Theoretical Model of an Electric Precipitator with Rotating Collecting Electrodes

Kasimova Botakoz Rakhmetollaevna

Abstract—We develop a theoretical model of an electric precipitator with rotating collecting electrodes. Collecting electrodes in the form of a rotating belt can help eliminate negative effects on the work of the precipitator, such as electric resistivity and the re-entrainment. We calculate the optimal speed of the rotating belt.

Keywords—Electric precipitator, corona charge, cleaning of stack gas, collecting electrode, wire plate

I. Introduction

Electrically purifying gases of suspended particles (such as dust, mist or smoke) is based on the following principle: ionizing the gas molecules with an electric discharge also charges particles contained in the gas, and then an electric field deposits these particles on the electrodes, thereby removing them from the gas stream.

II. Theoretical Model of an Electric Precipitator with Rotating Collecting Electrodes

To create an electric field that can cause a corona discharge between the electrodes, the latter must be connected to a direct current power supply of high voltage. The minimum voltage $U_0$ across the electrodes that produces the corona discharge is called the critical voltage or critical potential difference [1] and is given by (1):

$$ U_0 = E_0 r \left( \frac{nH}{d} - \ln \left( \frac{2mr}{d} \right) \right) $$

where $H$ is the distance from the wire to the collecting plate, $d$ is the particle diameter (m), $r$ is the wire plate radius, and $E_0$ is the critical field strength (the value at which the corona arises) defined by

$$ E_0 = 31\delta (1 + \frac{0.308}{\sqrt{\delta \cdot r}}) $$

In an environment close to atmospheric pressure, $\delta$ for a negative corona and round wire can be taken as

$$ \delta = \frac{3.92B}{273 + t}, $$

where $B$ is the barometric pressure, and $t$ is the temperature (°C).

During electric precipitation, problems arise such as re-entrainment, back corona, and resistivity. To solve these problems, we propose electrodes in the form of continuously rotating belts based on the shafts separated by a partition into two parts (one part connected to the gas duct and the other to a brush unit) installed on the bottom wall with the possibility of contact with the collecting electrodes, and a hopper for receiving dust [2].

The essence of the proposed electric precipitator is that the collecting electrodes have the form of a continuously rotating belt based on the shafts. Particles in gas passing through the wire plate electrode acquire an electrostatic charge and are deposited at the top of the collecting electrode. At the bottom of the wall a mounted brush cleans the collecting electrodes of solid dust particles, which then fall into a hopper.

Theoretical analysis using electric gas dynamics shows that the effectiveness of trapping and deposition of dust particles is most affected by the Coulomb force $F_{cf}$, aerodynamic force $F_a$, force of gravity $F_g$, strength of induction $F_i$, resistance force $F_c$ and to some degree the electric wind strength $F_{e.w.}$. We use all these forces to determine the velocity of dust particles and their time in the core of electric precipitator (Fig. 1) [3].

![Image](https://example.com/image)

Figure 1. Working of an electric precipitator.

In general, the corona process depends on the electrostatic voltage and current. To reduce the influence of electrical resistivity and re-entrainment, the belt should move at the speed at which the particles are removed from the electrode surface, given by
\[ \nu_1 = \frac{L}{16.7 \cdot S_n m_\eta / V z_m \eta}. \] (4)

where \( \nu_1 \) is the speed of the rotating band, \( L \) is the length of the precipitator, \( S_n \) is the deposition area of the field (m²), \( V \) is the amount of gas entering the field (m³/c), \( z_{int} \) is the dust at the entrance to the field (gr/m³), and \( \eta \) is the degree of gas cleaning electrostatic field.

