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Студенттер мен жас ғалымдардың  
**«ҒЫЛЫМ ЖӘНЕ БІЛІМ - 2018»**  
XIII Халықаралық ғылыми конференциясы

### **СБОРНИК МАТЕРИАЛОВ**

XIII Международная научная конференция  
студентов и молодых ученых  
**«НАУКА И ОБРАЗОВАНИЕ - 2018»**

The XIII International Scientific Conference  
for Students and Young Scientists  
**«SCIENCE AND EDUCATION - 2018»**



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the field  $\phi$  is monotonically increasing. For the field  $\xi$ , we discovered that it is increasing till the late time  $t \rightarrow \infty$ . Spacetime in this class is asymptotically considered as de Sitter for  $t \rightarrow \infty$ .

In this work we considered a formal framework of nonlocal f(T) theory of gravity and investigated the nonlocal f(T) theory through the Noether symmetry. We derived the Noether equations of the nonlocal f(T) theory in FLRW universe. We analysed the dynamics of the field in nonlocal f(T) gravity using the Noether symmetry. In summary, we observed classes of solutions that there exists the transition from a deceleration phase to the acceleration one in our present analysis.

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## CONSERVATION LAWS FOR THE INHOMOGENEOUS HIROTA AND THE MAXWELL-BLOCH SYSTEM

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**Introduction.** In recent years, nonlinear science has emerged as a powerful subject for explaining the mystery present in the challenges of science and technology today. Among nonlinear science, the interplay between dispersion and nonlinearity gives rise to several important phenomena in optical fibers, including parametric amplification, wavelength conversion, modulational instability(MI), soliton propagation and so on. Among all concepts, solitons, positons and rogons have been not only the subject of intensive research in oceanography [1, 2] but also it has been studied extensively in several areas, such as Bose-Einstein condensate, plasma, superfluid, finance, optics and so on [3-9].

An important ingredient in the development of the theory of soliton and of complete integrability has been the interplay between mathematics and physics. In 1973, Hasegawa and Tappert [10] modeled the propagation of coherent optical pulses in optical fibres by nonlinear Schrodinger (NLS) equation without the inclusion of fibre loss. They showed theoretically that generation and propagation of shape-preserving pulses called solitons in optical fibres is possible by balancing the dispersion and nonlinearity. The H-MB system has been shown to be integrable and also admits the Lax pair and other required properties for complete integrability [11].

In this paper, we will concentrate on the inhomogeneous Hirota and the Maxwell-Bloch (H-MB) system as following specific form [12, 13],

$$q_z = -\left(a_1(z)q_t + a_2(z)q + ia_3(z)q_u + a_4(z)q_{uu} + a_5(z)|q|^2 q_t + ia_6(z)|q|^2 q + a_7(z)p\right) \quad (1)$$

$$p_t = 2b_1(z)q\eta - 2ib_2(z)\omega p, \quad (2)$$

$$\eta_t = -b_1(z)(qp^* + q^* p), \quad (3)$$

with constraint

$$a_2 = \frac{\partial b_1}{\partial z}; a_5 = 6a_4 b_1^2; a_6 = 2a_3 b_1^3. \quad (4)$$

In the equations above,  $z$  and  $t$  represent the normalized distance and time respectively,  $E(z,t)$  denotes the slowly varying envelope axial field,  $p(z,t)$  is the measure of the polarization of the resonant medium,  $\eta(z,t)$  and represents the extent of the population inversion.  $a_1(z)$  results from the group velocity and  $a_2(z)$  describes the amplification or absorption. The coefficients  $a_3(z) - a_6(z)$  represent the group velocity dispersion (GVD), the third-order dispersion(TOD) [14], self-steepening (SS) [15], and self-phase modulation respectively.  $a_7(z)$  is the parameter describing the averaging with respect to inhomogeneous broadening of the resonant frequency.  $b_1(z)$  and  $b_2(z)$  depict the character of interactions between the propagation field and the erbium atoms. The real parameter  $\omega$  is a constant corresponding to the frequency, and the \* denotes the complex conjugate. If we set

