ALASHBAYEVA N.M., KERIMKULOV S.E.


M1 includes funds that are readily accessible for spending. It is known that M1 consists of: (1) notes and coins in circulation outside the U.S. Treasury, Federal Reserve Banks, as well as the vaults of depository institutions; (2) traveler's checks of nonbank issuers; (3) demand deposits; and (4) other checkable deposits (OCDs), which consist primarily of negotiable order of withdrawal (NOW) accounts at depository institutions and credit union share draft accounts. The basis for the measuring and modeling of chaotic and cyclic movement of M1 Money Stock for the U.S. can be information from sources: U.S. M1 Money Stock, Billions of Dollars, 1975-2012, Weekly (Federal Reserve Economic Data 2012).

Money stock of the USA accumulates the country's finances, both in real and financial sector of the economy. From the systematic approach the money stock represents a system of complex, dynamic, nonlinear and stochastic objects that make high demands on its research tools. Today, the basic research tools of money stock been established and their broad overviews are given in the works of Bernanke (1998), Laubach (2003), Nelson (2003), Peters (2000), Egorova (2011), etc.

1. Indicators, Data and Models

For the measuring and modeling of chaotic and cyclic movement of the indices of system of reserve money, in particular, for the U.S. M1 Money Stock, will be used the following well-known indicators.

Indicator moving average of time series $y_t$ of the $l$ time periods is:

$$MA(l)y_t = \frac{1}{l+1} \sum_{i=0}^{l} y_{t-i}, l = 0, \pm 1, \pm 2, ..., l = 0, 1, 2, .....$$

(1)

where time series is

$$\{y_t | t = 0, \pm 1, \pm 2, ...\}$$

(2)

Indicator moving standard deviation of time series $y_t$ of the $l$ time periods is:

$$MS(l)y_t = \sqrt{\frac{1}{l+1} \sum_{i=0}^{l} (y_{t-i} - MA(l)y_t)^2}, l = 0, \pm 1, \pm 2, l = 0, 1, 2, .....$$

(3)

Also, in this paper some few definitions of indicators were introduced for the first time – time periods and average with a constant measure of scattering for the measuring and modeling of chaotic and...
cyclic movement indicators of money market. Where here and throughout the paper we denote by $h$ the parameter of measure scattering, $0 < h \leq h_0$, $h_0$ – fixed positive number.

Indicator of time periods with a constant measure of scattering of the time series $y_t$ is:

$$l^*_t(h) = \min_{i=0,1,2,...} \left\{ i \mid h_{MS}(l) y_t \geq 1 \right\}, t = 0, \pm 1, \pm 2, ..., (4)$$

Indicator of average with a constant measure of scattering of the time series $y_t$ is:

$$A(h, y_t) = MA(l^*_t(h)) y_t, t = 0, \pm 1, \pm 2, ..., (5)$$

where $l^*_t(h)$ – time periods with a constant measure of scattering of the time series $y_t$ in accordance with (4).

For the measurement and modeling of chaotic and cyclic movements of the U.S. M1 Money Stock used information:

—source of primary information:

—source of derivative information:
  – Average with a constant measure of scattering (5) with parameters $h_1 = 0.00192$ (Fibonacci level), $h_2 = 0.000280$ (Fibonacci level) and $h_3 = 0.0000253$ (Fibonacci level) of the U.S. M1 Money Stock, Billions of Dollars, 1975-2012, Weekly (see Figure 1).
  – Average with a constant measure of scattering (5) with parameters $h_1 = 0.00192$, $h_2 = 0.000280$ and $h_3 = 0.0000253$ of Change from Year Ago of the U.S. M1 Money Stock, Billions of Dollars, 1975-2012, Weekly (see Figure 2).