In general, the motion of the particles depends on the gravity, electrostatic forces, and the aerodynamic forces because of interaction between the gas and particles along their trajectories. The aerodynamic forces can be represented as [4]:

\[
\begin{align*}
\frac{d\vec{u}_p}{dt} &= \vec{F}_D(\vec{u}_i - \vec{u}_p) + \frac{\vec{g}_i(\rho_p - \rho_i)}{\rho_p} + \vec{F}_s, \\
\frac{dx_i}{dt} &= u_i, i = x, y, z
\end{align*}
\] (5)

where \( \rho_p \) is the particle density, \( \vec{u}_p \) is the particle velocity, and \( \vec{g}_i \) is the intensity of gravity acting in the vertical direction. \( \vec{F}_s \) refers to the acceleration experienced by the particle from the electrostatic force:

\[ F_s = \frac{Eq}{m_p}. \] (6)

Where \( q \) and \( m_p \) are the electric charge and mass of the particle, respectively. \( \vec{F}_D(\vec{u}_i - \vec{u}_p) \) is the drag force per unit mass of particles corresponding to the relative velocity of the particle. Stokes' law of resistance for super-fine particles is:

\[ F_D = \frac{18 \mu}{\rho_p d_p^2} C_c(\lambda) \] (7)

where \( \vec{u} \) is the settling velocity (the velocity of the particle), \( \mu \) is the viscosity of air, \( \rho_p \) is the density of particles, \( d_p \) is the particle diameter, and \( C_c \) is the Cunningham correction factor. For dry air at atmospheric pressure, \( C_c \) is calculated as

\[ C_c(\lambda) = 1 + K_n \left[ 1.257 + 0.4 \exp \left( \frac{1.1}{K_n} \right) \right] \] (8)

with

\[ K_n = \frac{2 \lambda}{d_p} \] (9)

where \( K_n \) is the Knudsen number, and \( \lambda \) is the molecular mean free path (the average distance traveled by a gas molecule between collisions with other molecules). The molecular mean free path is defined in terms of Boltzmann’s constant \( k_B = 1.38 \times 10^{-23} \) J/K and the gas accommodation coefficient \( \sigma \) as

\[ \lambda = \frac{k_B T}{\sqrt{2 \pi \sigma^2 P}} \] (10)

The residence time of gas in the electrostatic precipitator is usually less than 10 seconds. During this time, a speck of dust, for example, less than 20 microns in diameter will fall only a few centimeters under the influence of gravity. Thus the effect of this force on the particle motion in the electrostatic precipitator can be ignored. We neglect the gravity and the Brown strength, while the electric force and air resistance are considered the two main forces acting on the particle.

This paper assumes that the particles deposited on the plate are removed before they can be discharged (because the plate is made as a continuously moving belt).

### TABLE I.

<table>
<thead>
<tr>
<th>Designation</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>Temperature</td>
<td>150T</td>
</tr>
<tr>
<td>N</td>
<td>The number of elementary charges in the 1m³</td>
<td>108</td>
</tr>
<tr>
<td>e</td>
<td>The electronic charge</td>
<td>1.6 \times 10^{-19}</td>
</tr>
<tr>
<td>r</td>
<td>The radius of the discharge electrode (C)</td>
<td>1.25 \times 10^{-3}</td>
</tr>
<tr>
<td>L</td>
<td>The length of the plate</td>
<td>10m</td>
</tr>
<tr>
<td>( \nu_\bar{a} )</td>
<td>Velocity of the gas</td>
<td>0.8m/sec</td>
</tr>
<tr>
<td>H</td>
<td>The distance between the corona and collecting electrodes</td>
<td>0.14m</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Viscosity of air (sm/sec)</td>
<td>2</td>
</tr>
</tbody>
</table>

Using the data in Table 1, we obtain

\[ \lambda = 7.7 \times 10^{-6}, \]
\[ K_n = 0.15, \]
\[ C_c = 1 + 0.15 \left[ 1.257 + 0.4 \exp \left( -\frac{1.1}{0.15} \right) \right] = 1.18, \]
\[ \frac{d\nu}{dt} = \frac{qE}{m} + 30.5 \cdot 10^7 \nu, \]
\[ q = 3 \pi l_r^2 \varepsilon_0 E, \]

