University as an analogue of the neural network

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Abstract. A mathematical model is proposed, which allows to estimate
the number of successful university graduates based on parameters
characterizing the effectiveness of vertical (lectures, seminars) and
horizontal (peer education) training. It is shown that with low effectiveness
of vertical learning, an effective means of improving the quality of
education in general is the targeted formation of horizontal groups within
which information is exchanged. It is shown that with extremely low
quality of vertical learning, the behavior of the "university" system is
characterized by phase transitions: with a smooth increase in the parameter
characterizing the intensity of horizontal learning, there is an abrupt
increase in the number of successful graduates. It has been established that
with the existence of pronounced links between individual lecture courses,
the "university" system becomes an analogue of a neural network.

1 University model as a system with phase transitions

It is now recognized that the effectiveness of horizontal education (peer education) is
much more effective than the effectiveness of traditional forms of education that are used in
universities, in particular classrooms [1-4]. This is due to the fact that peer education is
based on interpersonal contacts of students which are informal. There is every reason to
believe that, based on peer education, quite specific informal institutions can be formed,
which, if they are formed, complement the formal institutions that make up the higher
education system. It is worth noting that the term peer education itself is used relatively
recently, and evidence of increased learning efficiency based on interpersonal contacts
within student groups is also discussed in the literature recently. However, for centuries
there existed the notion that there is a university corporate spirit, there is a well-defined
university environment that educates the future specialist much more effectively than any
direct agitation.

In [5], attempts were made to construct a consistent mathematical theory that would
reflect the effectiveness of peer education. In these works, it was shown that even if the
effectiveness of direct education, in particular classroom training, is extremely small (as is
the case, say, in modern Kazakhstan [6]), there is, nevertheless, an opportunity to find a

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way out and even provide high efficiency of training in general, if methods of stimulating horizontal training are introduced.

Moreover, in the cited works [4] it was shown that if the effectiveness of peer education with low efficiency of direct training is high enough, then a phase transition is observed in this system. With such a transition, the system jumps abruptly from a state characterized by a low level of student preparation to a state in which learning efficiency approaches one.

The existence of such phase transitions suggests that for such countries as Kazakhstan, the so-called "miracle strategy" is implemented, with using relatively cheap measures lead to significant results.

However, phase transitions that may occur in institutions of higher education, allow us to draw a number of unexpected conclusions, the justification of which is the goal of this work.

Namely, in the cited works [4], where the existence of a phase transition in systems of the type under consideration was proved, only a transition along a ladder containing several successive levels was analyzed. This corresponds to the consideration of student learning in a single discipline. Here, the student learns an increasing number of information packages and having mastered the required number, acquires competence in this specialty.

In fact, a student is trained in several disciplines at once, and a phase transition of the type considered in the works cited above can be observed in relation to each of them, including separately. In other words, a student can effectively learn one discipline and poorly learn another. Of course, training in the disciplines that together make up the curriculum is a related process (the study of one discipline affects the learning of material in another).

However, the fact that the transition from one level to another in the study of a particular discipline is in the nature of a phase transition, allows us to prove that, in the aggregate, the curriculum of any university is an analogue of a neural network. The proof of this fact is the purpose of this work.

Moreover, this conclusion suggests that the curriculum, being an analogue of a neural network, to a certain extent provides recognition of the "image" that is formed by each of the specific students.

Following [7], we will build a university model based on the analogy with the Bass innovation promotion model [8]. The basic equation of this model can be written in the form

$$\frac{dN}{dt} = \alpha(N_0 - N)N + \beta(N_0 - N) \tag{1}$$

where the market potential of a particular product or service is taken as N_0, $N(t)$ is the number of customers held at time t. Traditionally, the proportional term α characterizes interpersonal influences (word-of-mouth), and the proportional term β characterizes the influence of the media and advertising; The coefficients α and β characterize the intensity of the information effects of the above types, which determine the advancement of innovations to the market.

