Суреттен байқағанымыздай гироскоп роторының β бұрылу бұрышының тербелісі уақыт өткен сайын өшпелі сипатта болады.

Қолданылған әдебиеттер тізімі

1. Кошляков В.Н. Задачи динамики твердого тела и прикладной теории гироскопов: Аналитические методы. – М.: Наука, 1985, – 286 с.
2. Тюреходжаев А.Н., Берсүгір М.Ә., Маматова Г.У. Гироскоптың кедергілі ортадағы қозғалысы туралы есептің аналитикалық шешімі // Материалы III Международной научной конференции «Актуальные проблемы механики и машиностроения». – Т.1. – Алматы, 2009. – С.127-132.

4.3 Математическое и компьютерное моделирование

UDC 338

GLOBAL OUTPUT TRACKING FOR INHERENTLY NON-LINEAR SYSTEMS BY CONTINUOUS STATE FEEDBACK CONTROLLERS

Bayan Nazarova
bakonai@gmail.com
L.N. Gumilyov Eurasian National University, Master's degree,
Astana, Kazakhstan

This paper considers the global practical tracking problem by state feedback for a class of high-order non-linear systems with more general uncertainties, to which the existing control methods are inapplicable. We successfully propose a new tracking control design scheme for the system studied by introducing sign function and necessarily modifying the method of adding a power integrator. It is shown that the designed controller guarantees that all states of the resulting closed-loop system are globally bounded and the tracking error remains prescribed arbitrarily small after a finite time.

Keywords: uncertain nonlinear systems, practical output tracking, sign function, state feedback

1. Introduction

This paper deals with the practical output tracking problem with a state feedback for a class of high-order nonlinear systems having the following form:

$$\begin{aligned} \dot{z}_1 &= z_2^{p_1} + \phi_1(t, z, u) \\ \dot{z}_2 &= z_3^{p_2} + \phi_2(t, z, u) \\ &\vdots \\ \dot{z}_{n-1} &= z_n^{p_{n-1}} + \phi_{n-1}(t, z, u), \\ \dot{z}_n &= u^{p_n} + \phi_n(t, z, u), \\ y &= z_1, \end{aligned} \qquad (1)$$

where $z = (z_1, \dots, z_n)^{\mathrm{T}} \in R^n$ and $u \in R$ are the system state and the control input, respectively. For $i = 1, \dots, n$, $\phi_i(t, z, u)$ are unknown continuous functions and $p_i \in R_{odd}^{\geq 1} := \{p/q \in R^+ : p$ and q are odd integers, $p \geq q\}$ $(i = 1, \dots, n-1)$ are said to be the high orders of the system.

We first introduce definition of the practical output tracking problem.

Consider the system (1) and assume that the reference signal $y_r(t)$ be a time-varying C^1-bounded on $[0, +\infty)$. Then, the *global practical output tracking problem* by a *state controller* is formulated as follows: For any given real number $\varepsilon > 0$, design a continuous controller having the following structure

$$u = u(z, y_r(t)), \tag{2}$$

such that

(i) all the states of the closed-loop system (1) and (2) are well-defined on $[0, +\infty)$ and globally bounded;

(ii) the global practical output tracking is achieved, that is, for every $z(0) \in R^n$ there is a finite time $T > 0$, such that the output $y(t)$ of the closed loop system (1) with (3) satisfies

$$|y(t) - y_r(t)| = |z_1(t) - y_r(t)| < \varepsilon, \qquad \forall t \geq T > 0. \tag{3}$$

In this paper, by introducing a combined sign function design and the method of adding a power integrator, we shall solve the above global practical output tracking problem. The main contributions of this paper are two- folds: (i) By comparison with the existing results in [4–7], the nonlinear growth condition is largely relaxed and a much weaker sufficient condition is given.(ii) By successfully over-coming some essential difficulties such as the weaker assumption on the system growth, the appearance of the sign function and the construction of a continuously differentiable Lyapunov function, a new method to solve the tracking problem of high-order nonlinear systems is given by state feedback, which can lead to more general results never achieved before.

