On the Threshold Nature of Erosive Burning

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A physical explanation for the existence of the threshold of erosive burning is proposed. It is shown that this type of combustion occurs when the thickness of the laminar sublayer in the turbulent boundary layer becomes smaller than the thickness of the laminar combustion zone. In this case, turbulent flame in the gas phase is formed. Relations are obtained linking the critical (threshold) velocity of the blowing flow and the critical Vilyunov number to the properties of the propellant and the gas resulting from propellant decomposition. Simple exponential dependences on the blowing velocity are found for the burning rate. The simplest representation of the erosive burning rate is obtained using the Bulgakov–Lipanov number, whose threshold value is equal to unity. A new mechanism for the occurrence of negative erosion is proposed, according to which the burning rate decreases during blowing because the boundary layer is displaced, resulting in a decrease in the heat flux from the flame zone to the solid-phase decomposition surface.

Key words: erosive burning, critical parameters, turbulent combustion, turbulent boundary layer, burning rate.

INTRODUCTION

It is known [1, 2] that erosive burning belongs to the class of threshold phenomena and is manifested in an increase in the burning rate \( u \) under the action of the hot gas flow blowing over the burning surface. This type of combustion, as a rule, occurs when the average (or maximum) gas flow velocity along the burning surface \( w_\infty \) exceeds the critical value \( w_* \). To determine the burning rate in the presence of the erosive effect \( u_w \), it is common to use the following expression (for a bounded range of \( w_\infty \)):

\[
u_w = u[1 + k_c(w_\infty - w_*)], \quad w_\infty \geq w_*, \quad k_c = \text{const};
\]

\[
u_w = u, \quad w_\infty < w_*.
\]

The ratio \( u_w/u = \varepsilon \) is called the erosion coefficient.

In the early stage of research, specialists did not pay attention to the threshold nature of erosive burning. In addition, the physics of the phenomenon was unclear. The situation was complicated by the fact that, at low blowing velocities, the opposite effect — a decrease in the burning rate — was observed. This phenomenon was called negative erosion.

According to the Vilyunov theory [3], the internal mechanism of the positive erosion effect involves activation of heat, momentum, and mass transfer processes in the gas-phase combustion zone under the action of turbulence. In aggregate, this results in an increase in the heat flux from the zone of gas-phase chemical reactions to the propellant decomposition surface. This is usually associated with a rise in the propellant surface temperature \( T_s \) [3], which indicates activation of surface and subsurface chemical reactions following the Arrhenius law. But this is not necessary. Thus, for example, if an increase in the heat flux from the gas phase leads to similar behavior of the temperature gradient in the solid phase, then favorable conditions for catastrophic thermomechanical failure of the propellant material may arise in the near-surface layer [4]. In this case, an increase in the burning rate will be determined primarily by the temperature gradient rather than by the temperature \( T_s \).
On the Threshold Nature of Erosive Burning

Here the main and determining parameter of the process is the Vilyunov number

\[ J = \sqrt{C_f \frac{\rho w_\infty}{\rho_c u}}, \]

where \( C_f \) is the drag coefficient, \( \rho_c \) is the propellant density, and \( \rho \) is the gas density. Accordingly, the formula recommended for practical use is written as [5]

\[ \varepsilon = \begin{cases} 1, & J < J_s, \\ 1 + k_c (J - J_s), & J \geq J_s, \end{cases} \]

where \( k_c = \text{const.} \) This constant, as well as that given above, is appropriate, as a rule, for a relatively narrow range of \( J \).

In the Vilyunov theory, the critical number \( J_s \) (or the critical velocity \( w_\infty \)) is not determined but is found by experiment. Therefore, the cause of the threshold nature of erosive burning remain obscure. As a consequence, without performing experiment, one cannot tell beforehand when and on what segment of the propulsion charge the erosive effect will manifest itself.

The first attempt [2, 6] to elucidate the nature of the criticality of erosive burning and to calculate the threshold values of the blowing velocity and Vilyunov number led to the following result:

\[ J_s = \frac{2T_b}{T_b \sqrt{C_f}}. \]

Here \( T_b \) is the flame temperature.