The analogy used [9] is as follows. Both the purchase of goods and the acquisition of knowledge can be viewed as the result of some information impact, which can be carried out through several channels. However, if, when considering the promotion of goods / services to the market, the information received (at least as a first approximation) can be correlated with a single information package, then with regard to the tasks of learning modeling it is necessary to use several information levels [7].

We emphasize that the approach proposed in this paper corresponds to the general concept of the analysis of complex systems as systems capable of processing information. Namely, in [10-12], on the basis of the concept of dialectical positivism, it was argued that

any system that meets the category of a philosophically complex term is a system of information processing. From the materials of cited works it also follows that the informational aspects of the interpretation of the category of complex becomes extremely important in order to reveal the essence of the concept of "intelligence", which also represents a system of information processing. Moreover, it follows from the materials of the cited works that human intelligence is not so much a sign of an individual as a collective sign of our biological species; it is a purely collective property. This conclusion compels us to argue that there are transpersonal levels of information processing that are responsible for the appearance of such phenomena as mentality, sociocultural code, etc. From this perspective, the study of any self-organizing systems in which mechanisms arise that provide information processing at a transpersonal level is also truly relevant.

The corresponding scheme is shown in Fig.1. Transitions "up" in this scheme correspond to the student's mastering of the following information package in the learning process; transitions "down" - the loss of this package (the student forgets material that is not used in practice). The upper level corresponds to the acquisition by a student of a certain competence.

The scheme in Figure 1 can be used in two ways: to describe the acquisition of qualifications in general (then individual levels reflect acquisitions of knowledge and skills related to a particular discipline within the training plan) and to describe the acquisition of a particular competence, say, the ability to program in a particular language.
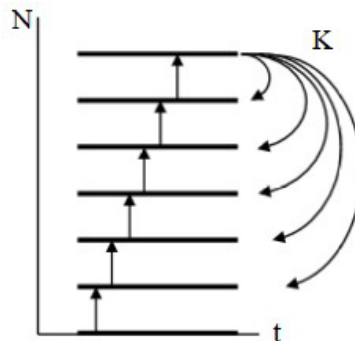

Fig. 1. K-level ladder: the influence of the upper level on induced transitions.

In accordance with [7], a new quality appears in the study of a K-level system containing at least three levels. In the cited work, equations were obtained that describe the system with precisely this number.

The system of equations describing transitions between levels in systems of this type in general has the form

$$\frac{dN_0}{dt} = -\alpha_0 N_0 N_K - \beta_0 N_0 + \frac{1}{\tau_1} N_1 \tag{2}$$

$$\frac{dN_k}{dt} = -\alpha_k N_k N_K + \alpha_{k-1} N_{k-1} N_K - \beta_k N_k + \beta_{k-1} N_{k-1} + \frac{N_{k+1}}{\tau_{k+1}} - \frac{N_k}{\tau_k} \tag{3}$$

$$\frac{dN_K}{dt} = \alpha_{K-1} N_K N_{K-1} + \beta_{K-1} N_{K-1} - \frac{1}{\tau_K} N_K, \tag{4}$$

where:

- N_k - the number of system elements that have reached level k (students who have mastered a certain amount of knowledge, consumers who have received relevant information about the innovative service, etc.);

- coefficients a_k reflect the informational impact of interpersonal communications, providing a transition between levels when obtaining additional information;
- coefficients b_j describe the effectiveness of direct informational impact in the classroom.

When writing equations (2) - (4), it was assumed that the transfer of knowledge is carried out only by those students who have already received a certain competence, i.e. purchased all necessary for this set of information packages, reaching the top level according to the scheme of Fig.1.

We also emphasize that with respect to the learning process, such an effect, described by coefficients a_k, is called peer education, it is analyzed in numerous works, for example [5]. As applied to the task of promoting goods / services to the market, this factor is interpreted as "word-in-mouth" and studied in great detail [9].