2. Mathematical Preliminaries

At first, we give the following notations which will be used in this study.

Notations: R^+ denotes the set of all the nonnegative real numbers and R^n denotes the real n-dimensional space. For any vector $x = (x_1, \ldots, x_n)^T \in R^n$, denote

$$\bar{x}_i := (x_1, \ldots, x_i)^T \in R^i, \ i = 1, \ldots, n, \quad \|x\| := \left(\sum_{i=1}^n x_i^2\right)^{1/2}.$$

A sign function $\mathrm{sgn}(x)$ is defined as: $\mathrm{sgn}(x) = 1$ if $x > 0$, $\mathrm{sgn}(x) = 0$ if $x = 0$, and $\mathrm{sgn}(x) = -1$ if $x < 0$. For any $\alpha \in R^+$ and $x \in R$, the function $[x]^\alpha$ is defined as $[x]^\alpha = \mathrm{sgn}(x)|x|^\alpha$. A function $f: R^n \to R$ is said to be C^k-function, if its partial derivatives exist and are continuous up to order k, $1 \leq k < \infty$. A C^0 function means it is continuous. A C^∞ function means it is *smooth*, that is, it has continuous partial derivatives of any order. Besides, the arguments of functions (or functionals) are sometimes omitted or simplified, whenever no confusion can arise from the context. For instance, we sometimes denote a function $f(x(t))$ by $f(x)$, $f(\cdot)$, or f.

In order to solve the global practical output tracking problem, we made the following assumption:

Assumption1

For $i = 1, \ldots, n,$ there are smooth functions $\psi_i(\bar{z}_i),$ and $\tau \geq -1/\sum_{l=1}^n p_1 \cdots p_{l-1}$ such that

$$|\phi_i(t, z, u)| \leq \psi_i(\bar{z}_i)\left(|z_1|^{(r_i+\tau)/r_1} + \cdots + |z_i|^{(r_i+\tau)/r_i}\right) \tag{4}$$

where

$$r_1 = 1, \quad r_{i+1} = (r_i + \tau)/p_i > 0, \quad i = 1, \ldots, n \tag{5}$$

and $\sum_{l=1}^n p_1 \cdots p_{l-1} = 1$ for the case of $l = 1$.

Remark: Assumption2.1, which gives the nonlinear growth condition n the system drift terms,encompasses the assumptions in existing results [1-7]. Specifically, when $\tau \geq 0$, it reduces to Assumptions in [4-6]. When τ is some ratios of odd integers in $\tau \in [-1/\sum_{l=1}^n p_1 \cdots p_{l-1}, 0]$, it becomes the condition used in [6]. This means that the system studied in this paper is less restrictive and allows for a much broader class of systems.

Now, we introduce six technical lemmas which will play an important role and be frequently used in the later control design.

Lemma1[2] For any real numbers $x \geq 0,\ y > 0$ and $m \geq 1$, the following inequality holds:

$$x \le y + (x/m)^m \left((m-1)/y\right)^{m-1}.$$

Lemma2[8] For all $x, y \in R$ and a constant $p \ge 1$ the following inequalities holds:

(i) $|x+y|^p \le 2^{p-1}|x^p + y^p|$, $(|x|+|y|)^{1/p} \le |x|^{1/p} + |y|^{1/p} \le 2^{(p-1)/p}(|x|+|y|)^{1/p}$

If $p \in R_{odd}^{\ge 1}$, then

(ii) $|x-y|^p \le 2^{p-1}|x^p - y^p|$ and $|x^{1/p} - y^{1/p}| \le 2^{(p-1)/p}|x-y|^{1/p}$.

