# Method of calculation for dynamic strength of complex configuration frame structures of locomotives for transport engineering

*Galina* Khromova[1], *Davran* Radjibaev[1,*], and *Aliya* Zabiyeva[2]

[1]Tashkent State Transport University, Tashkent, Uzbekistan
[2]Eurasian National university named after Gumilev,Astana, Kazakhstan

**Abstract.** The article presents a numerical and analytical method for studying the bending-longitudinal vibrations of the load-bearing frame of the locomotive frame of a complex configuration in the form of an elastic rod of variable cross-section with variable mass, bending and longitudinal stiffness, based on methods: piecemeal linear approximation, iterations and the Boundary Element Technology. Numerical studies are performed in C # and in MATHCAD 15.

## 1 Introduction

The evolution of the modern theory of vibrations of the load-bearing equivalent frame of locomotives and their spring suspension system is characterized by the simultaneous wide use of theoretical research methods with numerical processing of results on computers and experimental data on the stress-strain state of structures with dynamic strength calculations. In this case, numerical technologies are widely used, including the use of MATHCAD programming environment for calculations of the mechanical part of locomotives and the Boundary Element Technology for selection of rational parameters of assemblies and parts of the mechanical part of locomotives. [1-5].

Scientific researches directed on the solution of actual problems on improvement of methods of calculation of stress-strain state of rolling stock frames for transport machine building and development of technical means of their functional diagnostics are carried out in leading scientific centers and high educational establishments of the world, including in Massachusetts Institute of Technology (MIT) (USA), Washington State University (USA), Wessex Institute of Technology (Cambridge, UK, Great Britain), Departament of Mechanical Engineering University of Maine at Orono (Canada), University of Naples di Napoli (Italy), Technische University of Munchen, Technische University of Dresden (Germany), in Tokyo Institute of Technology (TIT) (Japan), ASEA (Sweden), Karabuk University (Turkey), Departament of Mechanical Engineering, Sree Vani Group of Education Society (India), Silesian University of Technology (Poland), Slovak University of Technology in Bratislava (Slovakia), in Russian University of Transport (Russia), in Emperor Alexander I St. Petersburg State Transport University (Russia), Omsk State Transport University (Russia),

* Corresponding author: davronmail@bk.ru

in Rostov State Transport University (Russia), in Almaty University of Power Engineering and Telecommunications (AUPET), Kazakh University of Railway Communications, (Almaty, Kazakhstan), in Tashkent State Transport University (Uzbekistan) and other educational and research centers of railway transport engineers [1÷7].

In Uzbekistan, the problems of dynamic strength calculation of rolling stock frames were studied by academician of the Academy of sciences of Republic of Uzbekistan, professor, d.s.c. A.D. Glushchenko, professors Sh.S. Fayzibaev, G.A. Khromova, Shermukhamedov A.A., Rakhimov R.V., Mukhamedova Z.G., Khamidov O.R., and their students [8, 11÷17].

Electric locomotives of VL80 series have been exploited by JSC "Uzbekistan Temir Yollari" in the Republic of Uzbekistan for more than 50 years. In operation, the main frame and the electric locomotive in general are subjected to alternating loads, a large temperature range, and a high level of humidity. All these factors lead to wear and damage of parts and assemblies of the electric locomotive, as well as the formation of fatigue cracks. In order to respond to the emerging necessity in the Republic of Uzbekistan, for electric locomotives of VL-80 series it was started to organize overhaul and restoration repair with extension of useful life for 8-10 years.

Implementation of the proposed methodology of theoretical-experimental research to justify the process of modernization of the main frame of the body of electric locomotive VL-80 in the conditions of operation in JSC "Uzbekistan Temir Yullari" will contribute to reduce the probability of their accidental destruction and will increase the useful life for 8-10 years after capital-recovery repair [11,12]. In this case, carrying out a properly calculated process of modernization of the main frames of electric locomotive bodies with the installation of reinforcing linings can increase their reliability and durability, and will also extend the useful life.

## 2 Objects and methods of research

Object of research are main body frame and bogie frames of locomotives in case of their modernization by installing reinforcing linings on the weakened sections with fatigue cracks.

The research methodology includes the analysis of systems of equations in partial derivatives that describe oscillations of sections of elastic curvilinear elements of variable bending stiffness with a spatial arrangement of sections, taking into account the influence of increased speeds, the solution of which is carried out by the methods of operational Laplace calculus and the further use of iterative methods (the method of piecewise linear approximations) on a computer based on the Fourier and Bubnov-Galerkin methods, numerical studies were performed in the MathCad 15 environment. For numerical calculation programs, 2 certificates of official registration of the computer program of the Republic of Uzbekistan (No. DGU 07664, 31.04.2020 and No. DGU 10286 of 24.02.2021) were received [9,10].

