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POLYNOMIAL SOLUTION FOR A SPIN MODEL WITH SELF-CONSISTENT POTENTIAL

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1.Introduction

Solutions of integrable nonlinear differential equations which could be a singular wave and have flexible properties of interaction with the same another arrangement have a assortment of applications in numerous regions of the characteristic sciences. Expository considers of forms of interaction of the singular waves is one of the most assignments of the hypothesis of solitons. The advancement of nonlinear hypothesis of attraction, in turn, has put the issue of building fundamental generalization of the Landau- Lifshitz equation with self-consistent potential.

One of such generalizations with self-consistent scalar potential was proposed in [1]. Different algebraic-geometric viewpoints of such models were considered in [2-5]. The generalized Landau-Lifshitz condition with self-consistent vector potential was gotten in [6], additionally their association with the development of bends and surfaces are established. The Landau-Lifshitz-Hilbert equation in ferromagnetism and the Landau-Lifshitz-Hilbert-Slonchevsky equation in nanomagnetic multilayers with spin transfer are among the fundamental equations that play a crucial role in understanding various physical properties of magnetic materials and the development of new technological innovations, such as microwave generation using the spin transfer effect . They are also closely related to the nonlinear family of Schrödinger equations through geometric (or Lakshmanan equivalence or L-equivalence) and the concept of gauge equivalence, and these systems often allow magnetic soliton solutions.

2.Generalized Landau-Lifshitz equation

The Generalized Landau-Lifshitz equation (GLL) equation with self-consistent potential reads as

$$iS_t + \frac{1}{2}[S, S_{xx}] + \frac{1}{a}[S, W] = 0, \qquad (2.1)$$

$$iW_x + a[S, W] = 0, \qquad (2.2)$$

where $a = const, S = \sum_{j=1}^{3} S_j(x,y,t)\sigma_j \quad S = \begin{pmatrix} S_3 & S^- \\ S^+ & -S_3 \end{pmatrix}$ is a matrix analogue of the spin vector , W – potenstial with the matrix form $W = \sum_{j=1}^{3} W_j(x,y,t)\sigma_j$, and they

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$

are Pauli matrices. The length of $\left|\vec{S}\right|^2 = 1$.

The GLL equation with self-consistent potential is integrable by the inverse scattering method and admits the following Lax representation:

$$\Psi_x = U\Psi , \tag{2.3}$$

$$\Psi_t = V\Psi , \tag{2.4}$$

where the matrix operators U and V are given as

$$U = -i\lambda S , \tag{2.5}$$

$$V = \lambda^2 V_2 + \lambda V_1 + (\frac{i}{\lambda + a} - \frac{i}{a})W , \tag{2.6}$$

Here

$$V_2 = -2iS, V_1 = SS_x . \tag{2.7}$$

The compatibility condition of equations (2.3) and (2.4) gives us equations (2.1)-(2.2).

3. Bilinearization

The bilinear form for equations (2.1) - (2.2) where given in [7] and has the form:

$$[iD_t + D_x^2](g \cdot f) - \frac{2}{a} p^* q = 0 , \tag{3.1}$$

$$D_x^2(f \cdot f) - \frac{2}{a} q^* q = 0 , \tag{3.2}$$

$$D_x(p \cdot f) - 2iag^* q = 0 , \tag{3.3}$$

$$D_x(q \cdot f) + 2iaqf = 0 , \tag{3.4}$$

$$p = \varphi_1 f e^{iax} , q = \varphi_2 f e^{iax} \tag{3.5}$$

where f(x,t) is a real functions, g(x,t), p(x,t), q(x,t) are complex functions. Here Hirota's operators characterized by

$$D_x^l D_t^n f(x,t) \cdot g(x,t) = (\partial_x - \partial_{x'})^l (\partial_t - \partial_{t'})^n f(x,t) \cdot g(x',t')\big|_{x=x',t=t'} . \tag{3.6}$$

