrt{2}c_1r + 3C\sqrt{2}c_1\big) \qquad (38)
\end{aligned}$$

Again, the qualitative behavior of the energy density and the pressure is plotted in Fig. 4, left panel, in log scale. Note that the energy density remains positive throughout the space-time. So the isotropic case satisfies the WEC. Further, as we observe in the right panel, the NEC is satisfied by the EM tensor components. The NEC along both the radial and the tangential directions are given by the positivity of the left-hand side of the following expressions of the (38). The non-violation of the NEC is for the tangential component of the pressure. So there exists at least one possibility to have a physically acceptable wormhole solution with barotropic radial fluid in our viable $f(T)$ model. In fact, in the barotropic case, obviously the NEC is satisfied throughout the space-time. In Fig. 5, we plot the ratio of pressure and energy density and show that these remain positive for asymptotic values of r.

9 Conclusion

We have considered Morris–Thorne wormholes, i.e., static and spherically symmetric traversable wormholes, in the Weitzenbock space-time with torsion. Basically, we discussed the possible wormhole solutions in a viable $f(T)$ model with form $f(T) = 2c_1\sqrt{-T} + \alpha T + c_2$. This model was preferentially chosen over other alternatives, purely because of its simplicity as far as numerical computation is concerned. Another advantage of this model is that the obtained results are easier to compare or differentiate from the corresponding results in GR. In connection with this model the cosmography of this $f(T)$ cosmology by using data from BAO, Supernovae Ia and WMAP shows that by choosing $c_2 = 0$, $\alpha = \Omega_{m0}$ and $c_1 = \sqrt{6}H_0(\Omega_{m0} - 1)$, it is possible to estimate the parameters of our proposed $f(T)$ model as functions of Hubble parameter H_0, the cosmographic parameters and the value of matter density parameter. We investigated three kinds of fluid: isotropic, anisotropic and finally the barotropic fluids, for which we considered the radial pressure satisfying a barotropic equation of state. We presented specific solutions with various choices of the shape function. In any case, we obtained the exact solutions which described the wormhole geometries. By checking the behavior of the weak and null energy conditions for each case we observed that it is violated for anisotropic while it is satisfied for isotropic and barotropic cases. So we can have both isotropic and barotropic wormhole solutions in this viable torsion-based model of the gravity.

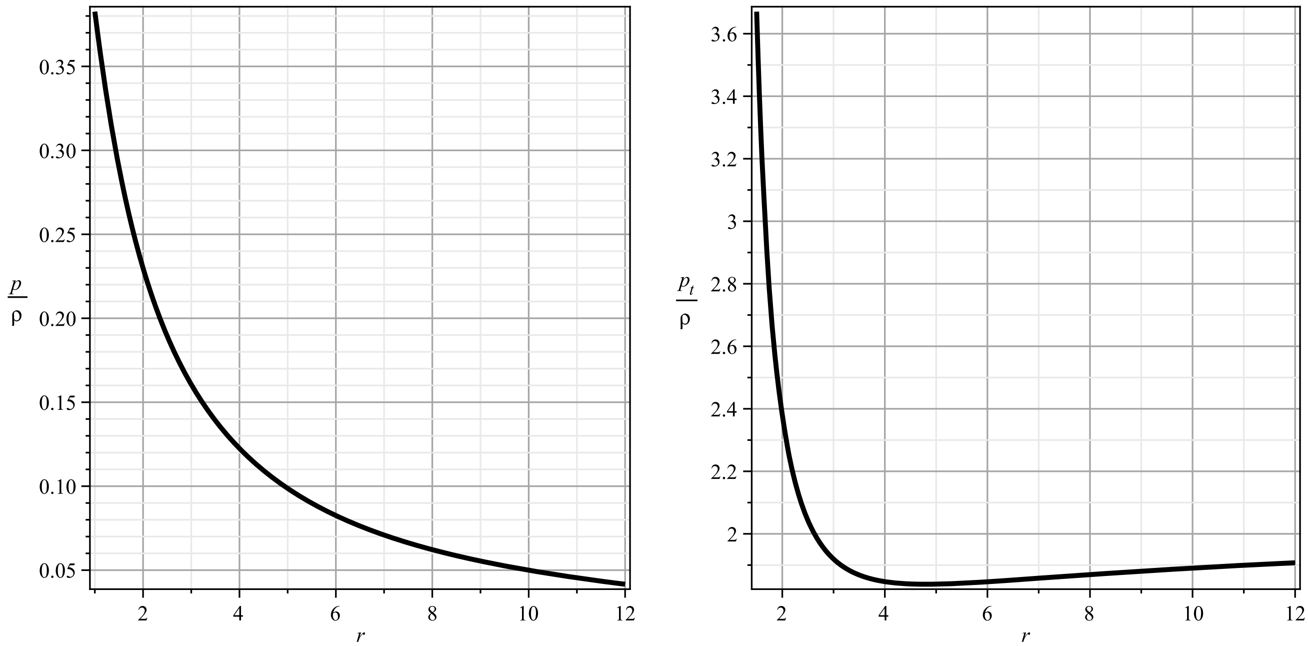


Fig. 5 (*Left*) The curve depicts the EoS parameter $w = \frac{p}{\rho}$ for the isotropic case. We have defined the following quantities: $r_0 = 1$, $n = 0.2$, $c_1 = 6^{1/2}/2H_0(\Omega_{m0} - 1)$, $c_2 = \Omega_{m0}$, $\alpha = 0$. (*Right*) Variation of the EoS parameter $w = \frac{p_t}{\rho}$ for barotropic case for the following quantities: $r_0 = 1$, $n = 0.2$, $c_1 = 6^{1/2}/2H_0(\Omega_{m0} - 1)$, $c_2 = \Omega_{m0}$, $\alpha = 0$

In the anisotropic case, our $f(T)$ model mimics the phantom energy since both energy conditions NEC and WEC are violated $r > r_0$. In the isotropic case, we have two special cases for the shape function. For the positive branch, both energy conditions are satisfied while they are violated in the negative branch case. Also the obtained wormhole solution is not asymptotically flat. For the barotropic case, again we have non-asymptotically flat solution and energy conditions are satisfied for transverse NEC and WEC. Moreover, we discussed the behavior of the EoS parameter $w = p/\rho$ for the isotropic and barotropic cases. Our numerical simulation shows that for the isotropic case, w remains positive. For the barotropic case, the same behavior occurs.

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