For the measurement and modeling of chaotic and cyclic movements of the U.S. M1 Money Stock will be used econometric models with the following specification of the regression for the time series $y_t$:

$$y_t = \beta_0 + \beta_1 A(h_1, y_t) + \cdots + \beta_j A(h_j, y_t) + \varepsilon_t, t = 0, \pm 1, \pm 2, ..., 0 < h_j \leq h_0, j = 1,2, ..., (6)$$
where $A(h_j, y_t)$ – factor variables of the model, i.e. indicators of the average with a constant of measure scattering for time series $y_t$; $\beta_j$ – the unknown parameters; $h_j$ – a given parameters of measures scattering of factor variables $A(h_j, y_t)$, $j = 1, 2, \ldots$; $h_0$ – a fixed positive number; $\varepsilon_t$ – a random errors such that for all $t, s = 0, \pm 1, \pm 2, \ldots$ and $t \neq s$ satisfy the following conditions:

$$
\begin{align*}
E[\varepsilon_t | \mathbf{X}] &= 0, \\
Var[\varepsilon_t | \mathbf{X}] &= \sigma^2, \\
Cov[\varepsilon_t, \varepsilon_s | \mathbf{X}] &= 0, \\
\varepsilon_t | \mathbf{X} &\sim N[0, \sigma^2 \mathbf{I}] ;
\end{align*}
$$

(7)

where $\mathbf{X}$ – a matrix of the observations compiled by of $A(h_j, y_t)$; $E[\cdot]$ – a expectation; $Cov[\cdot]$ – a covariance; $Var[\cdot]$ – a variance; $\mathbf{I}$ - identity matrix.


The choice of the structure of chaotic and cyclic movement and the identification of their characteristics allow using the category of deep chaos theory for solving many applied problems of system of reserve money, for example, measuring and modeling of chaotic and cyclic movements of the U.S. M1 Money Stock.

In particular, the choice of attractor of the U.S. M1 Money Stock – geometric structure that characterizes the behavior of participants' of money market in the phase space by sufficiently long period of time, that means an abstract space, which is represented by the set of all states of the system, so that each possible state of the system corresponds to the point of phase space whose coordinates are the degrees of freedom system.

In this case, the movement of the U.S. M1 Money Stock has two degrees of freedom. For example, in the case of M1 Money Stock: time series is the movement which is completely possible to determine with the initial moment of time and initial state of the volume, and in the case of the phase portrait – the initial state and change from year ago of volume of the U.S. M1 Money Stock.

At choice of measures of scattering (Fibonacci level) equal $h_1 = 0.00192$ (Fibonacci level), $h_2 = 0.000280$ (Fibonacci level), and $h_3 = 0.0000253$ (Fibonacci level) geometric illustration of the chaotic attractor of the U.S. M1 Money Stock as average with a constant measure of scattering are shown in Figure 1.
It should be noted that the chaotic attractors of the U.S. M1 Money Stock identify the creation of the following trends, uncertainties and cyclic movements, i.e. by $h_2 = 0.000280$ was obtained (see Figure 1):

– growth trend of the U.S. M1 Money Stock on the 560 billions of dollars (from the beginning of 1975 to mid 1990) with the transition to the zone of excess emissions of the monetary aggregate M1 Money Stock (from the beginning of 1987 to mid 1990) that during this period formed the uncertainty of the market money and U.S. economy led to a recession in 1990-1991;

– after the recession in 1990-1991 years again the volume of the U.S. M1 Money Stock had gained the growth trend on the 400 billions of dollars to the beginning of 1994, further a zone of the excess emission of M1 Money stock had been dominating up to the beginning of 2001, i.e. during the period from the beginning of 1994 to early 2001 there had been a high uncertainty in the money market, which ended by the U.S. economy recession (2001);

– after the recession in 2001 year again the volume of the U.S. M1 Money Stock had gained the growth trend up to 400 billions of dollars to the beginning of 2005, further there had been a dominant zone of the excess emission of M1 Money Stock to the beginning of 2008, i.e. during the period from the beginning of 2005 to early 2008 there had been a high uncertainty in the money market, which ended by the U.S. economy recession (2008-2009);
after the recession in 2008-2009 years there was formed only the growth phase of cyclic movements of the U.S. M1 Money Stock, more than two-fold volume, i.e. 900 billions of dollars, but the zone of excess emissions of the M1 Money Stock had not yet been formed, which is expected presence of this phase reconciling 3-4 years.


As a result of permanent divergence and convergence of the chaotic attractor a rapidly growing trend of uncertainty and cyclical movement of the U.S. M1 Money Stock, i.e. with the expiration of each point in time lose the ability to make accurate predictions. However, to assess trends in the differences, convergence, the initial state of monetary aggregate M1 Money Stock and they can be a measure of chaos, i.e. numerical expression of how the system is chaotic.

