ҚАЗАҚСТАН РЕСПУБЛИКАСЫ ҒЫЛЫМ ЖӘНЕ ЖОҒАРЫ БІЛІМ МИНИСТРЛІГІ

«Л.Н. ГУМИЛЕВ АТЫНДАҒЫ ЕУРАЗИЯ ҰЛТТЫҚ УНИВЕРСИТЕТІ» КЕАҚ

Студенттер мен жас ғалымдардың
«ǴYLYM JÁNE BILIM - 2023»
XVIII Халықаралық ғылыми конференциясының
БАЯНДАМАЛАР ЖИНАҒЫ

СБОРНИК МАТЕРИАЛОВ
XVIII Международной научной конференции
студентов и молодых ученых
«ǴYLYM JÁNE BILIM - 2023»

PROCEEDINGS
of the XVIII International Scientific Conference
for students and young scholars
«ǴYLYM JÁNE BILIM - 2023»

2023
Астана

УДК 001+37
ББК 72+74
G99

«ǴYLYM JÁNE BILIM – 2023» студенттер мен жас ғалымдардың XVIII Халықаралық ғылыми конференциясы = XVIII Международная научная конференция студентов и молодых ученых «ǴYLYM JÁNE BILIM – 2023» = The XVIII International Scientific Conference for students and young scholars «ǴYLYM JÁNE BILIM – 2023». – Астана: – 6865 б. - қазақша, орысша, ағылшынша.

ISBN 978-601-337-871-8

Жинаққа студенттердің, магистранттардың, докторанттардың және жас ғалымдардың жаратылыстану-техникалық және гуманитарлық ғылымдардың өзекті мәселелері бойынша баяндамалары енгізілген.

The proceedings are the papers of students, undergraduates, doctoral students and young researchers on topical issues of natural and technical sciences and humanities.

В сборник вошли доклады студентов, магистрантов, докторантов и молодых ученых по актуальным вопросам естественно-технических и гуманитарных наук.

УДК 001+37
ББК 72+74

ISBN 978-601-337-871-8

3. Alam, M.N., Akbar, M.A., Mohyud-Din, S.T., 2014b. A novel ðG0 =GÞ-expansion method and its application to the Boussinesq equation. Chin. Phys. B 23 (2). 020203.
4. Alam, M.N., Belgacem, F.B.M., 2016. Microtubules nonlinear models dynamics investigations through the expð/ðnÞÞ-expansion method implementation. Mathematics 4, 6.
5. Alam, M.N., Li, X., 2019. Exact traveling wave solutions to higher order nonlinear equations. J. Ocean Eng. Sci. 4 (3), 276–288.
6. Alam, M.N., Tunc, C., 2016. An analytical method for solving exact solutions of the nonlinear Bogoyavlenskii equation and the nonlinear diffusive predator-prey system. Alexandria Eng. J. 55, 1855–1865.
7. Alqurana, M., Jaradata, I., Baleanu, D., 2019. Shapes and dynamics of dual-mode Hirota-Satsuma coupled KdV equations: Exact traveling wave solutions and analysis. Chin. J. Phys. 58, 49–56.
8. Alquran, M., Jaradat, H., Syam, M.I., 2018. A modified approach for a reliable study of new nonlinear equation: two-mode Korteweg-de Vries-Burgers equation. Nonlinear Dynam. 91 (3), 1619–1626.
9. Fan, E., Zhang, J., Hon, B., 2001. A new complex line soliton for the two-dimensional KdV-Burgers equation. Phys. Lett. A 291, 376–380.