The threshold nature of erosive burning is associated with the effect of the friction induced shear stress \( \tau_s \) on the surface of contact of the gas with the propellant surface [6, 7]. The gaseous products of primary propellant decomposition flowing from the charge surface shift the hydrodynamic boundary layer. As a result, a gas layer of thickness \( \nu f / v \) (\( v \) is the injection velocity, \( \nu f \) is the kinematic viscosity of the gas) is formed near the propellant surface, in which motion is directed predominantly along the normal to the propellant decomposition surface (\( \tau_s \approx 0 \)). The turbulence in this area is weak. Outside the layer there is a boundary region of viscous flow, in which the direction and value of the gas velocity change rapidly, and on the outer boundary, the gas velocity coincides with the blowing velocity \( w_\infty \).

Next, a zone of intense turbulent flow is located.

Erosive burning begins, according to theory [6], after the injection parameter \( 2v / C_f w_\infty \) becomes smaller than the critical value equal to 4. Since an increase in the blowing velocity leads to an increase in the velocity of gas outflow from the burning surface, a further increase in the erosive effect is possible if \( v \) increases not faster than \( w_\infty \).

Numerical modeling [2, 8–10] of turbulent combustion has generally confirmed the results of [6]. Nevertheless, the problems of explaining the threshold nature of erosive burning, developing methods for calculating the critical number \( J_s \), and determining its relationship to the properties of propellant remain important and are considered in the present paper. The practical value of this knowledge is clear — it provides a possibility of controlling the burning behavior of rocket propellants and other high-energy materials.

1. CALCULATION OF PARAMETERS OF EROSIVE BURNING (POSITIVE EROSION)

The physical meaning of the Vilyunov parameter \( J \) is easily determined if \( \rho_c u \) in this parameter is replaced by \( \rho v \), using the mass conservation law for steady-state combustion in the form \( \rho_c u = \rho v \). Then, the parameter \( J \) is written in the form [11]

\[ J = \sqrt{C_f \frac{w_\infty}{v}}, \]

which reflects the presence of two competitive factors: the gas flowing from the propellant decomposition surface at velocity \( v \) tends to shift the turbulent boundary layer from the combustion zone. The appearance of this layer is due to the gas flow moving along the propellant surface at velocity \( w_\infty \) with the characteristic value \( w_\infty \), which is indicated by the presence of a factor with the hydrodynamic drag coefficient \( C_f \).

In the theory of turbulent boundary layer, the complex \( \sqrt{C_f w_\infty} \), to within a coefficient, is the root-mean-square value \( w' \) of the flow velocity pulsations [12], and the velocity \( v \) is the normal flame velocity \( v_n \) in the gas phase. Therefore, the Vilyunov parameter can be written as \( J = w' / v_n \), which as will be shown below, correctly reflects the physical meaning of (positive) erosive burning. The ratio \( w' / v_n \) is one of the main factors in the establishment of turbulent flame velocity in gases [13, 14], which is most likely indicative of the leading role of the gas phase in erosive burning.

We consider an arbitrary small region of a propellant charge in a rocket thrust chamber. Since the analysis refers to spatial scales much smaller than the characteristic sizes of geometrical nonuniformities of the propellant charge, the isolated region near the propellant surface is considered flat. The \( x \) axis is directed along the vertical to this surface (Fig. 1).

Below, as in the expressions given above, by the quantities affected by turbulent pulsations are meant their average values.

The coordinate of the propellant surface on which there is primary decomposition of the solid phase into gas will be denoted as \( x_s \). The gas-phase combustion zone, which is assumed to be infinitely thin, is located at
the point \( x_f \). Here by the zone of developed turbulent flow is meant its core and transition region.

In the consideration of the physical picture of erosive burning, the most important is the ratio of the thickness of the combustion zone with no turbulence \( x_f - x_s \) to the thickness of the laminar (viscous) sublayer of the turbulent boundary layer \([2, 15]\). The universal and key role of the ratio of these characteristic length scales in the description of the erosive burning mechanism is shown in \([2, 15]\). In the present work, unlike in the studies cited above, the following model of the erosion process is adopted.

If the thickness \( \delta_g \) of the laminar sublayer of the hydrodynamic boundary layer is larger than the thickness of the combustion zone \( x_f - x_s \) (Fig. 1a), turbulence has a weak effect on the combustion process. This aspect of erosion will be considered in detail below. In the laminar sublayer, the turbulence intensity is very low \([12]\), and, at distances \( x > x_f \) there is a flame zone in which chemical equilibrium has already established.