It also should be mentioned that the reason of the chaotic tendencies is the differences in initial conditions and time of the U.S. M1 Money Stock. Even a microscopic deviation of two trends of the monetary aggregate M1 at the first time the process of movement can lead to an exponential accumulation of errors and their respective stochastic divergence.

Indeed, the divergence and convergence of the chaotic attractor of the initial conditions of time and state of the U.S. M1 Money Stock is systematically replaced by new ones. Thus, when the descent trajectory becomes closer and begins to show the effect of short-sightedness – increases the uncertainty of large-scale information. And in the divergence of the trajectories is on the contrary, they diverge, the effect of farsightedness develops, when uncertainty of the small-scale information increases.

Thus, in order to estimate the parameters of the chaotic tendencies and the initial state of the U.S. M1 Money Stock, depending on the factor variables as average with a constant measure of scattering (5), respectively, parameters are: \( h_1 =0.00192, \ h_2 =0.000280, \ h_3 =0.0000253 \) an econometric model (6)–(7) will be used.

Table 1. Estimate of parameters of the trends and the initial state of the U.S. M1 Money Stock through the aid of average with a constant measure of scattering (5), respectively, with parameters \( h_1 =0.00192, \ h_2 =0.000280, \) and \( h_3 =0.0000253 \)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average M1</td>
<td>0.9979***</td>
<td>1.3132***</td>
<td>1.3182***</td>
<td>1.3182***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>with ( h_1 =0.00192 )</td>
<td>(0.001)</td>
<td>(0.024)</td>
<td>(0.033)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average M1 with ( h_2 = 0.000280 )</td>
<td>1.0076*** (0.003)</td>
<td>-0.3200*** (0.024)</td>
<td>1.3789*** (0.019)</td>
<td>-0.3291*** (0.044)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average M1 with ( h_3 = 0.0000253 )</td>
<td>1.0425*** (0.005)</td>
<td>-0.3880*** (0.020)</td>
<td>2.8150 (3.950)</td>
<td>8.8688*** (2.705)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>12.7218*** (1.486)</td>
<td>32.4951*** (2.626)</td>
<td>118.1226*** (7.933)</td>
<td>7.9394*** (1.656)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of observations</td>
<td>1 867</td>
<td>1 759</td>
<td>1 431</td>
<td>1 431</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R–square</td>
<td>0.9963</td>
<td>0.9893</td>
<td>0.9273</td>
<td>0.9960</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note.** The dependent variable – U.S. M1 Money Stock, 1975-2012, Weekly. In parentheses are standard errors. *, **, *** – estimate is significant at 10%, 5%, 1% level.

Thereby, ceteris paribus weekly movements of the U.S. M1 Money Stock during 1975-2012 years have the following estimates (see Table 1):

- on the basis of \( A(h_1, y_t) \) (average of M1 with \( h_1 = 0.00192 \)):
  - trend: \( \beta_1 = 0.9979 \);
  - initial state: \( \beta_0 = 12.7218 \);
- on the basis of \( A(h_2, y_t) \) (average of M1 with \( h_2 = 0.000280 \)):
  - trend: \( \beta_2 = 1.0076 \);
  - initial state: \( \beta_0 = 32.4951 \);
- on the basis of \( A(h_3, y_t) \) (average of M1 with \( h_3 = 0.0000253 \)):
  - trend: \( \beta_3 = 1.0425 \);
  - initial state: \( \beta_0 = 118.1226 \);
- on the grounds of \( A(h_1, y_t) \) and \( A(h_2, y_t) \):
  - trends, respectively: \( \beta_1 = 1.3132 \) and \( \beta_2 = -0.3200 \);
  - initial state: \( \beta_0 = 7.9394 \);
- on the grounds of \( A(h_1, y_t) \) and \( A(h_3, y_t) \):
  - trends, respectively: \( \beta_1 = 1.0855 \) and \( \beta_3 = -0.0941 \);
  - initial state: \( \beta_0 = 5.4907 \);
- on the grounds of \( A(h_2, y_t) \) and \( A(h_3, y_t) \):
  - trends, respectively: \( \beta_2 = 1.3789 \) and \( \beta_3 = -0.3880 \);
  - initial state: \( \beta_0 = 2.8150 \);
- on the basis of \( A(h_1, y_t) \), \( A(h_2, y_t) \) and \( A(h_3, y_t) \):
  - trends, respectively: \( \beta_1 = 1.3182 \), \( \beta_2 = -0.3291 \) and \( \beta_3 = 0.0037 \);
  - initial state: \( \beta_0 = 8.8688 \).


