S. E. KERIMKULOV, B. A. ZHUMATAEVA


Introduction. The U.S. Census Bureau produces the Annual Revision of Monthly Retail and Food Services to provide national estimates by kind of business of annual and monthly sales for establishments classified in the retail trade and food services industries. Estimates of end-of-month inventories, inventory-to-sales ratios, annual purchases, gross margin, gross margin as a percent of sales, end-of-year accounts receivable, and per capita sales are also provided, but only for retail trade. The basis for the measuring and modeling of chaotic and cyclic movement of Retail and Food Services Sales for the U.S. can be information from sources: U.S. Retail and Food Services Sales (2012), Millions of Dollars, 1992-2012, Monthly.

Retail and Food Services Sales of the USA accumulates the country's business and finances, both in real and financial sector of the economy. From the systematic approach the retail sales represents a system of complex, dynamic, nonlinear and stochastic objects that make high demands on its research tools. Today, the basic research tools of retail sales been established and their broad overviews are given in the works of Del (2008), Peters (2000), Stock (2002a, 2002b), Peters (2000), Egorova (2011), etc.

1. Indicators, Data and Models. For the measuring and modeling of chaotic and cyclic movement of the indices of system of retail sales, in particular, for the U.S. Retail and Food Services Sales, will be used the following well-known indicators.

Indicator moving average of time series \( y_t \) of the \( l \) time periods is:

\[
MA(l)y_t = \frac{1}{2l + 1} \sum_{i=t-l}^{t+l} y_i, \; t = 0, \pm 1, \pm 2, \ldots, l = 1, 2, \ldots, \tag{1}
\]

where time series is

\[
\{y_t | t = 0, \pm 1, \pm 2, \ldots \}. \tag{2}
\]

Indicator moving standard deviation of time series \( y_t \) of the \( l \) time periods is:
\[ MS(l)y_t = \sqrt{\frac{1}{2l} \sum_{i=t-l}^{t+l} (y_i - MA(l)y_t)^2}, \quad t = 0, \pm 1, \pm 2, \ldots, l \] 

(3)

Also, in this paper some few definitions of indicators were introduced for the first time – time periods and average with a constant measure of scattering for the measuring and modeling of chaotic and cyclic movement indicators of retail sales. Where here and throughout the paper we denote by \( h \) the parameter of measure scattering, \( 0 < h \leq h_0 \), \( h_0 \) – fixed positive number.

Indicator of time periods with a constant measure of scattering of the time series \( y_t \) is:

\[ l_t^*(h) = \min_{l=0,1,2,\ldots} \{l | hlMS(l)y_t \geq 1\}, \quad t = 0, \pm 1, \pm 2, \ldots \] 

(4)

Indicator of average with a constant measure of scattering of the time series \( y_t \) is:

\[ A(h, y_t) = MA(l_t^*(h))y_t, \quad t = 0, \pm 1, \pm 2, \ldots \] 

(5)

where \( l_t^*(h) \) – time periods with a constant measure of scattering of the time series \( y_t \) in accordance with (4).

For the measurement and modeling of chaotic and cyclic movements of the U.S. Retail and Food Services Sales used information:

–source of primary information:

–source of derivative information:
  –Average with a constant measure of scattering (5) with parameters \( h_1 = 0.0000253 \) (Fibonacci level), \( h_2 = 0.00000228 \) (Fibonacci level) and \( h_3 = 0.000000870 \) (Fibonacci level) of the U.S. Retail
and Food Services Sales, Millions of Dollars, 1992-2012, Monthly (see Figure 1).

Average with a constant measure of scattering (5) with parameters $h_1 = 0.0000253$, $h_2 = 0.00000228$ and $h_3 = 0.000000870$ of Change from Year Ago of the U.S. Retail and Food Services Sales, Millions of Dollars, 1992-2012, Monthly (see Figure 2).