The electric field for the plate at any point \( x \) is calculated as follows:

\[ E = \frac{U}{x}, \]
\[ \frac{d\nu}{dt} = \frac{qE}{m} + 30.5 \cdot 10^7 \nu, \quad (11) \]
\[ \frac{dx}{dt} = \nu, \]

or

\[ \ddot{x} = \frac{U^2}{x} \cdot 27.79 \cdot 10^{-24} + 30.5 \dot{x} \quad (12) \]

Using the Cauchy equation,

\[ \begin{cases} x_1 = x, \\ x_2 = \dot{x}. \end{cases} \quad (13) \]

the equation becomes:

\[ \begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= \frac{U^2}{x_1} \cdot 27.79 \cdot 10^{-24} + 30.5 x_2.
\end{align*} \quad (14) \]

Now we linearize the functions \( U^2, x_1^2 \) using the Taylor series expansion:

\[ f(x_0) - \hat{f}(x_0)(x - x_0) \quad (15) \]

where \( x_0 \) is the initial state. A critical voltage is calculated as follows:

\[ U_0 = E_o r(\pi H / d - \ln 2 \pi r / d) \quad (16) \]

Substituting the values we find:

\[ E_o = 683 \cdot 10^3 \quad (17) \]

and

\[ \frac{1}{x^2} = \frac{1}{x^2} + \frac{2}{x^3} (x - x_0) = 10^6 x - 0.6 \cdot 10^6 \quad (18) \]

We transform our equation:

\[ \begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= \frac{U^2}{x_1} \cdot 27.79 \cdot 10^{-24} + 30.5 x_2 = \\
&= 0.8 \cdot 10^{-9} x_1 - 0.5 \cdot 10^{-12} - 0.3 \cdot 10^{-9} U x_1 + \\
&+ 0.2 \cdot 10^{-12} U + 30.5 x_2.
\end{align*} \]

In general, the state equations have the forms:

\[ \dot{x} = Ax + Bu \]
\[ y = Cx + Du \quad (19) \]

At the output we observe the effectiveness of air purification. The effectiveness of air purification is given by

\[ \eta = 1 - e^{-\omega t}, \]
\[ f = \frac{L}{vH}, \quad (20) \]

where \( L \) is the length of the precipitator, \( v \) the gas flow rate, \( H \) the distance between the corona and collecting electrodes, and \( \omega \) the velocity of the particles given by

\[ \frac{dx}{dt} = \omega, \quad (21) \]

We substitute \( \omega = x_2 \). We expand \( e^{-x_2 t} \) in a Fourier series. The initial velocity is \( x_{20} = 0 \):
\[ e^{-x_2} = e^{-x_2f} + f e^{-x_2f} (x_2 - x_{20}) \mid_{x_{20}=0} = 1 + fx_2 \]

Now we substitute the value:

\[
\eta = 1 - (1 + fx_2) = fx_2,
\]

\[
f = \frac{L}{\nu H} = \frac{10}{0.8 \cdot 1.13} = 83
\]

As a result, our system is described by a transfer function in matrix form:

\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{pmatrix} =
\begin{pmatrix}
0 & 1 & 0 \\
0.8 \cdot 10^{-9} & 30.5 & 0.2 \cdot 10^{-12}
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix} + (0.53 \cdot 10^{-12}) \cdot u
\]

\[ y = \begin{pmatrix} 0 & 83 \end{pmatrix} \begin{pmatrix} x_1 \\ u \end{pmatrix} \]

We have shown that the greatest influences on the efficiency of an electrostatic precipitator with rotating electrodes are the voltage at the discharge electrode, electric field, and velocity of the particles from the corona electrode to the precipitation. Using a continually rotating precipitation electrode belt reduces the negative effects of re-entrainment and electrical resistivity. The derived mathematical model can be used for further studies of electric precipitators in various modes.

References


About Author (s):

Botagos Kasimova
Gumilov Eurasian National University
Kazakhstan