4.Solutions

We decompose in equation (3.8) - (3.11) g, f, p and q into formal series with an arbitrary constant ε:

$$g(x,t) = \varepsilon g_1(x,t) + \varepsilon^3 g_3(x,t) + ... ,$$

$$(4.1)\ f(x,t) = 1 + \varepsilon^2 f_2(x,t) + \varepsilon^4 f_4(x,t) + ... , \qquad (4.2)$$

$$p(x,t) = 1 + \varepsilon^2 p_2(x,t) + \varepsilon^4 p_4(x,t) + ... , \qquad (4.3)$$

$$q(x,t) = \varepsilon q_1(x,t) + \varepsilon^3 q_3(x,t) + ... \qquad (4.4)$$

In arrange to get one-soliton solution for equations (3.2) - (3.5), we select $g = \varepsilon g_1$

$$p = 1 + \varepsilon^2 p_2 \ , \ \ q = \varepsilon q_1 \ , \ \ f = 1 + \varepsilon^2 f_2 .$$

If we choose function g_1 as

$$g_1 = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \ \ a_4 x^4 + b_1 t - 24 i a_4 t x + a_5 b_2 x^2 t^2 ,$$

where $a_0, a_1, a_2, a_3, a_4, b_1, a_5, b_2$ are complex constants. Then we can construct solutions for f_2, p_2, and q_1 in forms $a_i, b_j, i = 0,1,...5, j = 1,2.$

$$q_1 = \frac{i}{2} a(2tx^2 a_5 b_2 - 24ixa_4 + b_1) + 2t^2 a_5 b_2 + 12x^2 a_4 + 6xa_3 + 2a_2 \qquad (4.5)$$

$$f_2 = \frac{a^2 k^3 + ak'}{-4k + 8ia^2 k} , \qquad (4.6)$$

$$p_2 = \frac{k}{(2ia+1)^2} - \frac{a^* k^*}{4(2ia+1)^2} + \frac{a^2(i-2a)}{2ia+1}(x^4 a^*{}_4 + x^3 a_3^* + (t^2 a^*{}_5 b_2^*)x^2 + (-24ta_4 + a_1^*)x + b_1^* t + a_0^*) \qquad (4.7),$$

where $k = i(2tx^2 a_5 b_2 - 24ixa_4 + b_1) + 2t^2 a_5 b_2 + 12x^2 a_4 + 6xa_3 + 2a_2$.

Components of the matrix S has the form:

$$S^+ = \frac{2fg}{f^2 + |g|^2}, S^- = \frac{2fg^*}{f^2 + |g|^2}, S_3 = \frac{f^2 - |g|^2}{f^2 + |g|^2} . \qquad (4.8)$$

A polynomial solution of equation (2.1)-(2.2) for the spin vector S is given by

$$\begin{aligned} S_1 = {} & \frac{t^2 x^2 a_5 b_2 + x^4 a_4 + x^3 a_3 - 24txa_4 + x^2 a_2 + a\ \ x + b_1 t + a_0}{a^2 k^3 a^* k^* + 16(-\frac{ak}{2} - ia^2 k)^2 (1 + (t^2 x^2 a_5 b_2 + x^4 a_4 + x^3 a_3 - 24txa_4 + } + \\ & + x^2 a_2 + a\ \ x + b_1 t + a_0)(t^2 x^2 a^*{}_5 b^*{}_2 + x^4 a^*{}_4 + x^3 a^*{}_3 - 24txa^*{}_4 \\ & + x^2 a^*{}_2 + a_1^* x + b_1^* t + a^*{}_0) \\ & + \frac{(t^2 x^2 a_5 b_2 + x^4 a_4 + x^3 a_3 - 24txa_4 + x^2 a_2 + a\ \ x + b_1 t + a_0) \cdot \cdot (t^2 x^2 a^*{}_5 b^*{}_2 + x^4 a^*{}_4 + x^3 a^*{}_3 - 24txa^*{}_4 + x^2 a^*{}_2 + a_1^* x + b_1^* t + a^*{}_0)}{16(\frac{ak}{2} - ia^2 k^2)^2} (a^2 k^3 a^* k^*). \end{aligned} \qquad (4.17)$$