For this case, the energy conservation law at the interface \( x = x_s \) between the solid and gas phases is written as

\[
\lambda_c \frac{\partial T_c}{\partial x} = \lambda_g \frac{\partial T}{\partial x} - L \rho_c u, \quad u = -\frac{dx_s}{dt} > 0, \quad (1)
\]

where \( \lambda_c \) and \( \lambda_g \) are the thermal conductivities of the propellant and gas, \( T_c \) and \( T \) are the temperatures of the solid and gas phases, and \( L \) is the heat effect of gasification. If gasification involves heat absorption, then, \( L > 0 \). The negative sign of \( L \) corresponds to an exothermic transformation of the solid phase to the gas.

The derivative on the left side of equality (1) can be written as

\[
\frac{\partial T_c}{\partial x} = \frac{u}{\alpha_c} (T_s - T_0),
\]

where \( \alpha_c \) is the thermal diffusivity of the solid phase and \( T_0 \) is the initial temperature of the propellant. The derivative with respect to the temperature on the right side of (1) is represented as

\[
\frac{\partial T}{\partial x} = A_1 \frac{T_b - T_s}{x_f - x_s},
\]

where \( A_1 = \text{const} \). Then, taking into account that \( \alpha_c = \lambda_c / \rho_c c_c \) (\( c_c \) is the heat capacity of the propellant) and the aforesaid, from equality (1) we obtain the equation

\[
c_c m (T_s - T_0) = A_1 \lambda_g \frac{T_b - T_s}{x_f - x_s} - L m, \quad (2)
\]

From this, the mass burning rate is expressed as

\[
m = A_1 \frac{\lambda_g}{c_c (T_s - T_0)} \frac{T_b - T_s}{x_f - x_s}. \quad (3)
\]

If the thickness of the laminar sublayer is smaller than the thickness of the (laminar) combustion zone, the position of the burning surface varies randomly about the average value of \( \delta_{b_T} \) (Fig. 1b), which will be called the averaged distance from the turbulent gas flame to the gasification surface. We determine the functional form of \( \delta_{b_T} \) based on the theory of similarity and dimension \([16]\). In our disposal there are only two comparable parameters: the thickness \( \delta_g \) of the laminar sublayer and the thickness of the combustion zone \( x_f - x_s \) in the absence of turbulence. Then, according to the theory of similarity and dimension, we write the relation

\[\text{Fig. 1. Relative arrangement of the combustion front and laminar sublayer: (a) } \delta_g > x_f; \quad (b) \delta_g < x_f \text{ (the scale is changed).} \]
On the Threshold Nature of Erosive Burning

\[ \delta_{br} = (x_f - x_s)F_\delta \left( \frac{\delta_g}{x_f - x_s} \right), \]

which contains the universal function \( F_\delta \) of the argument. We assume that this function has an exponential form as one of the simplest forms (below we show that this is justified):

\[ F_\delta = A_2 \left( \frac{\delta_g}{x_f - x_s} \right)^n, \quad A_2, n = \text{const}, \quad 0 \leq n < 1. \]

This exponential function is frequently encountered in various problems in the presence of an intermediate asymptotics [16]. Then, for the width of the turbulent combustion zone, we have

\[ \delta_{br} = A_2 \delta_g^n (x_f - x_s)^{1-n}. \]

The equality

\[ \delta_g = x_f - x_s \]

(4)
can be taken to be the condition for the onset of the erosive effect. This assumption is somewhat crude; it should be determined more rigorously to within a constant. This is of no significance, however, and, for simplification, the constant is dropped.

In the presence of erosive burning, Eq. (2) becomes

\[ \frac{c_w m_w(T_{s,w} - T_0)}{A_3 \lambda_g \frac{b_{\tau}}{\delta_{br}}} = \frac{\lambda_g}{\rho c_w} \frac{T_{b,w} - T_{s,w}}{m_w}, \]

where the subscript \( w \) indicates that the parameters depend on the blowing velocity. Accordingly, the mass rate of erosive burning \( m_w \) is expressed as

\[ m_w = A_3 \frac{\lambda_g}{c_c(T_{s,w} - T_0)} + L \frac{b_{\tau}}{\delta_{br}}. \]

Taking into account the representation for \( \delta_{br}, \) from (3) and (5) we obtain

\[ \frac{m_w}{m} = \frac{A_3}{A_1 A_2} \times \frac{c_c(T_s - T_0) + L}{c_c(T_{s,w} - T_0)} \frac{T_{b,w} - T_{s,w}}{L} \frac{(x_f - x_s)}{\delta_g}^n. \]

If equality (4) is valid, all fractions in the last expression containing physical quantities should become unity. Therefore, \( A_3/A_1 A_2 = 1. \) In addition, we note that the differences \( T_b - T_s \) and \( T_{b,w} - T_{s,w} \) are equal to the total heat effect \( Q \) of the gas-phase chemical reactions divided by the constant-pressure heat capacity \( c_p. \) This is valid provided that the chemical composition of the combustion products is not influenced by the axial component \( w \) of the gas velocity.