Equations (2) - (4) also take into account that the student (or consumer) can forget certain information, i.e. on the diagram in fig.1. there are not only direct, but also reverse transitions, which are described by members of the form N_j/t_j.

To consider the general case corresponding to a system with an arbitrary number of levels, we will form partial sums of the form

$$S_0 = N_0 \tag{5}$$

$$S_k = N_0 + N_1 + \ldots + N_k \tag{6}$$

$$S_K = N_0 + N_1 + \ldots + N_K = C \tag{7}$$

Where C is the total number of elements in the system.
Then

$$\frac{dS_0}{dt} = -(\alpha_0 N_K + \beta_0)N_0 + \frac{1}{\tau_1}N_1 \tag{8}$$

$$\frac{dS_k}{dt} = -(\alpha_k N_K + \beta_k)N_k + \frac{N_{k+1}}{\tau_{k+1}}, k = 2,3,\ldots,K-1 \tag{9}$$

$$\frac{dS_K}{dt} = 0 \tag{10}$$

The coefficients appearing in the written equations can be detailed. It is possible to assume that each of them is a product of the intensity of the informational impact on the susceptibility of the consumer (listener)

$$\alpha_k = r_k J_{\alpha k}; \beta_k = r_k J_{\beta k} \tag{11}$$

This allows to write down the used coefficients in the following form

$$\alpha_k = q_k \alpha_0; \beta_k = q_k \beta_0 \tag{12}$$

from where

$$\frac{dS_0}{dt} = -(\alpha_0 N_K + \beta_0)N_0 + \frac{1}{\tau_1}N_1 \tag{13}$$

$$\frac{dS_k}{dt} = -q_k(\alpha_0 N_K + \beta_0)N_k + \frac{N_{k+1}}{\tau_{k+1}}, k = 2,3,\ldots,K-1 \tag{14}$$

$$S_K = C \tag{15}$$

The resulting equations can be interpreted from the following point of view: the informational impact that determines the transitions between levels is the same (for example, the teacher gives a lecture in the classroom equally for all students), but the susceptibility to this effect may be different. In the specific example under consideration, it depends on the amount of knowledge already learned by the students, i.e. from the level number k according to the scheme of Fig.1. Define

$$G = \tau_1(\alpha_0 N_K + \beta_0) \tag{16}$$

Then for the equilibrium case the following recurrence relations take place

$$N_1 = GN_0 \tag{17}$$

$$N_{k+1} = q_k \frac{\tau_{k+1}}{\tau_1} GN_k, k = 2,3,\ldots,K-1 \tag{18}$$

The factor (16) is then determined from equation (15), which takes the form

$$N_0 \sum_0^K g_k G^k = C \tag{19}$$

Where

$$g_1 = 1, g_k = q_1 \frac{\tau_2}{\tau_1} \cdot q_2 \frac{\tau_3}{\tau_1} \cdot \ldots \cdot q_{k-1} \frac{\tau_k}{\tau_1}, k = 2,3,\ldots,K$$

Equation (19) should be supplemented by the equation relating the multiplier (16) and the population of the lower level of the system

$$G = \tau_1(\alpha_0 g_K G^K N_0 + \beta_0) \tag{20}$$

Expressions (19) and (20) together form a system of equations for two unknown quantities G and N0, which can be reduced to a single equation for the parameter G.

$$\frac{\sum_0^K g_k G^k}{g_K G^K} (G - \tau_1 \beta_0) = \tau_1 \alpha_0 C \tag{21}$$

Knowing the parameter G, it is easy to find all the concentrations directly on the basis of (19)

$$N_k = C \frac{g_k G^k}{\sum_n^K g_k G^k} \tag{22}$$

From expression (21), in particular, it is clear that it is advisable to determine the control parameter of the system (the reduced number of elements) as

$$c = \tau_1 \alpha_0 C \tag{23}$$

From the same expression (21) it can be seen that the asymptotic behavior of the solution for any values of the control parameters is the same: for large values of the aggregate information influence G in the fraction numerator on the left side of (21), we can ignore all the terms except the highest degrees, which implies

$$G - \tau_1 \beta_0 \approx c, G \to \infty \tag{24}$$

2 Results and discussion

Examples of the results of the numerical solution of equation (21) for the case of K = 3 are presented in Fig. 2 - Fig. 4. In particular, they show that for large values of the parameter c (the reduced number of particles in the system), the obtained curves do asymptotically approach straight lines with a single value of the slope tangent.