$$S_2=-\frac{i}{2}\Big(\frac{t^2x^2a_5b_2+x^4a_4+x^3a_3-24txa_4+x^2a_2+a\ x+b_1t+a_0}{a^2k^3a^*k^*+16(-\frac{ak}{2}-ia^2k)^2(1+(t^2x^2a_5b_2+x^4a_4+x^3a_3-24txa_4+ +x^2a_2+a\ x+b_1t+a_0)(t^2x^2a^*{}_5b^*{}_2+x^4a^*{}_4+x^3a^*{}_3-24txa^*{}_4 +x^2a^*{}_2+a_1{}^*x+b_1{}^*t+a^*{}_0)}+$$

$$+\frac{(t^2x^2a_5b_2+x^4a_4+x^3a_3-24txa_4+x^2a_2+a\ x+b_1t+a_0)\cdot \cdot(t^2x^2a^*{}_5b^*{}_2+x^4a^*{}_4+x^3a^*{}_3-24txa^*{}_4+x^2a^*{}_2+a_1{}^*x+b_1{}^*t+a^*{}_0)}{16(\frac{ak}{2}-ia^2k^2)^2}(a^2k^3a^*k^*)\Big), \quad (4.18)$$

$$S_3=\frac{a^2k^3a^*k^*+16(-\frac{ak}{2}-ia^2k)^2-(t^2x^2a_5b_2+x^4a_4+x^3a_3-24txa_4 +x^2a_2+a\ x+b_1t+a_0)(t^2x^2a^*{}_5b^*{}_2+x^4a^*{}_4+x^3a^*{}_3-24txa^*{}_4 +x^2a^*{}_2+a_1{}^*x+b_1{}^*t+a^*{}_0)(16(-\frac{ak}{2}-ia^2k)^2)^2}{a^2k^3a^*k^*+16(-\frac{ak}{2}-ia^2k)^2+(t^2x^2a_5b_2+x^4a_4+x^3a_3-24txa_4 +x^2a_2+a\ x+b_1t+a_0)(t^2x^2a^*{}_5b^*{}_2+x^4a^*{}_4+x^3a^*{}_3-24txa^*{}_4 +x^2a^*{}_2+a_1{}^*x+b_1{}^*t+a^*{}_0)(16(-\frac{ak}{}-ia^2k)^2)^2}. \quad (4.19)$$

Also taking into account that vector potential W has form (3.2) and in terms q and p (3.7) the following we obtain solution

$$W_1=\frac{8ia^{*2}k^*-4k^*}{8ia^{*2}k^*-4k^*+a^{*2}k^{*3}+a^*k^*}\left(1+\frac{k}{(2ia+1)^2}-\frac{a^*k^*}{4(2ia+1)^2}+\frac{a^2(i-2a)}{2ia+1}(x^4a^*{}_4+x^3a_3^*+(t^2a^*{}_5b_2^*)x^2+(-24ta_4+a_1^*)x+b_1^*t+a_0^*)\right) \quad (4.20)$$

$$\cdot\frac{ak\left(8ia^2k-4k\right)}{2(8ia^2k-4k+a^2k^3+ak^*)}+\frac{8ia^2k-4k}{2\left(8ia^2k-4k+a^2k^3+ak^*\right)}$$

$$\left(1+\frac{k}{(2ia+1)^2}-\frac{a^*k^*}{4(2ia+1)^2}+\frac{a^2(i-2a)}{2ia+1}(x^4a^*{}_4+x^3a_3^*+(t^2a^*{}_5b_2^*)x^2+(-24ta_4+a_1^*)x+b_1^*t+a_0^*)\right)$$

$$\frac{a^*k^*\left(8ia^{*2}k^*-4k^*\right)}{2(8ia^{*2}k^*-4k^*+a^{*2}k^{*3}+a^*k^{**})}.$$

$$W_2=\frac{8ia^{*2}k^*-4k^*}{8ia^{*2}k^*-4k^*+a^{*2}k^{*3}+a^*k^*}\left(1+\frac{k}{(2ia+1)^2}-\frac{a^*k^*}{4(2ia+1)^2}+\right.$$

$$\left.\frac{a^2(i-2a)}{2ia+1}(x^4a^*{}_4+x^3a_3^*+(t^2a^*{}_5b_2^*)x^2+(-24ta_4+a_1^*)x+b_1^*t+a_0^*)\right)\frac{ak\left(8ia^2k-4k\right)}{2(8ia^2k-4k+a^2k^3+ak^*)}-$$