On the basis of our earlier studies [11÷17] we propose a generalized analytical and numerical method for calculating the dynamic strength of the load-bearing frame of a locomotive frame of complex configuration in the form of a model of an elastic rod of variable cross-section with variable mass, bending and longitudinal strength.

## 3 Theoretical and numerical results

The equated load-bearing frame of a locomotive of complex configuration is modeled by an elastic rod of variable cross-section with variable mass, bending and extension stiffness. The difference of the proposed model from the existing ones [9,10,13] is that it takes into account the variability of cross-section, mass, bending and extension strength along the length of the equivalent beam, which complies with the real operating conditions. In existing calculation methods, for simplicity, it is assumed that the beam is of equal resistance or an approximate calculation is performed using a model with concentrated parameters without taking elasticity into account. These approximate models in dynamics can create errors up to 150 -200 % [10] from real deformations and stresses. Therefore, in practice, it is always necessary to perform experimental studies and introduce dynamic correction factors in the strength and stability calculations.

For the proposed model, the parameters of the equated load-bearing frame of the modernized locomotive body frame of complex configuration are taken as variable functions:

- variable mass by sections of the modernized frame of the locomotive body of complex configuration (kg/m) $m_{lb}\ (x)$, determined using the formula (1), in this case the complex character of loading is taken into account, calculated under quasi-static weighting of concentrated $Q_i$ and distributed loads $q_i$ (VL-80s electric locomotive was considered as a design example),

$$m_{lb}\ (x) = \begin{vmatrix} q_{11} & q_{12} & q_{13} & q_{14} \cdots & q_{1i} \\ q_{21} & q_{22} & q_{23} & q_{24} \cdots & q_{2i} \\ q_{31} & q_{32} & q_{33} & q_{34} \cdots & q_{3i} \\ q_{41} & q_{42} & q_{43} & q_{44} \cdots & q_{4i} \\ q_{51} & q_{52} & q_{53} & q_{54} \cdots & q_{5i} \\ q_{61} & q_{62} & q_{63} & q_{64} \cdots & q_{6i} \\ q_{71} & q_{72} & q_{73} & q_{74} \cdots & q_{7i} \\ q_{81} & q_{82} & q_{83} & q_{84} \cdots & q_{8i} \end{vmatrix} \quad (1)$$

Figure 1 shows the distribution of bending stresses from weight loading in the upper fibers of cross-sections of the sidewall of the main frame of the body of the electric locomotive VL-80 (for the reinforced frame sidewall – $\boldsymbol{\sigma}^{U}_{Rstat}$; as also before the reinforcement – $\boldsymbol{\sigma}^{U}_{stat}$). The stress epuray in the lower fibers is shown in Figure 2.

Figure 3 shows the graphs of variable mass variation by sections of the main frame of the VL-80s electric locomotive (t/m). Comparative analysis of the graphs obtained by quasi-static calculation and modeling of this function by spline shows that the polynomial degree $n = 8$ adopted in the calculation models the process with an error $\delta = 10^{-3}$

-variable cross-sectional area $F_{lb}(x)$ (determined according to formula (2), where, for example, the length of the main load-bearing frame of the body of an electric locomotive VL-80s is 15.2 meters, and the coordinate $x$ varies in the range of $0 \leq x \leq 15{,}2$ m

$$F_{lb}(x) = \begin{vmatrix} f_{11} & f_{12} & f_{13} & f_{14} \cdots & f_{1i} \\ f_{21} & f_{22} & f_{23} & f_{24} \cdots & f_{2i} \\ f_{31} & f_{32} & f_{33} & f_{34} \cdots & f_{3i} \\ f_{41} & f_{42} & f_{43} & f_{44} \cdots & f_{4i} \\ f_{51} & f_{52} & f_{53} & f_{54} \cdots & f_{5i} \\ f_{61} & f_{62} & f_{63} & f_{64} \cdots & f_{6i} \\ f_{71} & f_{72} & f_{73} & f_{74} \cdots & f_{7i} \\ f_{81} & f_{82} & f_{83} & f_{84} \cdots & f_{8i} \end{vmatrix} \quad (2)$$