From equation (21) it is also clear that when c = 0, the solution of this equation is exactly

$$G = \tau_1 \beta_0 \tag{25}$$

As one would expect, in this limiting case the intensity of the information impact is exactly equal to the expression describing vertical learning.
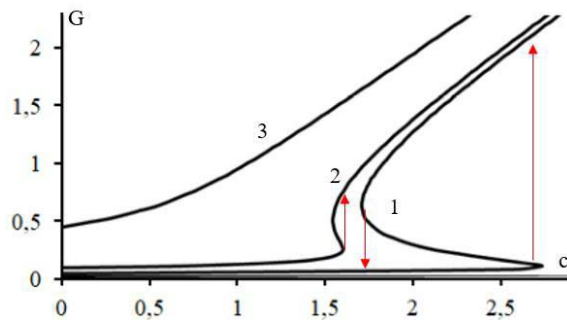
Fig. 2. The dependence of the intensity of the cumulative information impact G on the reduced number of elements of the system c; g2 = 2.5, t1b0 = 0.045 (1), 0.09 (2), 0.4 (3).

Figures 2-4 differ in the values of the parameter g2, which characterizes the probability of transition from the second level to the third. Each of these figures shows families of curves that differ in different values of the reduced intensity of direct learning $\tau_1\beta_0$.

It can be seen that for all studied values of the parameter g2, the same pattern of changes is observed associated with an increase in the reduced number of system elements c, namely, if the effectiveness of the reduced direct information impact is low, then phase transitions are observed in the system. In this case, the dependence of the parameter G on the parameter c becomes S-shaped. When a certain critical value of c is reached, the system from one state transitions to another abruptly, and the reverse transition occurs at other values of c.
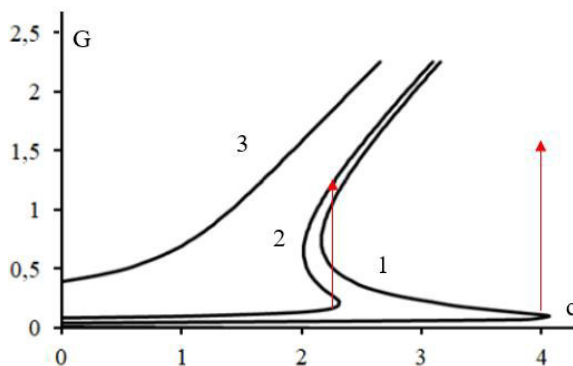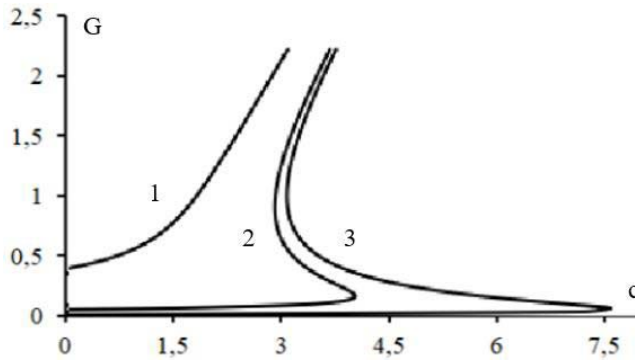

Fig. 3. The dependence of the intensity of the cumulative information impact G on the reduced number of elements of the system c; g2 = 1.5, t1b0 = 0.045 (1), 0.09 (2), 0.4 (3).