UDC 524.852, 524.834

STUDY OF THE COSMOLOGICAL MODEL BY METHODS OF THE $F(R, X, \varphi, Y, \psi, \bar{\psi})$ SYMMETRY THEORY

Amangeldiyev Alikhan Nurkasymovich[1], Kabibolla Dilnaz Talapkyzy[2]
aamangeldiev525@gmail.com, dilnazkabibolla@gmail.com
[1] 1-st grade master of the ENU named after L.N. Gumilyov
[2] 4-th grade student of the ENU named after L.N. Gumilyov
Astana, Kazakhstan
Scientific director: Yerzhanov K.K

The $F(R, X, \varphi, Y, \psi, \bar{\psi})$ gravity model is a theoretical framework that extends Einstein's general theory of relativity to include additional gravitational effects. This model proposes modifications to the traditional Einstein-Hilbert action by introducing new terms that include higher-order curvature invariants, scalar fields, and non-minimal matter coupling. The $F(R, X, \varphi, Y, \psi, \bar{\psi})$ model has been proposed as a possible solution to some open problems in modern cosmology such as dark energy and dark matter. This model has also received considerable attention in recent years due to its ability to unify the fundamental forces of nature. In this context, $F(R, X, \varphi, Y, \psi, \bar{\psi})$ gravity model has become an exciting area of research that could revolutionize our understanding of the universe [1-2].

We have Lagrangian the next form [3]:

$$L = a^3F - a^3F_R R - a^3F_R u - 6F_R\dot{a}^2 a - 6\dot{F}_R\dot{a}a^2 -$$
$$-a^3F_X\left(X - v - \frac{1}{2}\dot{\varphi}^2\right) - a^3F_Y\left(Y - \omega - \frac{1}{2}i\left(\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi\right)\right). \tag{1}$$

Here: R – curvature scalar, Y – kinetic term of the fermion field, X – kinetic term of the scalar field, φ – scalar field, ψ – fermion field [1].

$$X = v + \frac{1}{2}\dot{\varphi}^2, \tag{2}$$

$$Y = \omega + \frac{1}{2} i \left(\overline{\psi}\gamma^0\dot{\psi} - \dot{\overline{\psi}}\gamma^0\psi\right), \tag{3}$$

$$R = -u + 6\frac{\ddot{a}}{a} + 6\frac{\dot{a}^2}{a^2}. \tag{4}$$

The F can be write [2]:

$$F = (R + X + Y)^2 + R + X + Y + c\left(\psi, \overline{\psi}, \varphi\right). \tag{5}$$

Here:

$$F_R = 2(R + X + Y) + 1,$$
$$\dot{F}_R = 2\left(\dot{R} + \dot{X} + \dot{Y}\right),$$
$$\ddot{F}_R = 2\left(\ddot{R} + \ddot{X} + \ddot{Y}\right). \tag{6}$$

We rewrite functions u, ω, ν, ψ, φ and find derivative by time:

$$u = u_0 a^2, \quad v = \frac{1}{2} c a^2, \quad \omega = a^2, \quad \dot{\varphi} = ca,$$
$$\dot{u} = 2u_0\dot{a}a, \quad \dot{v} = c\dot{a}a, \quad \dot{\omega} = 2\dot{a}a, \quad \ddot{\varphi} = c\dot{a},$$
$$\ddot{u} = 2u_0\dot{a}^2 + 2u_0\ddot{a}a, \; \ddot{v} = c\dot{a}^2 + c\ddot{a}a, \; \ddot{\omega} = 2\ddot{a}a + 2\dot{a}^2, \; \dddot{\varphi} = c\ddot{a} \tag{7}$$

From (5) we have:

$$R + X + Y = -u_0 a^2 + 6\frac{\ddot{a}}{a} + 6\frac{\dot{a}^2}{a^2} + \frac{1}{2} c a^2 + \frac{1}{2} c^2 a^2 + a^2, \tag{8}$$

$$(R + X + Y)^2 = R^2 + X^2 + Y^2 + 2RX + 2RY + 2XY =$$
$$u_0{}^2 a^4 + 36\frac{\ddot{a}^2}{a^2} + 36\frac{\dot{a}^4}{a^4} - -12u_0\ddot{a}a - 12u_0\dot{a}^2 + 72\frac{\ddot{a}\dot{a}^2}{a^3} +$$
$$\frac{c^2a^4}{4} + \frac{c^3a^4}{2} + \frac{c^4a^4}{4} + a^4 - u_0 c a^4 + 6c\ddot{a}a + 6\dot{a}^2 - u_0 c^2 a^4 +$$
$$+6c^2\ddot{a}a + 6c^2\dot{a}^2 - 2u_0 a^4 + \frac{ca^4}{2} + \frac{c^2a^4}{2} \tag{9}$$