For the measurement and modeling of chaotic and cyclic movements of the U.S. Retail and Food Services Sales will be used econometric models with the following specification of the regression for the time series $y_t$:

$$y_t = \beta_0 + \beta_1 \times \begin{bmatrix} A(h_1, y_t) \\ A(h_1, \Delta y_t) \end{bmatrix} + \cdots + \beta_j \times \begin{bmatrix} A(h_j, y_t) \\ A(h_j, \Delta y_t) \end{bmatrix} + \epsilon_t,$$

$$t = 0, \pm 1, \pm 2, ..., 0 < h_j \leq h_0, j = 1,2, ...,$$

(6)

where $\begin{bmatrix} A(h_j, y_t) \\ A(h_j, \Delta y_t) \end{bmatrix}$ – factor variables of the model, i.e. indicators of the average with a constant of measure scattering for time series $\begin{bmatrix} y_t \\ \Delta y_t \end{bmatrix}$; $\beta_j$ – the unknown parameters; $h_j$ – a given parameters of measures scattering of factor variables $\begin{bmatrix} A(h_j, y_t) \\ A(h_j, \Delta y_t) \end{bmatrix}, j = 1,2, ...; h_0$ – a fixed positive number; $\epsilon_t$ – a random errors such that for all $t, s = 0, \pm 1, \pm 2, ...$ и $t \neq s$ satisfy the following conditions:

$$E[\epsilon_t | \mathbf{X}] = 0,$$
$$Var[\epsilon_t | \mathbf{X}] = \sigma^2,$$
$$Cov[\epsilon_t, \epsilon_s | \mathbf{X}] = 0,$$
$$\epsilon_t | \mathbf{X} \sim N[0, \sigma^2 \mathbf{I}];$$

(7)

$\mathbf{X}$ – a matrix of the observations compiled by of $\begin{bmatrix} A(h_j, y_t) \\ A(h_j, \Delta y_t) \end{bmatrix}$; $E[\cdot]$ – a expectation; $Cov[\cdot]$ – a covariance; $Var[\cdot]$ – a variance; $\mathbf{I}$ – identity matrix.

**2. Chaotic Attractor of the U.S. Retail and Food Services Sales: Time Series, 1992-2012, Monthly**

The choice of the structure of chaotic and cyclic movement and the identification of their characteristics allow using the category of deep chaos theory for solving many applied problems of system of retail
sales, for example, measuring and modeling of chaotic and cyclic movements of the U.S. Retail and Food Services Sales.

In particular, the choice of attractor of the U.S. Retail and Food Services Sales – geometric structure that characterizes the behavior of participants' of retail sales in the phase space by sufficiently long period of time, that means an abstract space, which is represented by the set of all states of the system, so that each possible state of the system corresponds to the point of phase space whose coordinates are the degrees of freedom system.

In this case, the movement of the U.S. Retail and Food Services Sales has two degrees of freedom. For example, in the case of Retail and Food Services Sales: time series is the movement which is completely possible to determine with the initial moment of time and initial state of the volume, and in the case of the phase portrait – the initial state and change from year ago of volume of the U.S. Retail and Food Services Sales.

At choice of measures of scattering (Fibonacci level) equal $h_1 = 0.0000253$ (Fibonacci level), $h_2 = 0.00000228$ (Fibonacci level), and $h_3 = 0.000000870$ (Fibonacci level) geometric illustration of the chaotic attractor of the U.S. Retail and Food Services Sales as average with a constant measure of scattering are shown in Figure 1.

![Figure 1. A Chaotic Attractor. Time Series: U.S. Retail and Food Services Sales, Millions of Dollars, 1992-2012, Monthly](image-url)
It should be noted that the chaotic attractors of the U.S. Retail and Food Services Sales identify the creation of the following trends, uncertainties and cyclic movements, i.e. for $h_1 = 0.0000253$, $h_2 = 0.00000228$, and $h_3 = 0.000000870$ was obtained (see Figure 1):

–after the recession in 1990-1991 years the volume of the U.S. Retail and Food Services Sales had gained the growth trend on the 290,000 Millions of Dollars to mid of 2000, with the transition to the zone of sharp reduction in the Retail and Food Services Sales (from the mid of 2000 to beginning 2003) that during this period formed the uncertainty of the retail sales and U.S. economy led to a recession in 2001;

–growth trend of the U.S. Retail and Food Services Sales had been on the 370,000 Millions of Dollars (from the beginning of 2003 to mid 2007), further a zone of the excess demand in the market of Retail and Food Services Sales had been dominating up to the beginning of 2001, which ended by the U.S. economy recession of 2008;

–after the recession in 2008-2009 years on the 315,000 Millions of Dollars there was formed only the growth phase of cyclic movements of the U.S. Retail and Food Services Sales, but the zone of excess demand in the market of the retail sales had not yet been formed, which is expected presence of this phase reconciling 3-4 years.