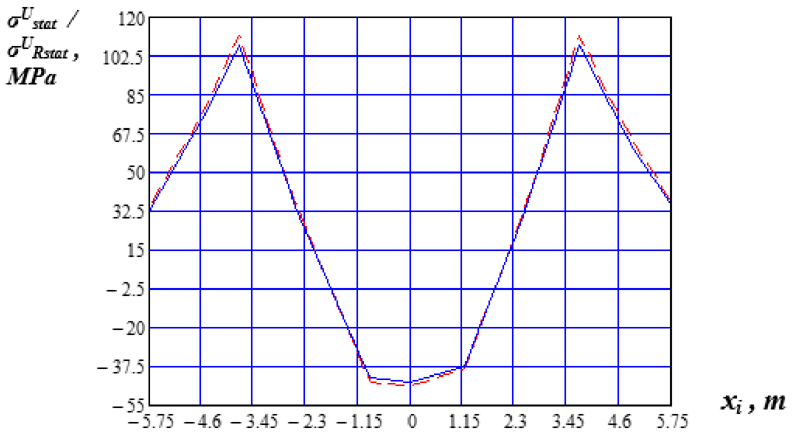


**Fig. 1**. Distribution of bending stresses from weight loading in the upper fibers of the main frame sidewall cross-sections of the body of electric locomotive VL-80s:
__________ for the reinforced frame sidewall – $\sigma^{U}{}_{Rstat}$ ; _ _ _ before the reinforcement – $\sigma^{U}{}_{stat}$

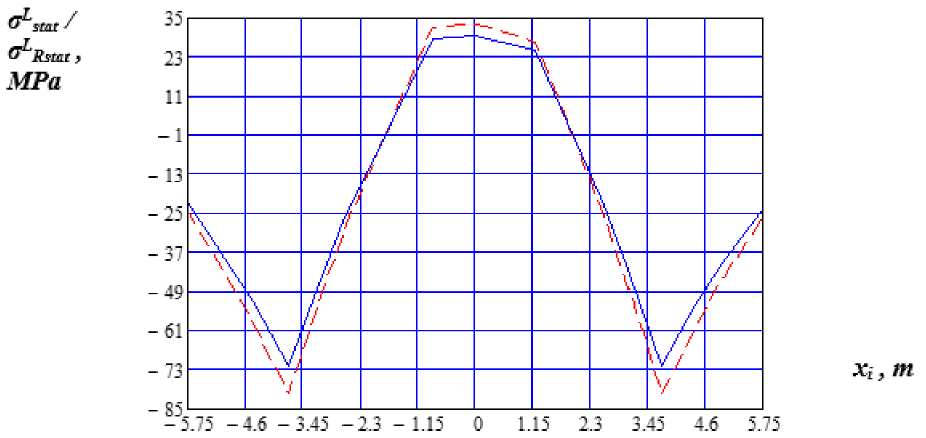


**Fig. 2**. Distribution of bending stresses from weight loading in the lower fibers of the main frame sidewall cross-sections of the body of electric locomotive VL-80s:
__________ for the reinforced frame sidewall $\sigma^{L}{}_{Rstat}$ ; ___ before the reinforcement $\sigma^{L}{}_{stat}$ - of the variable moment of inertia of the modernized frame sections along the axis *OX* - $I_x(x)$ *(cm⁴)*:

$$I_x(x) = \begin{vmatrix} j_{11} & j_{12} & j_{13} & j_{14} \cdots & j_{1i} \\ j_{21} & j_{22} & j_{23} & j_{24} \cdots & j_{2i} \\ j_{31} & j_{32} & j_{33} & j_{34} \cdots & j_{3i} \\ j_{41} & j_{42} & j_{43} & j_{44} \cdots & j_{4i} \\ j_{51} & j_{52} & j_{53} & j_{54} \cdots & j_{5i} \\ j_{61} & j_{62} & j_{63} & j_{64} \cdots & j_{6i} \\ j_{71} & j_{72} & j_{73} & j_{74} \cdots & j_{7i} \\ j_{81} & j_{82} & j_{83} & j_{84} \cdots & j_{8i} \end{vmatrix} \qquad (3)$$

- variable bending stiffness along the sections of the modernized frame

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$$G_{bend}(x) = E\,I_x(x) \tag{4}$$

where $I_x(x)$ is calculated by formula (3), and the optimal value of *n* (degree of polynomials) is selected with the use of ECM, by the method of piecewise linear approximation on the basis of real dimensions of frame sidewalls of electric locomotives. For the electric locomotive VL-80s in the numerical calculation the value was taken as $n = 8$ , (and the error was $\delta = 0{,}001$).