$$a_1 = a_2 = 0; a_3 = -\frac{1}{2}\alpha, a_4 = -\beta, a_5 = -6\beta, a_6 = -\alpha, b_1 = 1, b_2 = -1. \quad (5)$$

inhomogeneous H-MB equation will be reduced to H-MB equation as following

$$q_z = -i\alpha\left(\frac{1}{2}q_{tt} + |q|^2 q\right) + \beta\left(q_{ttt} + 6|q|^2 q_t\right) + 2p, \quad (6)$$

$$p_t = 2q\eta + 2i\omega p, \quad (7)$$

$$\eta_t = -(qp^* + q^* p), \quad (8)$$

We will call the inhomogeneous Hirota and the Maxwell-Bloch system when  $\alpha = 2$ ,  $\beta = -1$  the classical H-MB equation. The linear eigenvalue problem of IH-MB takes the form

$$\Phi_z = U\Phi, \quad (9)$$

$$\Phi_t = V\Phi, \quad (10)$$

where  $U$  and  $V$  can be expressed in following polynomials about complex constant eigenvalue parameter  $\lambda$

$$U = \begin{pmatrix} \lambda & b_1(z)q \\ -b_1(z)q^* & -\lambda \end{pmatrix} = \lambda\sigma_3 + U_0, \quad (11)$$

$$V = \lambda^3 \begin{pmatrix} -4a_4 & 0 \\ 0 & 4a_4 \end{pmatrix} + \lambda^2 \begin{pmatrix} -2ia_3 & -4a_4 b_1 q \\ 4a_4 b_1 q^* & 2ia_3 \end{pmatrix} + \lambda \begin{pmatrix} -a_1 - 2a_4 b_1^2 |q|^2 & -2ib_1(a_3 q - ia_4 q_t) \\ 2ib_1 a_3 q^* - 2ia_4 q_t & a_1 + 2a_4 b_1^2 |q|^2 \end{pmatrix} \quad (12)$$

$$+ \begin{pmatrix} -C_1 & -b_1 B_1 \\ -b_1 B_1^* & C_1 \end{pmatrix} + A_1 \begin{pmatrix} \eta & -p \\ -p^* & -\eta \end{pmatrix} = \lambda^3 V_3 + \lambda^2 V_2 + \lambda V_1 + V_0 + \frac{1}{\lambda + i\omega b_2} V_{-1},$$

$$V_{-1} = -\frac{b_1 a_7}{2} \begin{pmatrix} \eta & -p \\ -p^* & -\eta \end{pmatrix}, \quad (13)$$

$$A_1 = \frac{b_1 a_7}{2\lambda + i\omega b_2}, B_1 := 2a_4 b_1^2 q^* q^2 + a_1 q + ia_3 q_t + a_4 q_{tt}, \quad (14)$$

$$C_1 := ia_3 b_1^2 q q^* + a_4 b_1^2 (q^* q_t - q q_t^*) \quad (15)$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (16)$$

Defining  $\Gamma = \frac{\psi_2}{\psi_1}$ , we can derive a  $\Gamma$ -Riccati-type equation from the Lax pair of Eq. (20)-(21) as

$$\Gamma_t = -b_1(z)q^* - 2\lambda\Gamma - b_1(z)q\Gamma^2. \quad (28)$$

Substituting the expansion  $\Gamma = \frac{1}{q} \sum_{j=1}^{\infty} \Gamma_j \lambda^{-j}$  into equation (28) and equating the like powers of  $\lambda$  to zero, we can obtain the recursion formulas for  $\Gamma_n$

$$\Gamma_1 = -\frac{b_1(z)}{2} |q|^2 \quad (29)$$

$$\Gamma_2 = -\frac{1}{2} \left( \Gamma_{1,t} - \frac{q_t}{q} \Gamma_1 \right), \quad (30)$$

$$\Gamma_3 = -\frac{i}{2} \left( \Gamma_{2,t} - \frac{q_t}{q} \Gamma_2 + b_1(z) \Gamma_1^2 \right), \quad (31)$$

$$\Gamma_{j+1} = -\frac{i}{2} \left( \Gamma_{j,t} - \frac{q_t}{q} \Gamma_j + b_1(z) \sum_{k=1}^{j-1} \Gamma_k \Gamma_{j-k} \right), (j = 2, 3, 4, \dots), \quad (32)$$

