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EXACT SOLUTION COSMOLOGICAL MODEL WITH FERMION AND TACHYON FIELDS

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The phenomenon of accelerated expansion of the universe has come to the picture of the world in recent years of the last century, and has now been confirmed by various data sets, such as supernova Ia (SN Ia), baryon oscillations, clustering galaxy, relict radiation and weak lensing. As a result, a difficult problem emerged, to find out what is the driving force behind the accelerated expansion of the universe.

The metric signature used is (-,+,+,+) and units have been chosen so that $8\pi G = c = \hbar = 1$.

Set the action with fermion and tachyon fields [1], [2]

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}R+L_D+L_T\right], \tag{1}$$

where R is the curvature scalar. Lagrangian of the spinor field ψ has the form

$$L_D=\frac{i}{2}\left[\bar{\psi}\Gamma^{\mu}D_{\mu}\psi-\left(D_{\mu}\bar{\psi}\right)\Gamma^{\mu}\psi\right]-V\left(\bar{\psi}\psi\right), \tag{2}$$

with $V\left(\bar{\psi}\psi\right)$ denoting the self-interacting potential of the spinor field, which is supposed to be a function of the bilinear $\bar{\psi}\psi$.

Lagrangian of the tachyon field ϕ has the form

$$L_T=-V\left(\phi\right)\sqrt{1-\partial_{\mu}\phi\partial^{\mu}\phi}\,, \tag{3}$$

where $V\left(\phi\right)$ is the self-interaction potential of the tachyon field.

The action (1) we consider jointly with the Friedmann-Robertson-Walker metric

$$ds^2=-dt^2+a(t)^2\left(dx^2+dy^2+dz^2\right), \tag{4}$$

where $a\left(t\right)$ is scale factor dependent of time t.

Curvature scalar R

$$R=6\left(\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}\right). \tag{5}$$

Substituting (2), (3) and (5) in action (1), our Lagrangian is

$$L=-3\dot{a}^2a+a^3\frac{i}{2}\left[\bar{\psi}\gamma^0\dot{\psi}-\dot{\bar{\psi}}\gamma^0\psi\right]-a^3V_2\left(u\right)-a^3V_1\left(\phi\right)\sqrt{1-\dot{\phi}^2}\,, \tag{6}$$

where $u=\bar{\psi}\psi$.

Obtain the system of equations of motion

$$3H^2+2\dot{H}=-p, \tag{7}$$

$$3H^2=\rho, \tag{8}$$

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2}+3H\dot{\phi}+\frac{V_{1\phi}}{V_1}=0, \tag{9}$$

$$\dot{\psi}+\frac{3}{2}H\psi-iV_{2u}\gamma^0\psi=0, \tag{10}$$

$$\dot{\bar{\psi}}+\frac{3}{2}H\bar{\psi}+iV_{2u}\bar{\psi}\gamma^0=0, \tag{11}$$

where (7), (8) - Friedmann equations, (9) - Klein-Gordon equation, (10), (11) - Dirac equations.

The total energy density ρ and the total pressure p are given by

$$\rho = \frac{V_1}{\sqrt{1-\dot{\phi}^2}} + V_2 , \tag{12}$$

$$p = -V_2 + V_{2u}u - V_1\sqrt{1-\dot{\phi}^2} . \tag{13}$$

The total energy density of the sources of the gravitational field can be represented as the sum of the two contributions $\rho = \rho_D + \rho_T$, which are associated with the spinor and tachyon fields, respectively

$$\rho_D = V_2 , \tag{14}$$

$$\rho_T = \frac{V_1}{\sqrt{1-\dot{\phi}^2}} . \tag{15}$$

In the same way, we can represent the total pressure of the sources of the gravitational field as the sum of the pressure $p + p_D + p_T$ associated with the spinor and tachyon fields, respectively

$$p_D = -V_2 + V_{2u}u , \tag{16}$$

$$p_T = -V_1\sqrt{1-\dot{\phi}^2} . \tag{17}$$

In order to analyze cosmological solutions from the proposed model we have to enter new values for tachyon field ϕ

$$\phi = \phi_0 t^{\beta} , \tag{18}$$

where ϕ_0 and β are some constants.

Self-interacting potential of the tachyon field

$$V_1 = \frac{V_{10}}{\phi^2} , \tag{19}$$

where V_{10} - is a constant.