As a result of permanent divergence and convergence of the chaotic attractor a rapidly growing trend of uncertainty and cyclical movement of the U.S. Retail and Food Services Sales, i.e. with the expiration of each point in time lose the ability to make accurate predictions. However, to assess trends in the differences, convergence, the initial state of monetary aggregate Retail and Food Services Sales and they can be a measure of chaos, i.e. numerical expression of how the system is chaotic.

It also should be mentioned that the reason of the chaotic tendencies is the differences in initial conditions and time of the U.S. Retail and Food Services Sales. Even a microscopic deviation of two trends of the retail sales at the first time the process of movement can lead to an
exponential accumulation of errors and their respective stochastic divergence.

Indeed, the divergence and convergence of the chaotic attractor of the initial conditions of time and state of the U.S. Retail and Food Services Sales is systematically replaced by new ones. Thus, when the descent trajectory becomes closer and begins to show the effect of short-sightedness – increases the uncertainty of large-scale information. And in the divergence of the trajectories is on the contrary, they diverge, the effect of farsightedness develops, when uncertainty of the small-scale information increases.

Thus, in order to estimate the parameters of the chaotic tendencies and the initial state of the U.S. Retail and Food Services Sales, depending on the factor variables as average with a constant measure of scattering (5), respectively, parameters are: $h_1 = 0.0000253$, $h_2 = 0.00000228$, $h_3 = 0.000000870$ an econometric model (6)–(7) will be used.

There by, ceteris paribus Monthly movements of the U.S. Retail and Food Services Sales during 1992-2012 years have the following estimates (see Table 1):

– on the basis of $A(h_1, y_t)$ (average of R&FS Sales with $h_1 = 0.0000253$):
  – trend: $\beta_1 = 0.9955$;
  – initial state: $\beta_0 = 1355.206$;

– on the basis of $A(h_2, y_t)$ (average of R&FS Sales with $h_2 = 0.00000228$):
  – trend: $\beta_2 = 1.0010$;
  – initial state: $\beta_0 = -435.732$;

Table 1. Estimate of parameters of the trends and the initial state of the U.S. Retail and Food Services Sales through the aid of average with a constant measure of scattering (5), respectively, with parameters $h_1 = 0.0000253$, $h_2 = 0.00000228$, and $h_3 = 0.000000870$

<table>
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<th>(1)</th>
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<tr>
<td>$A(h_1, y_t)$</td>
<td>0.9955*** (0.019)</td>
<td>0.4253** (0.214)</td>
<td>0.6751*** (0.161)</td>
<td>0.4774** (0.229)</td>
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<tr>
<td>$A(h_2, y_t)$</td>
<td>1.0010*** (0.007)</td>
<td>0.5828*** (0.215)</td>
<td>1.1326*** (0.298)</td>
<td>0.5109*** (0.419)</td>
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<tr>
<td>$A(h_3, y_t)$</td>
<td>0.9990*** (0.026)</td>
<td>0.3214** (0.162)</td>
<td>-0.1382 (0.298)</td>
<td>-0.0083 (0.304)</td>
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–on the basis of $A(h_3, y_t)$ (average of R&FS Sales with $h_3=0.000000870$):
  –trend: $\beta_3 = 0.9900$;
  –initial state: $\beta_0 = 2,526.399$;
–on the grounds of $A(h_1, y_t)$ and $A(h_2, y_t)$:
  –trends, respectively: $\beta_1 = 0.4253$ and $\beta_2 = 0.5828$;
  –initial state: $\beta_0 = -2,240.381$;
–on the grounds of $A(h_1, y_t)$ and $A(h_3, y_t)$:
  –trends, respectively: $\beta_1 = 0.6751$ and $\beta_3 = 0.3214$;
  –initial state: $\beta_0 = 978.426$;
–on the grounds of $A(h_2, y_t)$ and $A(h_3, y_t)$:
  –trends, respectively: $\beta_2 = 1.1326$ and $\beta_3 = -0.1382$;
  –initial state: $\beta_0 = 1,547.291$;
–on the basis of $A(h_1, y_t)$, $A(h_2, y_t)$ and $A(h_3, y_t)$:
  –trends, respectively: $\beta_1 = 0.4774$, $\beta_2 = 0.5109$ and $\beta_3 = 0.0083$;
  –initial state: $\beta_0 = 990.025$.