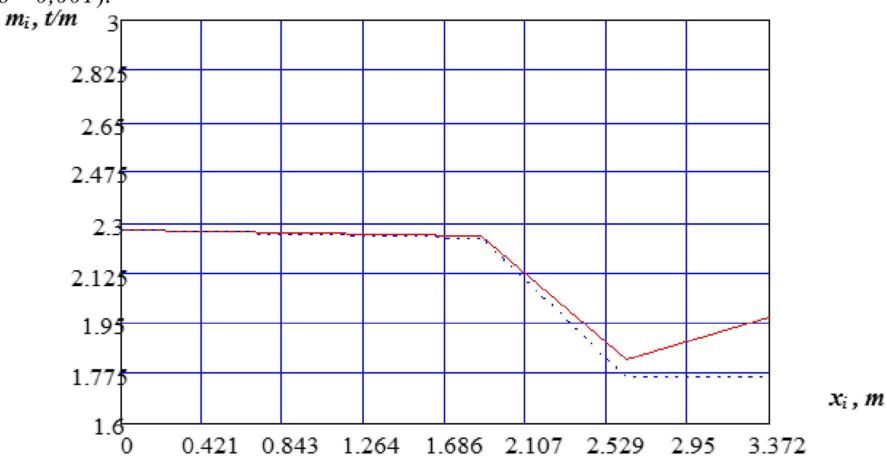


**Fig. 3**. Comparative analysis of functions obtained by quasi-static frame calculation and spline-function modeling for variable mass by sections of electric locomotive VL-80s ( _______ for standard frame sidewall construction, _ _ _ _ for reinforced frame sidewall with reinforcing plates)

At justification of the dynamic model the following assumptions are accepted: the modernized main frame of the locomotive body is represented as an elastic rod (beam) with constant modulus of elasticity of the material $E = const$ and density $\rho = const$, which has some static initial deflection radius *R*. The equations of bending-longitudinal vibrations for such a model are taken by analogy with the monographs [8, 11].

To analyze the stress-strain state of the equated frame of a modernized locomotive body frame of complex configuration, differential equations of bending and continuation vibrations of straight rods of variable cross-section are used (considering torsional vibrations as small compared to the other components).

$$m_{lb}(X)\frac{\partial^2 U_{\text{к}}(X,t)}{\partial t^2} - E\frac{\partial F(X)}{\partial X}\cdot\frac{\partial U_{\text{к}}(X,t)}{\partial X} - EF(X)\frac{\partial^2 U_{\text{к}}(X,t)}{\partial X^2} = N_{dyn}(X,t) + E\frac{\partial I_X(X)}{\partial X}\cdot\frac{1}{R^2} + 2EI_X(X)\frac{1}{R}\frac{\partial^3 W_{\text{к}}(X,t)}{\partial X^3} \tag{5}$$

$$m_{lb}(X)\frac{\partial^2 W_{\text{к}}(X,t)}{\partial t^2} + EI_X(X)\frac{\partial^4 W_{\text{к}}(X,t)}{\partial X^4} + E\frac{\partial^2 I_X(X)}{\partial X^2}\cdot\frac{\partial^2 W_{\text{к}}(X,t)}{\partial X^2} = P_{dyn}(X,t) + \frac{E}{R}\left[\frac{\partial^2 I_X(X)}{\partial X^2} + 2I_X(X)\cdot\frac{\partial^3 U_{\text{к}}(X,t)}{\partial X^3}\right] \tag{6}$$

The system of differential equations with variable coefficients (5) - (6) is solved with linearization by Simpson's method, then the Fourier method is applied to differential equations with constant coefficients with further application of the operational Laplace transform in time, numerical studies were carried out by methods of piecewise linear approximation and boundary elements similar to the methods of [11-18] in the *Mathcad 15* programming environment. The initial conditions are assumed to be zero, and the boundary conditions are in the form of elastic fixation of the ends.

For a complete description of all the steps of the developed new method, Figure 4 shows a block diagram for numerical studies on the dynamic model of the modernized frame structure of the main body frame and bogie frames of locomotives of complex configuration at increased speeds for transport engineering.