From the (10), (11) find the type of bilinear function $u = \bar{\psi}\psi$

$$u = \frac{C}{a^3} , \tag{20}$$

where C is the constant of inegration.

Using equation (9), we find the Habble parameter H

$$H = \frac{\dot{a}}{a} = \frac{2}{3}\frac{1}{\phi_0^2\beta t^{2\beta-1}} - \frac{(\beta-1)t}{3(1-\phi_0^2\beta t^{2(\beta-1)})} , \tag{21}$$

and scale factor a

$$\ln a = \frac{2}{3\phi_0^2\beta}\frac{t^{-2\beta+2}}{(-2\beta+2)} - \frac{\beta-1}{3}\int\frac{t}{1-\phi_0^2\beta^2 t^{2(\beta-1)}}dt . \tag{22}$$

The function of fermion field ψ we will look in the form

$$\psi_k = A_k(t)e^{-iD_k(t)}. \tag{23}$$

Using equation (10), we obtain

$$A_k(t) = a^{-\frac{2}{3}}A_{k0}, \tag{24}$$

where A_{k0} - is integration constant $(k = 0,1,2,3)$.

$$\begin{aligned} D_k = \int V_{2u}dt = -\frac{4a^3}{3C\phi_0^2\beta t^{2\beta-1}} + \int \frac{a^3}{C}\frac{V_{10}}{\phi_0^2 t^{2\beta}}\left[\sqrt{1-\phi_0^2\beta^2 t^{2(\beta-1)}} - \frac{1}{1-\phi_0^2\beta^2 t^{2(\beta-1)}}\right]dt \\ +\int \frac{a^3}{C}\frac{V_{10}}{\phi_0^2 t^{2\beta}}\left[\frac{2(\beta-1)}{3\left(1-\phi_0^2\beta^2 t^{2(\beta-1)}\right)} + \frac{4(\beta-1)^2\phi_0^2\beta^2 t^{2(\beta-1)}}{3\left(1-\phi_0^2\beta^2 t^{2(\beta-1)}\right)}\right]dt \end{aligned}, \tag{25}$$

where $k = 0,1$.

$$\begin{aligned} D_i = \int V_{2u}dt = \frac{4a^3}{3C\phi_0^2\beta t^{2\beta-1}} - \int \frac{a^3}{C}\frac{V_{10}}{\phi_0^2 t^{2\beta}}\left[\sqrt{1-\phi_0^2\beta^2 t^{2(\beta-1)}} - \frac{1}{1-\phi_0^2\beta^2 t^{2(\beta-1)}}\right]dt \\ -\int \frac{a^3}{C}\frac{V_{10}}{\phi_0^2 t^{2\beta}}\left[\frac{2(\beta-1)}{3\left(1-\phi_0^2\beta^2 t^{2(\beta-1)}\right)} + \frac{4(\beta-1)^2\phi_0^2\beta^2 t^{2(\beta-1)}}{3\left(1-\phi_0^2\beta^2 t^{2(\beta-1)}\right)}\right]dt \end{aligned}, \tag{26}$$

where $i = 2,3$.

From the equation (12), we find potential of the fermion field in terms of t

$$\begin{aligned} V_2(t) = \frac{1}{1-\phi_0^2 t^{2\beta}}\left[\frac{4}{3\phi_0^2\beta^2 t^{2(\beta-1)}} - \frac{V_{10}}{1-\phi_0^2\beta^2 t^{2(\beta-1)}}\right] + \\ +\frac{(\beta-1)t}{3\left(1-\phi_0^2\beta^2 t^{2(\beta-1)}\right)}\left[\frac{(\beta-1)t}{1-\phi_0^2\beta^2 t^{2(\beta-1)}} - \frac{4}{\phi_0^2\beta t^{2\beta-1}}\right] \end{aligned}. \tag{27}$$