$$-\frac{8ia^2k-4k}{2\left(8ia^2k-4k+a^2k^3+ak^*\right)}-1+\frac{k}{(2ia+1)^2}-\frac{a^*k^*}{4(2ia+1)^2}+\frac{a^2(i-2a)}{2ia+1}(x^4a^*{}_4+x^3a_3^*+(t^2a^*{}_5b_2^*)x^2+ \quad (4.21)$$

$$+(-24ta_4+a_1^*)x+b_1^*t+a_0^*)\frac{a^*k^*\left(8ia^{*2}k^*-4k^*\right)}{2(8ia^{*2}k^*-4k^*+a^{*2}k^{*3}+a^*k^{**})}$$

$$W_3 = \frac{8ia^2k-4k}{2\left(8ia^2k-4k+a^2k^3+ak^*\right)}\frac{8ia^{*2}k^*-4k^*}{8ia^{*2}k^*-4k^*+a^{*2}k^{*3}+a^*k^*}$$
$$\left(1+\frac{k}{(2ia+1)^2}-\frac{a^*k^*}{4(2ia+1)^2}+\frac{a^2(i-2a)}{2ia+1}(x^4a^*{}_4+x^3a_3^*+(t^2a^*{}_5b_2^*)x^2+(-24ta_4+a_1^*)x+b_1^*t+a_0^*)\right) \quad (4.22)$$
$$\left(1+\frac{k^*}{(2ia^*+1)^2}-\frac{a^*k^*}{4(2ia^*+1)^2}+\frac{a^2(i-2a^*)}{2ia+1}(x^4a^*{}_4+x^3a_3^*+(t^2a^*{}_5b_2^*)x^2+(-24ta^*{}_4+a_1^*)x+b_1^*t+a_0^*)\right)$$
$$+\left(\frac{aka^*k^*}{2}\right).$$

The behavior of the spin matrix S and potential W is clearly visible in the figure below. The graphs of the obtained solutions are given in the time interval from 0 to 20. For figure 1 we obtain visualization of solution (4.17) and (4.19), with parameters:

$$a = 25-i, a_1 = 6, a_2 = 5, a_3 = 5, a_4 = 5, a_5 = 5, b_1 = 2, b_2 = 25-i, a_0 = 3.$$

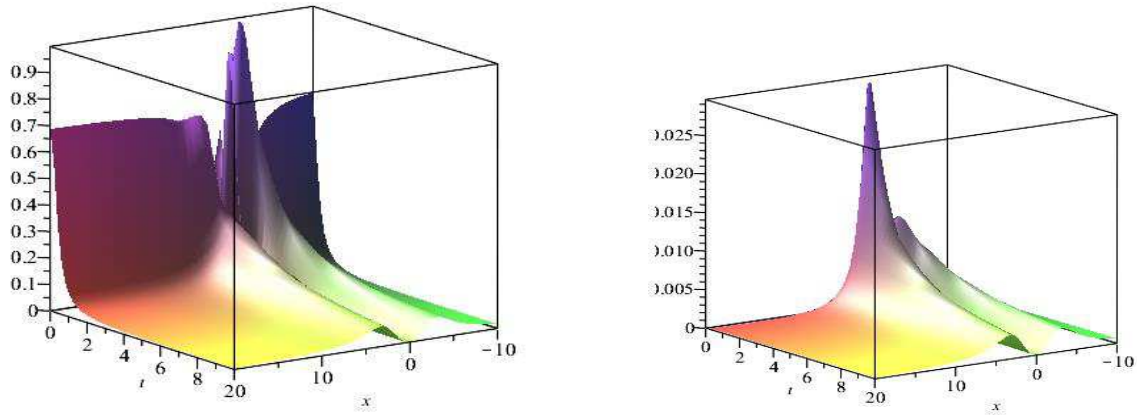

Figure 1 – Solution for components of spin vector $S_1, S_2, x = [-10...20], t = [0...20]$