Fig. 4. The dependence of the intensity of the cumulative information impact G on the reduced number of elements of the system c; g2 = 0.8, t1b0 = 0.045 (1), 0.09 (2), 0.4 (3).

On the contrary, if the intensity of direct training is sufficiently high, then the dependence of G on c is monotonic. It can be concluded that even with extremely low direct education there is an opportunity to achieve high-quality education at the expense of peer education. This conclusion is also directly confirmed by Figures 5–7, each of which shows a family of curves describing the population of the upper level of the reduced number of elements c.

It is also clearly seen here that with low efficiency of direct education, there is an abrupt transition from a state in which almost all students have not reached the top level, to a state in which the relative number of successful students approaches one, that is, 100%.
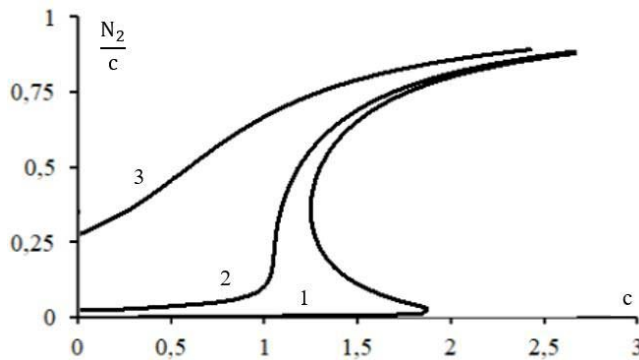
Fig. 5. Dependence of the reduced population of the upper level N2/C on the reduced number of system elements c; g2 = 4.0, t1b0 = 0.04 (1), 0.09 (2), 0.35 (3).
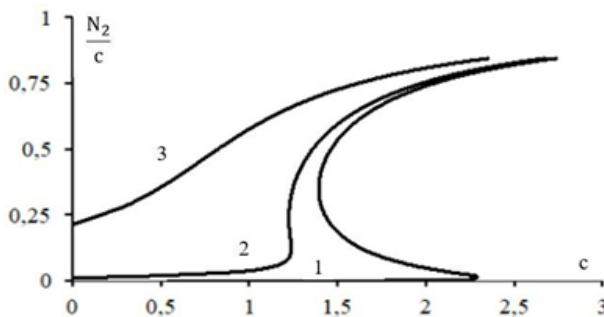
Fig. 6. Dependence of the reduced population of the upper level N2/C on the reduced number of system elements c; g2 = 3.0, t1b0 = 0.04 (1), 0.09 (2), 0.35 (3).
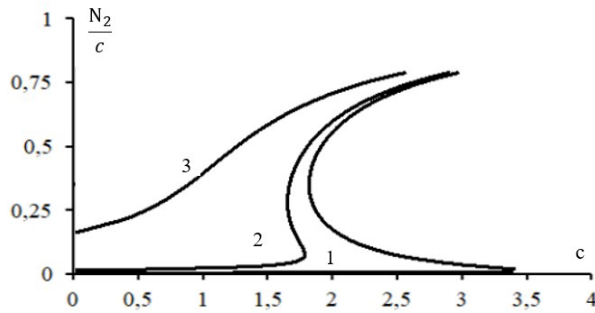
Fig. 7. Dependence of the reduced population of the upper level N2/C on the reduced number of system elements c; g2 = 2.0, t1b0 = 0.04 (1), 0.09 (2), 0.35 (3).

Moreover, these figures show that, with the same values of s, it is possible to achieve more effective training precisely through peer education. We emphasize that here dependencies on the reduced number of system elements were considered. This parameter characterizes the effectiveness of interpersonal communications of students, thereby these graphs clearly show that in order to increase academic performance, it is desirable to stimulate interpersonal contacts that cause the phase transition in question.