Thus, we have the formula

\[ \frac{m_w}{m} = \frac{c_c(T_s - T_0) + L}{c_c(T_{s,w} - T_0)} \left( \frac{x_f - x_s}{\delta_g} \right)^n. \]

The thickness of the laminar sublayer is related to the average velocity \( w_\infty \) (in the flow core) as follows [12]:

\[ \delta_g = 32.5 \frac{\nu_g}{\sqrt{C_f w_\infty}}. \]

Then, equality (4) defines the critical (threshold) velocity

\[ w_* = \frac{32.5 \nu_g}{\sqrt{C_f (x_f - x_s)}}. \]

In view of this expression, formula (6), whose left part is the erosion coefficient, can be written in terms of the velocity of blowing over the propellant surface:

\[ \varepsilon = \frac{m_w}{m} = \frac{c_c(T_s - T_0) + L}{c_c(T_{s,w} - T_0) + L} \left( \frac{w_\infty}{w_*} \right)^n. \]

Here \( \varepsilon = 1 \) if \( w_\infty < w_* \).

On the other hand, the burning rate, as a rule, is determined by the decomposition surface temperature according to the Arrhenius law with the activation energy of the solid-state chemical reaction \( E_c. \) Therefore, we adopt the equality [3]

\[ \varepsilon = \frac{m_w}{m} = \exp \left[ \frac{E_c}{2RT_s} \left( 1 - \frac{T_s}{T_{s,w}} \right) \right]. \]

(\( R \) is the universal gas constant). Formula (9) also follows from the Merzhanov–Dubovitskii expression for the burning rate [17] and the expression obtained in the combustion model [18, 19] for evaporating propellants taking into account only the strong exponential temperature dependence of the burning rate.

Formulas (7)–(9) give a solution of the problem of erosive burning. It should be noted that the question is so-called positive erosion, which is pronounced at high blowing flow velocities. At relatively low velocities \( w_\infty \) (at high initial temperatures \( T_0 \) [20]), a weak negative erosion effect is observed, resulting in a slightly reduced burning rate.

2. COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

Let us estimate of the threshold velocity \( w_* \). At temperatures \( T_s \approx 600-800 \text{ K}, \) the viscosity of the typical gaseous decomposition products of ballistite solid rocket propellants (\( \text{O}_2, \text{CO}, \text{Cl}_2, \text{NO}, \text{H}_2\text{O}, \text{etc.} \)) is within \( (3.2-4.6) \cdot 10^{-5} \text{ m}^2/\text{sec} \) [21]. The thickness of the combustion zone \( x_f - x_s \) (at atmospheric pressure) is usually \( (0.6-1.1) \cdot 10^{-4} \text{ m} [1, 22]. \) Averaging their values over the indicated intervals and setting \( C_f \approx 0.03 [3, 12], \) we obtain \( w_* \approx 100 \text{ m/sec. This} \)
value of the critical velocity agrees in order of magnitude with well-known experimental data [1, 3]. Accordingly, \( J_v \approx 1 \).

The distance \( x_f - x_s \) between the flame zone and the propellant surface depends appreciably on the pressure and decreases with its growth.

If the activation energy is high enough (\( E_c/RT_s \gtrsim 16 \)), the first fraction in (8) varies only slightly and can be set equal to unity. Then, for the erosion coefficient, we obtain the exponential function

\[
\varepsilon = \begin{cases} 
1, & w_\infty < w_s, \\
(w_\infty/w_s)^n, & w_\infty \geq w_s,
\end{cases}
\]

(10)

where negative erosion, as a weak effect, is neglected.