Lemma3[8] Let c, d be positive constants. Then, for any real-valued function $\gamma(x,y) > 0$, the following inequality holds:

$$|x|^c |y|^d \le \frac{c}{c+d}\gamma(x,y)|x|^{c+d} + \frac{d}{c+d}\gamma^{-c/d}(x,y)|y|^{c+d}.$$

Lemma4[9] For $x, y \in R$ and $0 < p \le 1$ the following inequality holds

$$(|x|+|y|)^p \le |x|^p + |y|^p.$$

When $p = a/b \le 1$, where $a > 0$ and $b > 0$ are *odd* integers

$$|x^p + y^p| \le 2^{1-p}|x+y|^p.$$

Lemma5[10] If $p = a/b \in R_{\text{odd}}^{\ge 1}$ with $a \ge b \ge 1$ being some real numbers, then for any $x, y \in R$

$$|x^p - y^p| \le 2^{1-1/b}\left|\text{sgn}(x)|x|^a - \text{sgn}(y)|y|^a\right|^{1/b}$$

Lemma6[3]. If $f: [a,b] \to R$ $(a \le b)$ is monotone continuous and satisfies $f(a) = 0$, then

$$\left|\int_a^b f(x)dx\right| \le |f(b)| \cdot |b-a|.$$

3. Main Result

In this section, we are ready to state our own main theorem below

Theorem 1. Let $y_r(t)$ be a reference signal whose derivative $\dot{y}_r(t)$ is also bounded. Then, under Assumption1, the global practical output tracking problem of the system (1) is solvable by a continuous state feedback controller of the form (2).

***Sketch of Proof*:** The inductive proof relies on the simultaneous construction of a C^1 Lyapunov function which is positive define and proper, as well as a homogeneous-like controller at each iteration.

Let $x_1 = z_1 - y_r$ and given $x_i = z_i$, $i = 2, \ldots, n$. Then, we have

$$\begin{aligned}
\dot{x}_1 &= x_2^{p_1} + \phi_1(t, x_1 + y_r, x_2, \ldots, x_n, u) - \dot{y}_r(t), \\
\dot{x}_2 &= x_3^{p_2} + \phi_2(t, x_1 + y_r, x_2, \ldots, x_n, u), \\
&\vdots \\
\dot{x}_{n-1} &= x_n^{p_{n-1}} + \phi_{n-1}(t, x_1 + y_r, x_2, \ldots, x_n, u), \\
\dot{x}_n &= u + \phi_n(t, x_1 + y_r, x_2, \ldots, x_n, u), \\
y &= x_1 + y_r.
\end{aligned} \tag{6}$$

In this section, we will present a recursive design approach to construct the tracking control for system (1) is solvable, the proof is omitted for page limitation. A continuous state feedback controller of the form

$$u = x_{n+1}^* = -\beta_n^{r_{n+1}/\sigma}(\bar{x}_n)[\xi_n]^{r_{n+1}/\sigma} \tag{7}$$

with the C^1, proper and positive definite Lyapunov function $V_n(x_1, x_2 \ldots, x_n)$ constructed via the inductive procedure, we arrive at

$$\dot{V}_n(x_1, \ldots, x_n) \le -\sum_{i=1}^n \xi_i^2 + n\delta. \tag{8}$$

Noting that $\tau \ge -1\Big/\sum_{l=1}^{n} p_1 \cdots p_{l-1}$, $r_{k+1}p_k = r_k + \tau$ **and** $\sigma \ge \max_{1\le i\le n}\{1, \tau + r_i\}$, **we have** $0 < r_{k+1}p_k/\sigma \le 1$**. Moreover, recall that** $V(x_1,\ldots,x_n) = \sum_{k=1}^{n} W_k(x_1,\ldots,x_k)$**, where** W_k**'s are defined in (13). Then, it follows from Lemmas 5-6, we have**