It is of interest to consider how the other levels behave depending on the parameter change being studied with. Figures 8-12 show this. In these figures, it is clearly seen that in the area where there is a pronounced phase transition, there are practically no students with intermediate competence, that is, under these conditions there is an abrupt transition from the state with minimal competence to state with maximum competence. In this sense, the behavior of the system under consideration can be likened to a formal neuron that is part of any artificial neural network. A state with low competence can be put in accordance with a logical zero, and a state with high competence can be put in a logical one.
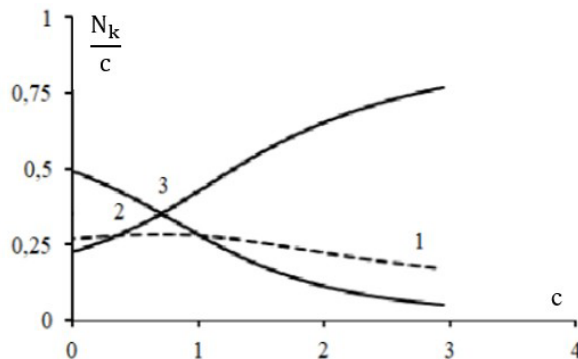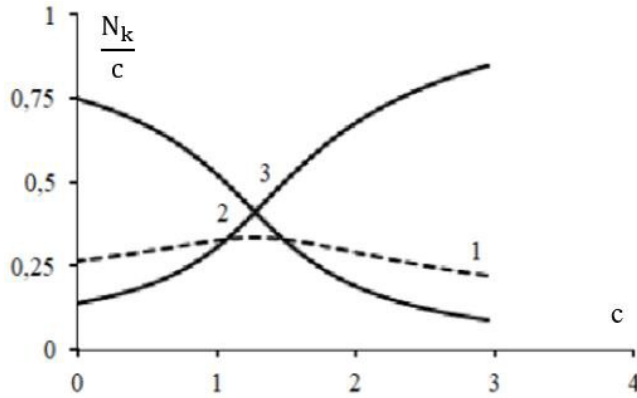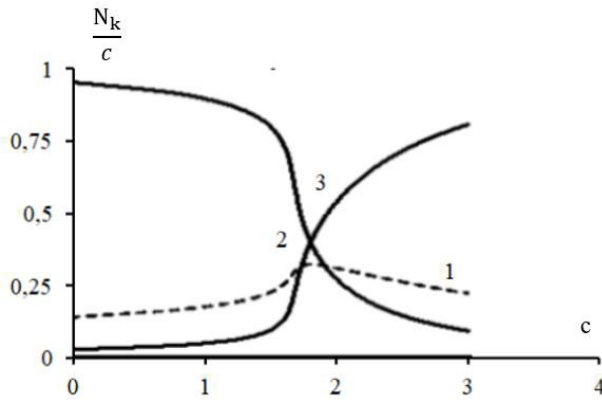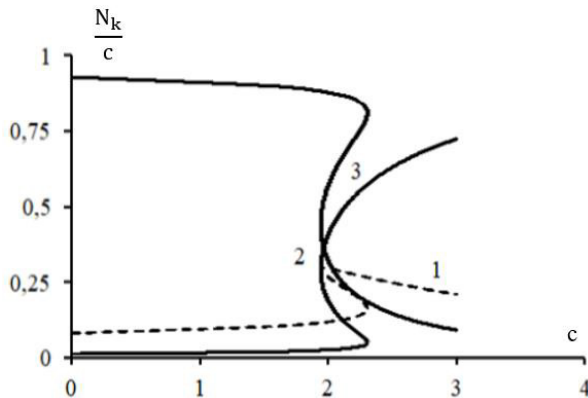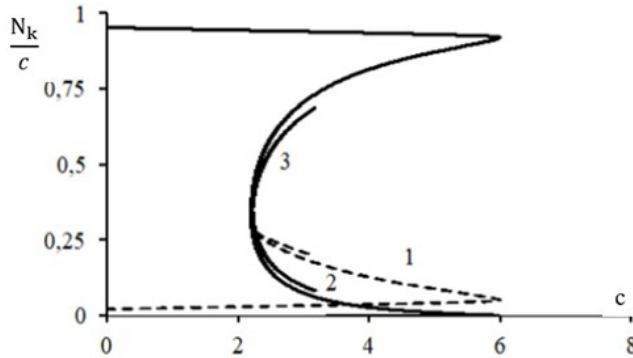

Fig. 8. The dependence of the reduced population levels of the system $\frac{N_0}{C}$ (1); $\frac{N_1}{C}$ (2); $\frac{N_2}{C}$ (3) of the reduced number of elements of the system c; g2 = 1.5, t1b0 = 0.55.