Figure 2 gives theoretical results obtained using formula (10), data taken from [3], and experimental data for the JPN propellant [3] (pressure 19.6 atm) consisting mainly of nitrocellulose (51.5%) and nitroglycerin (43.0%). It should be noted that in constructing the dependence using (10) (solid curve), we adjusted only the exponent \( n \). The value of the critical velocity was taken from experiment because insufficient data are available for its calculation.

Alemasov et al. [23] give experimental curves for two types of nitroglycerin powder, and, as is evident from Fig. 3, they are well approximated by exponential functions of the form (10). The difference between the theoretical and experimental curves is so small (\( \lesssim 4\% \)), that it makes no sense to give all of them. Therefore, the curves in Fig. 3 were constructed only using formula (10), and experimental values are shown by points.

The detected good agreement between the theoretical and experimental data may be accidental. Therefore, the further verification of the validity of formulas (8)–(10) is an urgent problem to be solved in the future.

In addition, the exponential function adopted above for the thickness of the combustion zone \( \delta_b \tau \) in the presence of turbulence is an intermediate asymptotic relation. This follows from the fact that the exponent \( n \) should be influenced not only by the properties of the propellant but also by the velocity \( w_\infty \). At the same time, as we saw, the quantity \( n \) can be considered a constant parameter over its wide range.

The velocity ratio \( w_\infty/w_s \) can be written in terms of the Vilyunov number by using formula (7) and invoking the equality \( c_p \rho_0 g \approx \lambda_g \) and the relation

\[
x_f - x_s = \frac{w_s}{u} Z,
\]

(11)

where the constant \( Z \) depends on the properties of the propellant:

\[
Z = \frac{\sigma \rho_c}{\rho_0} \left[ \frac{T_b - T_s}{T_s} + \frac{T_0 - L/c_c}{T_s} \ln \frac{Q}{c_c(T_s - T_0) + L} \right],
\]

\[
\sigma = \frac{D \rho^2}{x_c \rho_0^2}
\]

(\( \rho_0 \) is the gas density on the propellant surface and \( D \) is the diffusion coefficient of the chemically reacting component in the gas phase).
On the Threshold Nature of Erosive Burning

The expression for $Z$ is obtained in combustion models [22], in which the chemical reaction zone in the solid and gas phases is assumed to be infinitely thin. A method for its derivation is given in Appendix.

The parameter $\sigma$ has the meaning of the ratio of the relaxation times of the transfer processes in the gas and the heat transfer in the solid phase. It is proportional to the pressure like the density $\rho_0$. Therefore, their ratio does not depend on the pressure.

After simple transformations, we obtain

$$\frac{w_{\infty}}{w_s} = AJ, \quad A = \frac{Z}{32.5} \frac{c_{p\lambda_c}}{c_c \lambda},$$

Then, the reciprocal of the parameter $A$ is nothing but the critical value of the Vilyunov number: $J_s = 1/A$. Uncovering the meaning of $A$ as a function of the parameters of the gas and solid phases by using the representations of $Z$ and $\sigma$ we obtain the approximate expression

$$A = \frac{\text{Le}}{32.5} \left[ \frac{T_b - T_s}{T_s} + \frac{c_L}{c_c} \ln \frac{Q}{c_c(T_s - T_b) + L} \right],$$

where $\text{Le}$ is the Lewis number.

The physical quantities included in the definition of the parameter $A$ depend weakly on the pressure. However, according to equalities (7) and (11), a pressure rise (at a constant velocity $w_{\infty}$) leads to an increase in the threshold velocity

$$w_s = \frac{32.5}{\sqrt{C_f}} \frac{\nu_{st}}{\delta_c} Z^{-1},$$

which means the displacement of the region of developed turbulence from the propellant surface due to an increase in the burning rate.

The ratio $(x_f - x_s)/\delta_g = \Theta$ is a new criterial parameter introduced in [2, 15]. Its critical value, according to (4), is equal to unity. Therefore, the expression for the erosion coefficient at high activation energies $E_c$ in terms of the number $\Theta$ is simplified:

$$\varepsilon = \begin{cases} 1, & \Theta < 1, \\ \Theta^n, & \Theta \geq 1. \end{cases}$$

We approximately calculate the width of the gaseous-phase combustion zone, knowing experimental values of the threshold velocity. From formula (7) we express

$$\varepsilon = \begin{cases} 1, & \Theta < 1, \\ \Theta^n, & \Theta \geq 1. \end{cases}$$

For all propellants listed above (Figs. 2 and 3), we set $\nu_{st} \approx 3.8 \cdot 10^{-3} \text{ m}^2/\text{sec}$ and $C_f \approx 0.03$. As a result, we have:

$$w_s, \text{ m/sec} \quad x_f - x_s, 10^{-4} \text{ m}$$

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These data are within the observed values of the combustion-zone thickness $x_f - x_s$.