$$\begin{aligned} W_k(\bar{x}_k) &\le \left|x_k - x_k^*\right|\left|\xi_k\right|^{(2\sigma-\tau-r_k)/\sigma} \le 2^{1-\frac{r_k}{\sigma}}\left|\text{sgn}(x_k)\left|x_k\right|^{\sigma/r_k} - \text{sgn}(x_k)\left|x_k^*\right|^{\sigma/r_k}\right|^{r_k/\sigma}\left|\xi_k\right|^{(2\sigma-\tau-r_k)/\sigma} \\ &= 2^{1-\frac{r_k}{\sigma}}\left|\left[x_k\right]^{\sigma/r_k} - \left[x_k^*\right]^{\sigma/r_k}\right|^{r_k/\sigma}\left|\xi_k\right|^{(2\sigma-\tau-r_k)/\sigma} \\ &= 2^{1-\frac{r_k}{\sigma}}\left|\xi_k\right|^{r_k/\sigma}\left|\xi_k\right|^{(2\sigma-\tau-r_k)/\sigma} \\ &\le 2\left|\xi_k\right|^{(2\sigma-\tau)/\sigma}. \end{aligned} \quad \textbf{(9)}$$

So we have the following estimate:

$$V_n(\bar{x}_n) = \sum_{k=1}^{n} W_k(\bar{x}_k) \le 2\sum_{k=1}^{n}\left|\xi_k\right|^{(2\sigma-\tau)/\sigma}. \quad \textbf{(10)}$$

Let $\lambda = (2\sigma-\tau)/\sigma$. By $\tau \ge -1\Big/\sum_{l=1}^{n} p_1 \cdots p_{l-1}$ and $\sigma \ge \max_{1\le i\le n}\{1, \tau + r_i\}$, $1/\lambda \in (0,1)$. With (8) and (10) in mind, by Lemma4, it is not difficult to obtain that

$$\dot{V}_n(\bar{x}_n) \le -\left(V_n(\bar{x}_n)/2\right)^{1/\lambda} + n\delta \quad \textbf{(11)}$$

It will show that the state $x(t)$ of closed-loop system (6) is well-defined on $[0,+\infty)$ and globally bounded. First, introduce the following set

$$\Omega := \left\{ x(t) \in R^n \,\middle|\, V_n(\bar{x}_n) \ge 2\left(2n\delta\right)^{\lambda} \right\}, \quad (12)$$

and let $x(t)$ be the trajectory of (6) with an initial state $x(0)$. If $x(t) \in S$, then it follows from (12) that

$$\dot{V}_n(\bar{x}_n) \le -\left(V_n(\bar{x}_n)/2\right)^{1/\lambda} + n\delta \le -n\delta < 0. \quad (13)$$

This implies that, as long as $x(t) \in \Omega$, $V_n(x(t))$ is strictly decreasing with time *t*, and hence $x(t)$ must enter the complement set $R^n - \Omega$ in a finite time $T \ge 0$ and stay there forever. Thus, the solution $x(t)$ of the system (6) is well-defined and globally bounded on $[0,+\infty)$. Next, it will be shown that

$$\left|y(t) - y_r(t)\right| = \left|z_1(t) - y_r(t)\right| < \varepsilon, \qquad \forall t \ge T > 0. \quad (14)$$

This is also easily shown from (11) and by tuning the parameter δ:

$$\left|y(t) - y_r(t)\right| = \left|x_1(t)\right| \le V_n(x(t)) \le 2\left(2n\delta\right)^{\lambda} < \varepsilon.$$

Therefore, for any $\varepsilon > 0$, there is globally practical output-tracking such that (14) holds. This completes the proof of Theorem 1.

5. Conclusions

In this paper, a state feedback output tracking controller is presented for a class of high-order nonlinear systems under weaker condition. The controller guarantees that the states of the closed-loop system are globally bounded, while the tracking error can be bounded by any given positive number after a finite time. It should be noted that the proposed controller can only work well when the whole state vector is measurable. Therefore, a natural and more interesting problem is how to design output feedback output tracking controller for the systems studied in the paper if only partial state vector being measurable, which is now under our further investigation.