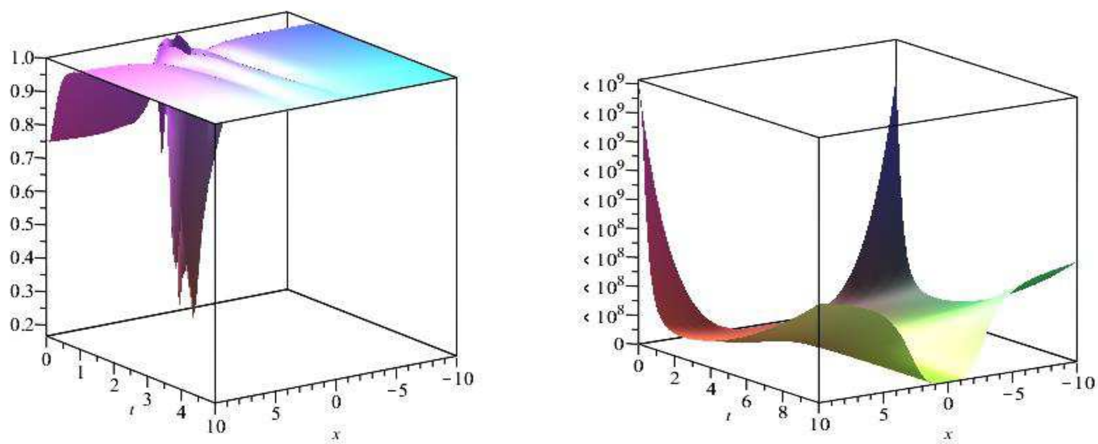

Figure 2– Solutions for components of spin vector S_3 and vector potential W_3 with parameters x=[-10,10], t=[0,10]

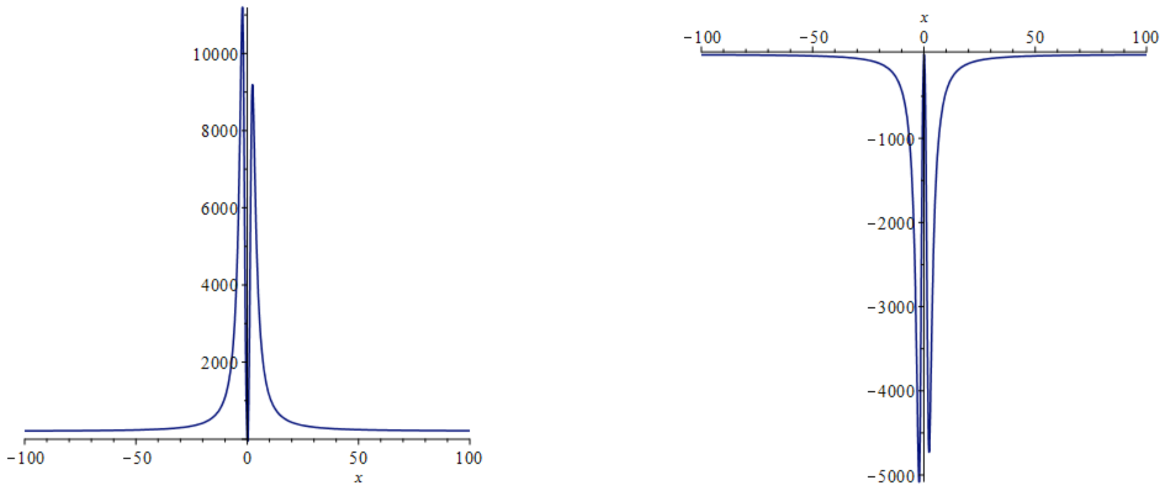

Figure 3– Solutions for S_1, S_2 and potential W_1, W_2, $x=[-100...100], t=25$.

5. Conclusion

Spin systems are intriguing nonlinear dynamical systems. In specific integrable spin systems have pertinence in connected ferromagnetism and nanomagnetism. More interests, integrable spin systems have near association with nonlinear Schrodinger family of equations. In this work, we have a steady continuation of our past investigate within the field of indispensable spin systems, in specific, the generalized Landau-Lifshitz equations with a self-consistent vector potential. Subsequently, the generalized Landau-Lifshitz equations with a self-consistent vector potential, which we considered, isn't constrained to the conclusions and regularities defined for the spin demonstrate with a scalar potential. The main result of this paper finding a polynomial solution of GLL with self-consistent potential.

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