Chaotic attractors of change to the corresponding period from year ago (more strength) of the U.S. M1 Money Stock identifies creation of the following trends, uncertainties and cyclic movements, i.e. by \( h_2 = 0.000280 \) was obtained (see Figure 2):

- growth trend of strength of the U.S. M1 Money Stock on the 80 billions of dollars (from the beginning of 1975 to mid 1987) with the transition to trend down on the 100 billions of dollars (from mid 1987 to mid 1990) that during this period formed the uncertainty of the market money and U.S. economy led to a recession in 1990-1991;
- after the recession in 1990-1991 years again of strength of the U.S. M1 Money Stock is gaining upward trend on the 100 billions of
dollars to the beginning of 1994, then dominated by trends in the reduction on the 150 billions of dollars to the middle of 1997;

–from the middle of 1997 to the beginning of 2004 of strength of the U.S. M1 Money Stock is gained growth trends on the 130 billions of dollars, it should be noted that the U.S. economy experienced a recession (2001). During the period from early 2004 to early 2008 there is a downward trend, which ended in the U.S. by economy recession (2008-2009);

–after the recession 2008-2009 years it only the growth phase of cyclic movements of strength of the U.S. M1 Money Stock was formed, more than two-fold volume, i.e. on the 370 billions of dollars, but still the downward trend had not been formed of strength of M1 Money Stock that are expected presence of this phase reconciling 3-4 years.


As in the fifth section for the estimation of parameters of chaotic trends and initial state of change from year ago of the U.S. M1 Money Stock in depend ingot variables factors, average with a constant measure of scattering (5), respectively, parameters are: $h_1 = 0.00192$, $h_2 = 0.000280$, $h_3 = 0.0000253$ an econometric model (6)–(7) will be used.

**Table 2.** Estimate of parameters of the trends and the initial state of Change from Year Ago (strength) of the U.S. M1 Money Stock through the aid of average with
a constant measure of scattering (5), respectively, with parameters $h_1 = 0.00192$, $h_2 = 0.000280$, and $h_3 = 0.0000253$

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average of Change of M1 with $h_1$</td>
<td>1.0255***</td>
<td>1.2487***</td>
<td>0.9993***</td>
<td>1.1799***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.018)</td>
<td>(0.015)</td>
<td>(0.025)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average of Change of M1 with $h_2$</td>
<td></td>
<td>-0.3209***</td>
<td></td>
<td></td>
<td>-0.2773***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.022)</td>
<td></td>
<td></td>
<td>(0.032)</td>
<td></td>
</tr>
<tr>
<td>Average of Change of M1 with $h_3$</td>
<td></td>
<td>4.7744***</td>
<td>0.2421***</td>
<td>0.4831***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.223)</td>
<td>(0.108)</td>
<td>(0.107)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.2284</td>
<td>1.3627</td>
<td>-124.9490***</td>
<td>-5.5307*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.647)</td>
<td>(1.612)</td>
<td>(9.031)</td>
<td>(3.869)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-11.2216***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(3.756)</td>
<td></td>
</tr>
<tr>
<td>Number of observations</td>
<td>1761</td>
<td>1489</td>
<td>786</td>
<td>786</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-square</td>
<td>0.8965</td>
<td>0.5995</td>
<td>0.3691</td>
<td>0.9083</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9163</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note.** The dependent variable – U.S. M1 Money Stock, Change from Year Ago, 1975-2012, Weekly. In parentheses are standard errors. *, **, *** - estimate is significant at 10%, 5%, 1% level.