Similarly, by using  $(\ln \psi_1)_{tx} = (\ln \psi_1)_{xt}$ , and symbolic computation, the infinitely many conservation laws for equations (17) - (19) can be constructed

$$i \frac{\partial}{\partial t} \rho_i = \frac{\partial}{\partial x} J_i, (i = 1, 2, 3, \dots)$$

where  $\rho_i$  and  $J_i, (i = 1, 2, 3, \dots)$  are the conserved densities and associated fluxes, respectively, then we multiply both sides to  $\lambda + i\omega b_2$ , while the first three conservation laws are shown as

$$\begin{aligned} \rho_1 &= -\frac{i}{2} (b_1(z))^2 |q|^2, \\ J_1 &= -i\omega b_2 C_1 - \frac{b_1 a_7}{2} \eta + 4ia_4 b_1 \left( -\frac{(b_1(z))^3}{8} |q|^4 - \frac{b_1(z)}{8} q q_{tt}^* \right) + \frac{b_1(z)}{4} q q_t^*, \\ &\cdot \left( -4a_4 b_1 i \omega b_2 - \frac{2ib_1}{q} (a_3 q - ia_4 q_t) \right) - \left( \frac{2\omega b_1 b_2}{q} (a_3 q - ia_4 q_t) - \frac{b_1 B_1}{q} \right) \frac{(b_1(z))^3}{2} |q|^2, \\ \rho_2 &= \frac{i}{4} (b_1(z))^2 q_t^* q + \omega \frac{(b_1(z))^2}{2} b_2 |q|^2, \\ J_2 &= \left( -4a_4 b_1 i \omega b_2 - \frac{2ib_1}{q} (a_3 q - ia_4 q_t) \right) \left( -\frac{(b_1(z))^3}{8} |q|^4 - \frac{b_1(z)}{8} q q_{tt}^* \right) + \frac{b_1(z)}{4} q q_t^*, \\ &\cdot \left( \frac{2\omega b_1 b_2}{q} (a_3 q - ia_4 q_t) - \frac{b_1 B_1}{q} \right) + \left( -\frac{i\omega b_2 b_1}{q} B_1 + \frac{b_1 a_7}{2q} p \right) \left( -\frac{b_1(z)}{2} |q|^2 \right), \\ \rho_3 &= -\frac{i}{8} (b_1(z))^4 |q|^4 - \frac{i}{8} (b_1(z))^2 q_{tt}^* q - \frac{i}{4} (b_1(z))^2 b_2 \omega q q_t^*, \\ J_3 &= \left( \frac{2\omega b_1 b_2}{q} (a_3 q - ia_4 q_t) - \frac{b_1 B_1}{q} \right) \left( -\frac{(b_1(z))^3}{8} |q|^4 - \frac{b_1(z)}{8} q q_{tt}^* \right) + \frac{b_1(z)}{4} q q_t^* \cdot \left( -\frac{i\omega b_1 b_2}{q} B_1 + \frac{b_1 a_7}{2q} p \right). \end{aligned}$$

The existent of infinite conservation laws helpfully indicates the completely integrable property of equation (3).

**Conclusion.** In this paper we consider the creation of an infinite number of conservation laws for the inhomogeneous Hirota and the Maxwell-Bloch (H-MB) system. Creating an infinite number of conservation laws for the inhomogeneous Hirota and the Maxwell-Bloch (H-MB) system -Lax pair and Riccat equation. This means that the system of equations, which we discussed above, is sufficiently integrated.

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### ЖАЛПЫЛАНГАН ЛАНДАУ-ЛИФШИЦ ТЕНДЕУЛЕРИНІҢ ЕКІ СОЛИТОНДЫ ШЕШІМІ

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