Fig. 9. The dependence of the reduced population levels of the system $\overline{\frac{N_0}{c}}$ (1); $\overline{\frac{N_1}{c}}$ (2); $\overline{\frac{N_2}{c}}$ (3) of the reduced number of elements of the system c; g2 = 1.5, t1b0 = 0.35.

Fig. 10. The dependence of the reduced population levels of the system $\overline{\frac{N_0}{c}}$ (1); $\overline{\frac{N_1}{c}}$ (2); $\overline{\frac{N_2}{c}}$ (3) of the reduced number of elements of the system c; g2 = 1.5, t1b0 = 0.15.

Fig. 11. The dependence of the reduced population levels of the system $\overline{\frac{N_0}{c}}$ (1); $\overline{\frac{N_1}{c}}$ (2); $\overline{\frac{N_2}{c}}$ (3) of the reduced number of elements of the system c; g2 = 1.5, t1b0 = 0.085.

Fig. 12. The dependence of the reduced population levels of the system $\frac{N_0}{c}$ (1); $\frac{N_1}{c}$ (2); $\frac{N_2}{c}$ (3) of the reduced number of elements of the system c; g2 = 1.5, t1b0 = 0.03.

All the above drawings related to the situation when the parameter c was changed, that is, how much the intensity of interpersonal exchanges influenced the quality of learning was analyzed.

From the point of view of the objectives of this work, it is of interest to analyze how the system will react to a change in the $\tau 1\beta 0$ parameter that characterizes direct learning. The fact is that this parameter can collectively characterize not only the direct training of this particular discipline, but also the impact that the presence of other disciplines in the curriculum has on the nature of learning information in this particular discipline.

This is illustrated in Figure 13, which schematically shows the mutual influence of training in three separate disciplines.
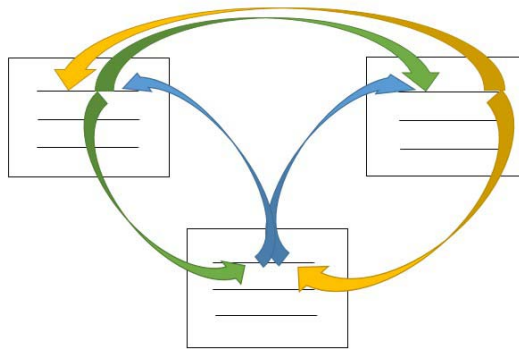

Fig. 13. The curriculum as an analogue of the neural network: the scheme of the mutual influence of training in three disciplines.

In this case, we are talking about the fact that those students who have learned this particular discipline with high efficiency (reached the top level in the model under consideration) may otherwise perceive the material from other disciplines. At the same time, the influence of studying one discipline on the study of others can be both positive and negative.

In particular, if these disciplines are complementary to each other, then good training in one of them will facilitate the learning of the material in another, and thus the parameter $\beta 0$ will be more. On the contrary, if these disciplines are not complementary to each other, and the material on them is weakly connected, then the mutual influence can be negative. Let's

say a student has an interest in one discipline, he spends more time on it and he, especially in the conditions of time shortage, does not have time to adequately study another subject.

As figures 14 through 19 show, the dependences of the parameters discussed above on the parameter $\tau 1\beta 0$, which characterizes the intensity of direct training, can also be non-linear. Moreover, the curves under consideration can also acquire an S-shaped character, by virtue of which each item can be assigned to a neuron of some kind of artificial neural network.