The following feature of formula (10) is of interest. If it is written in terms of gas-phase parameters using the relation $m = \rho v$, the closeness of the Prandtl number to unity, and the approximate relation between the turbulent pulsation velocity $w'$ and the average velocity $w_{\infty}$,

$$w' \sim \sqrt{C_f} w_{\infty},$$

then formula (10) becomes

$$u_t \sim v_n \left( \frac{w'}{v_n} \right)^n,$$  \hspace{1cm} \text{(12)}

where $u_t$ is the turbulent flame velocity in the gas.

Expression (12) is known as the asymptotic $(w'/v_n \gg 1)$ Vilyunov–Klimov formula [24, 25]. The exponent $n$ is the difference between the fractial ($d_f$) and topological ($d_t$) dimensions of the burning surface [26]: $n = d_f - d_t$, which gives a geometrical interpretation of the number $n$. Formula (12) is well confirmed in experiments for a wide class of combustible gas mixtures [13, 14]. From this it follows that the results given above can be obtained from formulas (12) using the same reasoning in reverse order.

We note that, using the scheme of solution of the erosive burning problem proposed above, it is possible to obtain all results [2, 15] of numerical modeling that confirm the validity of the representation of the erosive burning rate as of a function of the shearing friction stress $\tau_s$ or the ratio $(x_f - x_s)/\delta_g$ since this ratio is directly related to $\tau_s$.

3. POSSIBLE MECHANISM OF OCCURRENCE OF NEGATIVE EROSION

Below, we consider some issues related to chemical reactions near the contact of the blowing flow with solid surfaces permeable to gases. Gas injection (forced or natural) through this surface is an effective means for producing thermal protective coatings for spacecraft [7]. The mass flux $j_n = \rho v$ of the injected gas along the normal to the surface of the shielding material interacts with the high-temperature flux $j_t = \rho w_{\infty}$ and, for a large value, removes it from the outer surface of the thermal protective coating. The dependence of the
Stanton number (St) on these fluxes can be approximately written as [7]

\[ \frac{St}{St_0} = \left(1 - \frac{b}{b_{cr}}\right)^2, \quad b = \frac{1}{St_0} \frac{j_0}{\nu_t} \]  

(13)

where the proportionality coefficient of the order of unity is dropped; \(St_0\) is the Stanton number on the impermeable surface for identical Reynolds numbers and the same law of velocity variation along the length of the outer surface of the boundary layer at \(T_s = \text{const}\); and \(b_{cr}\) is the critical value \(b (b_{cr} < 10)\).

Erosive burning of condensed materials is largely similar to the burnout of permeable thermal protective coatings: in both cases there is gas injection from the surface of the solid material into the blowing flow. This analogy is also manifested in similarity between the relations of the erosive coefficient for them [4]. Therefore, the effect of decreasing the heat flux from the gas to the burning surface predicted by formula (13) should also be present in the determination of the dependence of the burning rate of high-energy materials on the blowing flow. We make some estimates of the order of magnitude of this effect assuming that the gas phase is the rate controlling one.

In the presence of exact similarity between the momentum and energy transfer, \(St_0 = C_f/2\) and \(b_{cr} \approx 4\) [7]. Using typical values \(C_f \approx 0.03, \omega_\infty \approx 100 \text{ m/sec}\), and \(v \approx 1 \text{ m/sec}\), we obtain \(2b/Cfb_{cr} \approx 0.17\). Thus, the occurrence of positive erosion is characterized by a weak displacement of the boundary layer. The characteristic distance at which the displacement is manifested is \(\nu_d/v \approx 10^{-5} \text{ m}\). Although this value is comparable to the thickness of the gas-phase combustion zone, the heat flux to the propellant surface is determined mainly by the average location of the burning surface \(\delta_{b\tau}\) in the presence of turbulence.