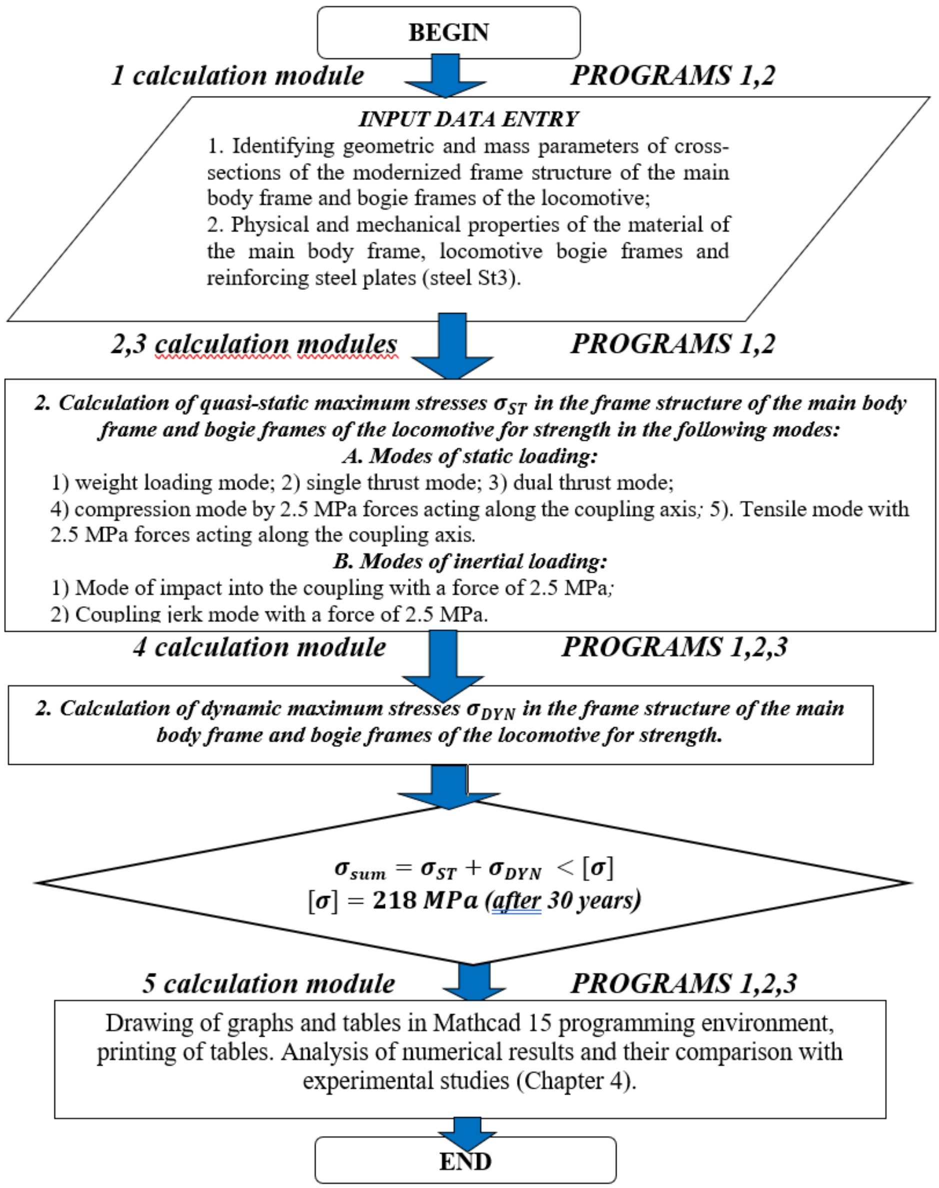


**Fig. 4.** Block diagram for computational research on dynamic model of modernized frame structure of main body frame and bogie frames of locomotives of complex configuration at increased speeds for transport engineering.

## 4 Conclusion

On the basis of analytical and numerical studies [11-17] and comparative analysis with our experimental studies [11], the following generalizing conclusions can be formed:

1. A generalized method of dynamic strength calculation of locomotive frame structures of complex configuration for transport engineering has been developed, taking into account the influence of contact dynamic and thermal loads, as well as longitudinal, transverse and torsional components of traction forces at increased speeds, specifically, for the modernized main frame and bogie frames of the VL-80 electric locomotive [9-15].

2. The proposed engineering calculation method allows for quasi-static and dynamic modeling of the stress-strain state of the modernized main body frame and bogie frames of locomotives with a reinforced load-bearing frame, on the basis of programs for numerical calculation, for which 2 certificates on official registration of computer programs of the Republic of Uzbekistan have been issued (№ DGU 07664, 31.04.2020 y. and № DGU 10286 from 24.02.2021 y.).

3. On the basis of train tests, which are planned to be carried out in 2023-2024, the results of theoretical calculations on selection of rational parameters of reinforcing plates with complex configuration for parts of modernized main frames of VL-80 electric locomotive bodies will be specified.

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