Using Friedmann equations (7) and (8), we obtain energy density and pressure for our model

$$\rho = \frac{4}{3\phi_0^4\beta^2 t^{2(2\beta-1)}} + \frac{(\beta-1)t}{3\left(1-\phi_0^2\beta^2 t^{2(\beta-1)}\right)}\left[\frac{(\beta-1)t}{1-\phi_0^2\beta^2 t^{2(\beta-1)}} - \frac{4}{\phi_0^2\beta t^{2\beta-1}}\right], \tag{28}$$

$$\begin{aligned} p = \frac{4}{3\phi_0^2\beta t^{2\beta}}\left[\frac{1}{\phi_0^2 t^{2(\beta-1)}} + (-2\beta+1)\right] - \frac{(\beta-1)t^2}{3\left(1-\phi_0^2\beta^2 t^{2(\beta-1)}\right)^2} + \\ +\frac{(\beta-1)}{3\left(1-\phi_0^2\beta^2 t^{2(\beta-1)}\right)}\left[\frac{4}{\phi_0^2\beta t^{2\beta-1}} + 2 + \frac{4\phi_0^2\beta(\beta-1)t^{2\beta-1}}{1-\phi_0^2\beta^2 t^{2(\beta-1)}}\right] \end{aligned}. \tag{29}$$

Energy density of the spinor and tachyon field (14), (15) taking into scale factor (22)

$$\rho_D=\frac{1}{\phi_0^2t^{2\beta}}\left[\frac{4}{3\phi_0^4\beta^2t^{2(2\beta-1)}}-\frac{V_{10}}{1-\phi_0^2\beta^2t^{2(\beta-1)}}\right]+\frac{(\beta-1)t}{3\left(1-\phi_0^2\beta^2t^{2(\beta-1)}\right)}\left[\frac{(\beta-1)t}{1-\phi_0^2\beta^2t^{2(\beta-1)}}-\frac{4}{\phi_0^2\beta t^{2\beta-1}}\right], \tag{30}$$

$$\rho_T=\frac{V_{10}}{\phi_0^2t^{2\beta}\sqrt{1-\phi_0^2\beta^2t^{2(\beta-1)}}}. \tag{31}$$

Pressure of the spinor and tachyon field (16), (17) taking into scale factor (22)

$$p_D=\frac{(\beta-1)^2}{3\left(1-\phi_0^2\beta^2t^{2(\beta-1)}\right)^2}\left[-t^2+4\phi_0^2\beta^2t^{2(\beta-1)}\right]+\frac{2(\beta-1)}{3\left(1-\phi_0^2\beta^2t^{2(\beta-1)}\right)}\left[\frac{2t}{\phi_0^2\beta t^{2\beta-1}}+1\right]-$$
$$-\frac{1}{\phi_0^2t^{2\beta}}+\left[\frac{4}{3\phi_0^2\beta^2t^{2(\beta-1)}}+\frac{4(-2\beta+1)}{3\beta}-V_{10}\sqrt{1-\phi_0^2\beta^2t^{2(\beta-1)}}\right], \tag{32}$$

$$p_T=\frac{V_{10}\sqrt{1-\phi_0^2\beta^2t^{2(\beta-1)}}}{\phi_0^2t^{2\beta}}. \tag{33}$$

The boson and fermion fields in the early epoch are responsible for the accelerated mode, but since the total pressure tends to zero at a later time, the transition to the slow mode occurs. Tachyon field is crucial only in the early times. Isotropization, which occurs in the late epoch, is associated with the presence of a boson and fermion field as sources of a gravitational field.

The equation of state parameter ω [3], [4]

$$\omega(t)=\frac{p}{\rho}=-1-\frac{2}{3}\frac{\dot{H}}{H^2}, \tag{34}$$

it should be $\omega\approx-1$ [5]-[9]. For our models

$$\omega=-1-\frac{(1-2\beta)\left(2-4\phi_0^2\beta^2t^{2(\beta-1)}+2\phi_0^4\beta^4t^{4(\beta-1)}\right)+\phi_0^2\beta t^{2\beta}(1-\beta)-\phi_0^4\beta^3t^{2(\beta-1)}\left(1-5\beta-2\beta^2\right)}{4\left(1-\phi_0^2\beta^2t^{2(\beta-1)}\right)+\phi_0^2\beta t^{2(\beta+1)}\left(1-\beta-\beta^2\right)+4(1-\beta)\left(t^2-\phi_0^2\beta^2t^{2\beta}\right)} \tag{35}$$

Thus, we have considered exact solution for cosmological model with fermion and tachyon fiels together with a homogeneous, isotropic and flat Friedman-Robertson-Walker universe. For this model, we have found the exact solution, found a scale factor, restored the potential of the fermion field. The parameter of the equation of state ω, whose value corresponds to the accelerated expansion of the universe for $\beta>1$.

Model which we consider, the fermion field corresponds to the accelerated mode in the late period, and the tachyon field contributes only to the early period.

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