If the blowing velocity is low \((\omega_\infty \lesssim 10 \text{ m/sec})\), the second term in the bracket of formula (13) becomes considerable. Then, one might expect that the effect of boundary-layer displacement dominates over the turbulence effect. In the absence of blowing, the gas velocity component \(v\) normal to the solid-phase surface is due to the phase transformation at the interface and heat expansion. The presence of blowing at velocity \(w\) leads to an increase in the gas outflow velocity by a value \(\Delta v \approx C_f \omega_\infty\), which entails the removal of the flame zone from the propellant surface. In this case, the solid-phase decomposition surface is cooled [3], resulting in a reduction in the burning rate. This can be treated as the onset of negative erosion. This mechanism of burning rate reduction during blowing differs from the mechanisms proposed in [6, 11]. The complex \(2b/Cfb_{cr}\) decreases with increasing initial temperature.

Therefore, in the negative erosion mechanism proposed here, one of the key experimental facts [20] — a reduction in the negative erosion effect due to a decrease in the initial propellant temperature — is explained by the displacement of the boundary layer. In this connection, it should be noted that Bulgakov and Lipanov [6] actually propose two mechanisms for negative erosion, although they do not indicate the key difference between them. According to the first mechanism, the gas-phase flame stretches, which should lead to a reduction in the burning rate. The second mechanism is, in fact, equivalent to that proposed above: because of the displacement of the boundary layer, the flame recedes from the solid-phase surface and the temperature of this surface decreases. This is supported by the results [6] of numerical modeling of turbulent propellant combustion.

From the results obtained here, we propose a semiempirical formula for the erosive coefficient which is valid for both negative and positive erosion. This formula should meet the following requirements:

— at high blowing velocities \((\omega_\infty > \omega_\ast)\), it should have the form of (10);
— contain a parameter depending on which negative erosion could vanish asymptotically;
— assume the values \(\varepsilon (\omega_\infty = 0) = \varepsilon (\omega_\infty = \omega_\ast) = 1\);
— contain the minimum number of free parameters;
— have a simple functional form.

This requirements are satisfied by the expression

\[ \varepsilon = \frac{1 + aB(\omega_\infty/\omega_\ast)^{\alpha+n}}{1 + B(\omega_\infty/\omega_\ast)^{\alpha}}, \]  

(14)

where \(a, B, \) and \(\alpha\) are numerical parameters and \(a \approx 1\). For \(\alpha \to \infty\), this formula leads to discontinuous be-
behavior of the erosive coefficient: \( \varepsilon = 1 \) in the range \( 0 \leq w_\infty < w_s \), and, for \( w_\infty > w_s \), relation (10) is valid. Therefore, the disappearance of negative erosion at low temperatures, and the absence of erosive burning at low blowing velocities in some cases can be associated primarily with this parameter. The constant \( B \) is required for a small correction of the values \( \varepsilon \). Obviously, there is no great need for the parameter \( a \) for small correction of the values \( \varepsilon \) by changing \( B \) between (14) and experimental data can be corrected by changing \( w_s \).

The curve constructed by formula (14) is in satisfactory agreement with the experimental data (Fig. 4) taken from [6] for powder N and recalculated for velocities \( w_\infty \) and \( w_s \).

4. EFFECT OF BLOWING FLOW ON THE UNSTEADY BURNING RATE OF HIGH-ENERGY MATERIALS

In the phenomenological theory of unsteady combustion of powders and other high-energy materials with zero relaxation time for gas-phase processes (\( t_c \)-approximation), is necessary to know the dependence of the surface temperature of the solid phase, the temperature gradient in it, and the burning rate on the blowing velocity. The propellant surface decomposition temperature, as follows from (9) and (10), is equal to

\[
T_{s,w} = \frac{T_s}{1 - 2 n \frac{R T_s}{E_c} \ln \frac{w_\infty}{w_s}}.
\]

The activation energy \( E_c \) is related to the phenomenological coefficients \( k \) and \( r \) by the equality [22]

\[
\frac{E_c}{2 k R T_s} \approx \frac{T_{s0}}{T_{s0} - T_0} k,
\]

where \( T_{s0} \) is the steady-state value of the temperature \( T_s \). Then, we obtain

\[
T_{s,w} = \frac{T_s}{1 - 2 n \frac{T_{s0} - T_0}{T_{s0}} r \ln \frac{w_\infty}{w_s}}.
\]