Thus, other things being equal, weekly traffic of change from year ago (more strength) of the U.S. M1 Money Stock during 1975-2012 have the following estimates (see Table 2):

– on the basis of $A(h_1, y_t)$ (average of change from year ago of M1 with $h_1 = 0.00192$):
  – trend: $\beta_1 = 1.0255$;
  – initial state: $\beta_0 = 0.2284$;

– on the basis of $A(h_2, y_t)$ (average of change from year ago of M1 with $h_2 = 0.000280$):
  – trend: $\beta_2 = 1.0668$;
  – initial state: $\beta_0 = 1.3627$;

– on the basis of $A(h_3, y_t)$ (average of change from year ago of M1 with $h_3 = 0.0000253$):
  – trend: $\beta_3 = 4.7744$;
  – initial state: $\beta_0 = -124.9490$;

– on the grounds of $A(h_1, y_t)$ and $A(h_2, y_t)$:
  – trends, respectively: $\beta_1 = 1.2487$ and $\beta_2 = -0.3209$;
  – initial state: $\beta_0 = 3.9444$;

– on the grounds of $A(h_1, y_t)$ and $A(h_3, y_t)$:
  – trends, respectively: $\beta_1 = 0.9993$ and $\beta_3 = 0.2421$;
  – initial state: $\beta_0 = -5.5307$;

– on the grounds of $A(h_2, y_t)$ and $A(h_3, y_t)$:
  – trends, respectively: $\beta_2 = 0.9673$ and $\beta_3 = 1.0760$;
  – initial state: $\beta_0 = -29.7945$;

– on the basis of $A(h_1, y_t)$, $A(h_2, y_t)$ and $A(h_3, y_t)$:
  – trends, respectively: $\beta_1 = 1.1799$, $\beta_2 = -0.2773$ and $\beta_3 = 0.4831$;
  – initial state: $\beta_0 = -11.2216$. 

The choice of chaotic attractor of the U.S. M1 Money Stock: Phase portrait – a geometric structure consisting of all states as the degree of freedom system, whose coordinates are determined by the initial state and change from year ago of volume of the U.S. M1 Money Stock.

Chaotic attractor of the U.S. M1 Money Stock: Phase portrait as average with a constant measure of scattering (5) with a parameter (Fibonacci level), for example by \( h_2 = 0.000280 \) has geometric illustration in Figure 3, and identifies creation of the following trends, uncertainties and cyclic movements was obtained (see Figure 3.):

– divergence trends around the triangular area A, which was formed as a result of chaotic change of the U.S. M1 Money Stock in the intervals 250-800 billions of dollars and strength in the intervals (–)10-120 billions of dollars;

– chaotic closed curve (the zone around the point B), which was formed as a result of chaotic change of the U.S. M1 Money Stock in the intervals 950-1 200 billions of dollars and strength in the intervals (–)70-40 billions of dollars;

– convergence trends to around the triangular zone C, which was formed as a result of chaotic change of the U.S. M1 Money Stock in the intervals 950-1 500 billions of dollars and strength in the intervals (–)70-135 billions of dollars;

– convergence trends to around the triangular area D, which was formed as a result of chaotic change of the U.S. M1 Money Stock in the intervals 1 350-2 100 billions of dollars and strength in the intervals (–)30-320 billions of dollars.
The measuring and modeling of chaotic and cyclic movement of the U.S. M1 Money Stock confirms the applicability of an indicator of the average with a constant measure of scattering, for example in the study of applied problems in the system of money reserve. In particular, the use of this indicator of the average with a constant measure of scattering allows taking effective management decisions and provides meaningful interpretable quantitative assessment of the activities and management of money reserve system of the United States.

There was given an estimation of the parameters of the trends and the initial state of an econometric model of the U.S. M1 Money Stock depending on the parameter measure of scattering (Fibonacci level). Chaotic attractors weekly cyclic movements of the U.S. M1 Money Stock were constructed and its change from year ago, both in the form of time series, and the phase portrait, as well as trends in stochastic divergence and convergence, and other uncertainty characteristics.

Indeed, for chaotic attractors cyclic movements of the U.S. M1 Money Stock were observed and estimated three full-cycle trends of divergence and level convergence (see Figure 1). For the fourth cycle movements were defined only trend of divergence, but levels of convergence still not defined.

In the case, for chaotic attractor’s cyclic movements of change from year ago of the U.S. M1 Money Stock were observed and estimated for one cycle and were estimated trends and level convergence (see Figure
2). For a cycle movements that began in 2008 were identified only trend of divergence, but levels of convergence still not defined.

In conclusion, for the phase portrait of chaotic attractor’s cyclic movements and change from year ago of the U.S. M1 Money Stock were observed and estimated by two full-cycle trends and their corresponding levels of convergence (see Figure 3). For the third cycle, only the breakdowns of trends were identified divergence, but levels of convergence still not defined.

References