By this we have:

$$R = -u_0 a^2 + 6\frac{\ddot{a}}{a} + 6\frac{\dot{a}^2}{a^2}, \tag{10}$$

$$\dot{R} = -2u_0\dot{a}a + \frac{6\dddot{a}a - 6\ddot{a}\dot{a}}{a^2} + \frac{12\ddot{a}\dot{a}a^2 - 12\dot{a}^3 a}{a^4} = -2u_0\dot{a}a^7 + 6\dddot{a}a^3 + 6\ddot{a}\dot{a}a^2 - 12\dot{a}^3 a, \tag{11}$$

$$\ddot{R} = -2u_0\dot{a}^2a^6 - 2u_0\ddot{a}a^7 + 6\ddddot{a}a^5 - 24\dddot{a}\dot{a}a^4 - 6\ddot{a}^2a^4 +$$
$$+12\ddot{a}\dot{a}a^4 + 24\ddot{a}^2\dot{a}a^6 - 12\dddot{a}\dot{a}^2a^6 - 24\ddot{a}\dot{a}^3 - 84\ddot{a}\dot{a}^2a^5 + 36\dot{a}^4a^4, \tag{12}$$

$$X = \frac{1}{2} c a^2 + \frac{1}{2} c^2 a^2, \tag{13}$$

$$\dot{X} = c\dot{a}a + c^2\dot{a}a, \tag{14}$$

$$\ddot{X} = c\dot{a}^2 + c\ddot{a}a + c^2\ddot{a}a + c^2\dot{a}^2, \tag{15}$$

$$Y = a^2, \tag{16}$$

$$\dot{Y} = 2\dot{a}a, \tag{17}$$

$$\ddot{Y} = 2\ddot{a}a + 2\dot{a}^2. \tag{18}$$

Now, we find F:

$$F = -4u_0a^6 + 24\ddot{a}a^3 + 48\dot{a}^2a^4 + 2ca^6 + 2c^2a^6 + 4a^6 + 4u_0{}^2a^8 + 144\ddot{a}^2a^6 + 144\dot{a}^4 - 48u_0\ddot{a}a^5 - 48u_0\dot{a}^2a^4 + 288\ddot{a}\dot{a}^2a + c^2a^8 + 2c^3a^8 + c^4a^8 + 4a^8 - 4u_0ca^8 + 24c\ddot{a}a^5 - 4u_0c^2a^8 + 24c^2\ddot{a}a^5 + 24c^2\dot{a}^2a^4 - 8u_0a^8 + 2ca^8 + 2c^2a^8 + 4ca^4. \tag{19}$$

$$F - 2\frac{\ddot{a}}{a}F_R - 4\frac{\dot{a}^2}{a^2}F_R + 4\frac{\dot{a}}{a}\dot{F}_R + 2\ddot{F}_R = 0. \tag{20}$$

If we use (20), we have next equation:

$$
\begin{gathered}
-4u_0a^6+24\ddot{a}a^3+48\dot{a}^2a^4+2ca^6+2c^2a^6+4a^6+4{u_0}^2a^8+144\ddot{a}^2a^6+144\dot{a}^4-\\
48u_0\ddot{a}a^5-48u_0\dot{a}^2a^4+288\ddot{a}\dot{a}^2a+c^2a^8+2c^3a^8+c^4a^8+4a^8-4u_0ca^8+24c\ddot{a}a^5-\\
4u_0c^2a^8+24c^2\ddot{a}a^5+24c^2\dot{a}^2a^4-8u_0a^8+2ca^8+2c^2a^8+4ca^4+4u_0\ddot{a}a-24\frac{\ddot{a}^2}{a^2}-\\
24\frac{\ddot{a}\dot{a}^2}{a^3}-2c\ddot{a}a-2c^2\ddot{a}a-4\ddot{a}a-2\frac{\ddot{a}}{a}+8u_0\dot{a}^2-48\frac{\ddot{a}\dot{a}^2}{a^3}-48\frac{\dot{a}^4}{a^4}-4c\dot{a}^2-4c^2\dot{a}^2-4\dot{a}^2-\\
4\frac{\dot{a}^2}{a^2}-16u_0\dot{a}^2a^6+48\dddot{a}\dot{a}a^2+48\ddot{a}\dot{a}^2a-96\dot{a}^4+8c\dot{a}^2+8c^2\dot{a}^2+16\dot{a}^2-4u_0\dot{a}^2a^6-\\
4u_0\ddot{a}a^7+12\ddddot{a}a^5-48\dddot{a}\dot{a}a^4-12\ddot{a}^2a^4+24\ddot{a}\dot{a}a^4+48\ddot{a}^2\dot{a}a^6-24\ddot{a}\dot{a}^2a^6-48\ddot{a}\dot{a}^3-\\
168\ddot{a}\dot{a}^2a^5+72\dot{a}^4a^4+2c\dot{a}^2+2c\ddot{a}a+2c^2\ddot{a}a+2c^2\dot{a}^2+4\ddot{a}a+4\dot{a}^2=0.
\end{gathered}
\tag{21}
$$

And we have:

$$a=a_0e^{H_0t},\ \dot{a}=a_0H_0e^{H_0t},\ \ddot{a}=a_0{H_0}^2e^{H_0t},\ \dddot{a}=a_0{H_0}^3e^{H_0t},\ \ddddot{a}=a_0{H_0}^4e^{H_0t}. \tag{22}$$

Substitute (22) to (21) we have system of equation:

$${a_0}^2e^{2H_0t}\left(12u_0{H_0}^2+6c{H_0}^2+6c^2{H_0}^2+16{H_0}^2\right)=0, \tag{23}$$

$${a_0}^4e^{4H_0t}(4c+384{H_0}^4+24{H_0}^2)=0, \tag{24}$$

$${a_0}^6e^{6H_0t}\begin{pmatrix}12{H_0}^4+24{H_0}^3+36{H_0}^2+24c^2{H_0}^2+\\+24c{H_0}^2-96u_0{H_0}^2+4+2c^2+2c-4u_0\end{pmatrix}=0, \tag{25}$$

$${a_0}^8e^{8H_0t}\begin{pmatrix}-144{H_0}^4-24u_0{H_0}^2+3c^2+2c-8u_0-\\-4u_0c^2-4u_0c+4+c^4+2c^3+144{H_0}^2+4{u_0}^2\end{pmatrix}=0. \tag{26}$$

From (24) we have next linear equation:

$$4c+384{H_0}^4+24{H_0}^2=0. \tag{27}$$

By solving this equation we will find scale factor *a*:

$$a=a_0e^{\sqrt{\frac{-3\pm\sqrt{9-96c}}{96}}t}. \tag{28}$$

In this paper, we have considered extended modified gravity with curvature, scalar and fermionic fields, and their kinetic terms. This is a modification of Myrzakulov's gravity F(R, X). We got a solution, that is, we found a scale factor.

The author expresses his gratitude to his supervisor, Ph.D., Yerzhanov K.K. for setting the task.

This research has been funded by AP14870191 "Investigation of the cosmology of gravitational theories in non-Riemannian geometry"

References

1. K. Yerzhanov, G. Bauyrzhan, A. Altaibayeva, R. Myrzakulov. Inflation from the Symmetry of the Generalized Cosmological Model // Autumn – 2021 p. 5 – 9.
2. E.N. Saridakis, Sh. Myrzakul, K. Myrzakulov, K. Yerzhanov. Cosmological applications of Myrzakulov gravity // 2020 p. 4 – 7.

3. R. Myrzakulov, K. Yerzhanov, G. Bauyrzhan, B. Meirbekov. F(R, T, X, φ) cosmology via Noether symmetry // 23 april 2021 p. 2 – 5.