Chaotic attractors of change to the corresponding period from year ago (more strength) of the U.S. Retail and Food Services Sales identifies creation of the following trends, uncertainties and cyclic movements, i.e. for $h_1 = 0.0000253$, $h_2 = 0.00000228$, and $h_3 = 0.000000870$ was obtained (see Figure 2):

–after the recession in 1990-1991 years the volume of strength of the U.S. Retail and Food Services Sales had been the uncertainty on the level 14,000 Millions of Dollars to end of 2000, with the transition to the zone of growth on the level 22,000 Millions of Dollars to begin of 2005;
Figure 2. A Chaotic Attractor. Time Series: U.S. Retail and Food Services Sales, Change from Year Ago, Millions of Dollars, 1992-2012, Monthly

– further was been the sharp reduction of strength of the Retail and Food Services Sales (from the beginning of 2005 to mid 2008) on the level –17 000 Millions of Dollars that during this period formed the uncertainty of the retail sales and U.S. economy led to a recession in 2008;

– after the recession in 2008-2009 years there was formed only the growth phase of cyclic movements of strength of the U.S. Retail and Food Services Sales, but the zone of excess demand in the market of the retail sales had not yet been formed, which is expected presence of this phase reconciling 2-3 years.

5. Econometric Measurement of the Chaotic Tendencies and the Initial State of Change from Year Ago of the U.S. Retail and Food Services Sales, 1992-2012, Monthly

As in the fifth section for the estimation of parameters of chaotic trends and initial state of change from year ago of the U.S. Retail and Food Services Sales in depend ingot variables factors, average with a constant measure of scattering (5), respectively, parameters are: \( h_1 = 0.0000253 \), \( h_2 = 0.00000228 \), \( h_3 = 0.000000870 \) an econometric model (6)–(7) will be used.

Thus, other things being equal, Monthly traffic of change from year ago (more strength) of the U.S. Retail and Food Services Sales during 1992-2012 have the following estimates (see Table 2):
– on the basis of $A(h_1, \Delta y_t)$ (average of change from year ago of R&FS Sales with $h_1=0.0000253$):
  – trend: $\beta_1 = 1.0504$;
  – initial state: $\beta_0 = -640.841$;
– on the basis of $A(h_2, \Delta y_t)$ (average of change from year ago of R&FS Sales with $h_2=0.00000228$):
  – trend: $\beta_2 = 1.3076$;
  – initial state: $\beta_0 = -2224.396$;
– on the basis of $A(h_3, \Delta y_t)$ (average of change from year ago of R&FS Sales with $h_3=0.000000870$):
  – trend: $\beta_3 = 1.6271$;
  – initial state: $\beta_0 = -5992.841$;
– on the basis of $A(h_1, \Delta y_t)$ and $A(h_2, \Delta y_t)$:
  – trends, respectively: $\beta_1 = 1.1052$ and $\beta_2 = -0.0915$;
  – initial state: $\beta_0 = -329.624$;
– on the grounds of $A(h_1, \Delta y_t)$ and $A(h_3, \Delta y_t)$:
  – trends, respectively: $\beta_1 = 1.0537$ and $\beta_3 = -0.0530$;
  – initial state: $\beta_0 = -241.398$;
– on the grounds of $A(h_2, \Delta y_t)$ and $A(h_3, \Delta y_t)$:
  – trends, respectively: $\beta_2 = 1.9062$ and $\beta_3 = -1.7120$;
  – initial state: $\beta_0 = 8193.360$;
– on the basis of $A(h_1, \Delta y_t) y_t, A(h_2, \Delta y_t)$ and $A(h_3, \Delta y_t)$:
  – trends, respectively: $\beta_1 = 1.1621, \beta_2 = -0.2399$ and $\beta_3 = 0.1943$;
  – initial state: $\beta_0 = -1434.606$.