Taking into account the aforesaid, the right side of the energy conservation equation at the interface

\[
\lambda \frac{\partial T_c}{\partial x} = A_3 \lambda_g \frac{T_{b,w} - T_{s,w}}{\delta_{br}} - L m_w
\]

written for the case of erosive burning, can be rearranged as follows:

\[
A_3 \lambda_g \frac{T_{b,w} - T_{s,w}}{\delta_{br}} - L m_w
\]

\[
= A_1 \frac{\lambda_g}{c_p} \frac{Q}{x_f - x_g} \left( \frac{w_\infty}{w_s} \right)^n - L m_w
\]

The energy conservation condition now takes the same form as in the absence of the erosion effect but with the effective thermal conductivity \( \lambda_{c,w} \) of the solid phase:

\[
\lambda_{c,w} \frac{\partial T_c}{\partial x} = \lambda_g \frac{\partial T}{\partial x} - L \rho_e u, \quad \lambda_{c,w} = \lambda_c \left( \frac{w_s}{w_\infty} \right)^n.
\]

As is obvious, during erosive burning, the heat flux into the depth of the solid phase in the phenomenological theory of unsteady combustion in the \( t_c \)-approximation (zero relaxation time of gas-phase processes) can be taken into account correctly by considering only the effective thermal conductivity of the propellant.

CONCLUSIONS

The above analysis of erosive burning suggest the following explanation for its threshold nature. This type of combustion arises in the case where the (conditional) thickness of the laminar sublayer above the burning propellant surface becomes smaller than the thickness of the combustion zone formed in the absence of the blowing gas flow. In other words, at high blowing velocities, turbulence penetrates into the combustion zone, which, before this, was under laminar conditions. Activation of transfer processes leads to the formation of a turbulent combustion zone of smaller thickness, resulting in increases in the heat flux to the propellant decomposition surface, its temperature and burning rate. The increasing gas flow from the solid-phase surface tends to displace the turbulent flow region and restore the laminar combustion regime.

At low blowing velocities, the effect of boundary-layer displacement becomes significant and can be considered to be responsible for the occurrence of negative erosion.

The results of the present work leads to the following conclusions:

- An increase in the heat flux from the gas flame to the propellant surface is due to a decrease in the thickness of the combustion region due to turbulence;
- The threshold occurs when the laminar-sublayer thickness becomes smaller than the thickness of the combustion zone in the absence of turbulence (or blowing); in this case, a turbulent flame arises in the gas phase;
- The threshold (critical) blowing velocity is proportional to the gas viscosity and is inversely proportional to the product of the thickness of the laminar
combustion zone into the square root of the hydrodynamic resistance coefficient;

- The critical blowing velocity increases with increasing pressure;
- The critical Vilyunov number depends weakly on the pressure;
- The critical Bulgakov–Lipanov number is equal to unity;
- For the erosive burning rate, an exponential function is preferred over a linear function.

APPENDIX

Since all values in the gas phase are determined in the mass Lagrangian coordinate $\xi$ [22, 27], before calculating the coefficient $Z$, we convert to the Euler coordinate $x$. These two coordinates are linked by the relation

$$
\xi = \frac{u}{\sigma \rho c} \int_{x_s}^{x} \rho dx', \quad \frac{\partial \xi}{\partial x} = \frac{u \rho}{\sigma \rho c}.
$$

(A1)

Here the parameter $\sigma \ll 1$ has the meaning of the ratio of the relaxation time of hydrodynamic and thermal-diffusion processes to the thermal relaxation time of the propellant solid phase [22, 27].

The density is determined in the Lagrangian coordinate system and is linked to the gas temperature $T$ by the relation (which follows from the ideal gas law) $\rho = \rho_0 T_s / T = \rho_0 / \theta$, where $\theta = T / T_s$. Substituting it into the second equality from relation (A1) and taking into account that $\theta = \theta_0 - l + (1 - \theta_0 + l) \exp(\xi)$, we perform integration:

$$
\int_{0}^{\xi_i} [\theta_0 - l + (1 - \theta_0 + l) \exp(\xi)] d\xi = \frac{u \rho_0}{\sigma \rho c} (x_f - x_s),
$$

where $\theta_0 = T_0 / T_s$, $l = L / T_s c$, and

$$
\xi_f = \ln \frac{Q}{c} (T_s - T_0) + L
$$

is the position of the leading edge of the flame in the Lagrangian coordinate system [15]. Simple calculations yield formula (11).

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REFERENCES