Literature

1. A. Isidori, C.I. Byrnes, "Output regulation of nonlinear system", IEEE Trans. on Automatic Control, 35 ,(1990), pp. 31-140.

2. C.J. Qian, W. Lin, "Practical output tracking of nonlinear systems with uncontrollable unstable linearization", IEEE Trans. on Automatic Control, 47, (2002), pp. 21−36.

3. Z.Y. Sun, Y.G. Liu, "Adaptive practical output tracking control for high-order nonlinear uncertain systems", Acta Automatica sinica, 34, (2008), pp. 984-989.

4. K. Alimhan, H. Inaba, “Practical output tracking by smooth output compensator for uncertain nonlinear systems with unstabilisable and undetectable linearization”, Int. J. of Modelling, Identification and Control, 5, (2008), pp.1-13.
5. K. Alimhan, H. Inaba, “Robust practical output tracking by output compensator for a class of uncertain inherently nonlinear systems”, Int. J. of Modelling, Identification and Control, 4, (2008), pp.304-314.
6. W.P. Bi, J.F. Zhang, “Global practical tracking control for high-order nonlinear uncertain systems”, Proc. of the Chinese Control and Decision Conference, (2010), May. 26–28, Xuzhou, China, pp.1619-1622.
7. K. Alimhan, N. Otsuka, A. A. Adamov, M. N. Kalimoldayev, Global practical output tracking of inherently non-linear systems using continuously differentiable controllers, Mathematical Problems in Engineering, Article ID 932097, 2015.
8. J. Polendo, C. Qian, “A generalized homogeneous domination approach for global stabilization of inherently non-linear systems via output feedback”, Int. J. of Robust Nonlinear Control, 17, (2007), pp.605–629.
9. J. Polendo, C. Qian, “A universal method for robust stabilization of nonlinear systems: unification and extension of smooth and non-smooth approaches”, Proc. of the American Control Conference, (2006), pp.4285-4290.
10. Huang, X., Lin, W. and Yang, B., “Finite-time stabilization in the large for uncertain nonlinear systems”, Proc. of the American Control Conference, (2004), pp.1073–1078.

UDK 338

MODELING HUMAN INTENTION USING BAYESIAN NETWORK

Oteniyaz G.S.
goteniyaz@nu.edu.kz
Bachelor student, Nazarbayev University, Astana
Supervisor – Martin Lukac

During some game event on the field or stadium, spectators can always view and follow what is occurring by turning the head to the right direction. However when person watches the game on TV, he can only rely on camera movement, that shows what is happening on the field. Game events that are usually showed on TV does not always satisfy user with the content. Cameras in the pitch cannot always capture some important moments. In this project football was chosen to represent game event. In order to solve that problem, we built a model that estimate human intention and provide desirable content.

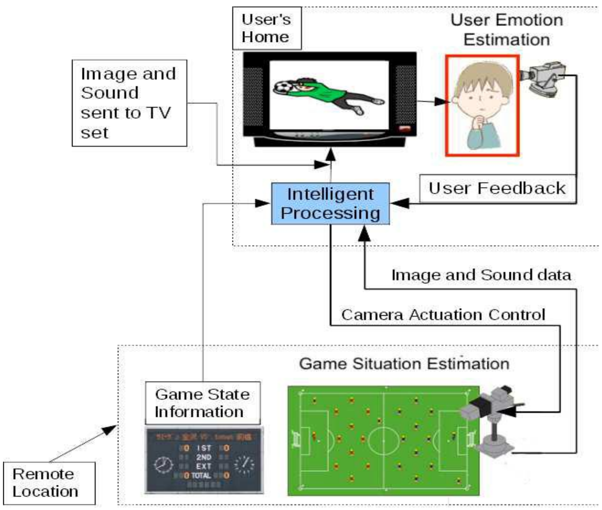

Fig. 1 Live feeling communication platform