**Table 2.** Estimate of parameters of the trends and the initial state of Change from Year Ago (strength) of the U.S. Retail and Food Services Sales through the aid of average with a constant measure of scattering (5), respectively, with parameters $h_1 = 0.0000253$, $h_2 = 0.00000228$, and $h_3 = 0.000000870$

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<tbody>
<tr>
<td>$A(h_1, \Delta y_t)$</td>
<td>1.0504***</td>
<td>1.1052***</td>
<td>1.0537***</td>
<td>1.1621***</td>
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<td></td>
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<td>(0.061)</td>
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<td>$A(h_2, \Delta y_t)$</td>
<td>1.3076***</td>
<td>-0.0915</td>
<td>1.9062***</td>
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<td>$A(h_3, \Delta y_t)$</td>
<td>1.6272***</td>
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<td>-1.7120***</td>
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The choice of chaotic attractor of the U.S. Retail and Food Services Sales: Phase portrait – a geometric structure consisting of all states as the degree of freedom system, whose coordinates are determined by the initial state and change from year ago of volume of the U.S. Retail and Food Services Sales.

Chaotic attractor of the U.S. Retail and Food Services Sales: Phase portrait as average with a constant measure of scattering (5) with a parameter (Fibonacci level), for example by $h_1 = 0.0000253$, $h_2 = 0.00000228$, and $h_3 = 0.000000870$ has geometric illustration in Figure 3, and identifies creation of the following trends, uncertainties and cyclic movements was obtained (see Figure 3.):

– divergence trends around the triangular area A, which was formed as a result of chaotic change of the U.S. Retail and Food Services Sales in the intervals 145 000-320 000 Millions of Dollars and strength in the intervals (−)50 000-28 000 Millions of Dollars;

– chaotic closed curve (the zone around the point B), which was formed as a result of chaotic change of the U.S. Retail and Food Services Sales in the intervals 320 000-390 000 Millions of Dollars and strength in the intervals (−)40 000-15 000 Millions of Dollars.

The research was supported by the Committed Science of Ministry Education and Sciences of the Republic of Kazakhstan for Scientific Research (grant № 1072/ГФ2-ОТ).
CONCLUSION

The measuring and modeling of chaotic and cyclic movement of the U.S. Retail and Food Services Sales confirms the applicability of an indicator of the average with a constant measure of scattering, for example in the study of applied problems in the system of retail sales. In particular, the use of this indicator of the average with a constant measure of scattering allows taking effective management decisions and provides meaningful interpretable quantitative assessment of the activities and management of retail sales of the United States.

There was given an estimation of the parameters of the trends and the initial state of an econometric model of the U.S. Retail and Food Services Sales depending on the parameter measure of scattering (Fibonacci level). Chaotic attractors weekly cyclic movements of the U.S. Retail and Food Services Sales were constructed and its change from year ago, both in the form of time series, and the phase portrait, as well as trends in stochastic divergence and convergence, and other uncertainty characteristics.

Indeed, for chaotic attractors cyclic movements of the U.S. Retail and Food Services Sales were observed and estimated three full-cycle trends of divergence and level convergence (see Figure 1). For the fourth
cycle movements were defined only trend of divergence, but levels of convergence still not defined.

In the case, for chaotic attractors cyclic movements of change from year ago of the U.S. Retail and Food Services Sales were observed and estimated for one cycle and were estimated trends and level convergence (see Figure 2). For a cycle movements that began in 2008 were identified only trend of divergence, but levels of convergence still not defined.

In conclusion, for the phase portrait of chaotic attractors cyclic movements and change from year ago of the U.S. Retail and Food Services Sales were observed and estimated by two full-cycle trends and their corresponding levels of convergence (see Figure 3). For the third cycle, only the breakdowns of trends were identified divergence, but levels of convergence still not defined.

REFERENCES
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