Under these conditions, a state of low competence corresponds to a logical zero, and from a high level to a logical one. There is a relationship between the items mentioned above. This allows us to consider their totality as an analog of a neural network, specifically as an analog of a Hopfield neural network (Fig.20), which is proved by direct comparison of Fig. 13 and fig.20.
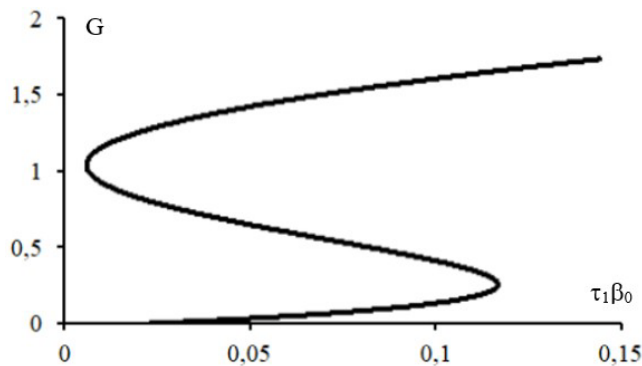

Fig. 14. The dependence of the intensity of the cumulative informational influence G on the reduced effectiveness of direct training t1b0; g2 = 0.9, c = 3.2.
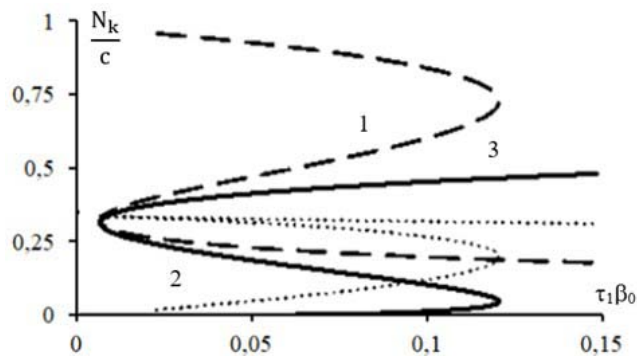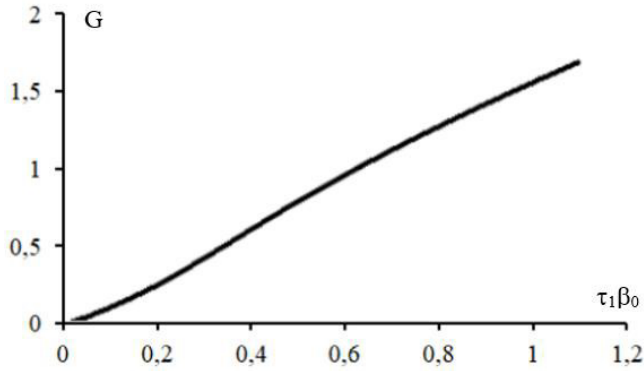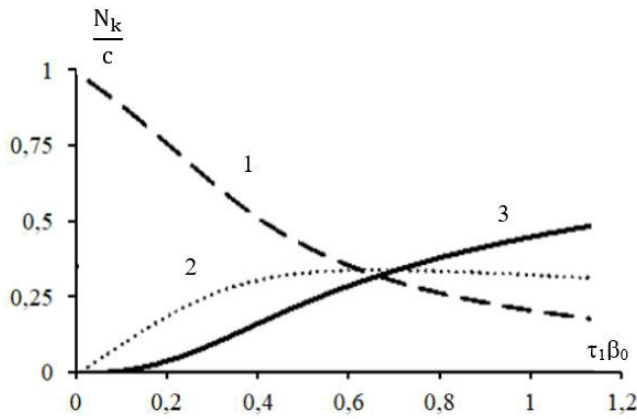

Fig. 15. The dependence of the reduced population levels of the system $\frac{N_0}{c}$ (1); $\frac{N_1}{c}$ (2); $\frac{N_2}{c}$ (3) the reduced effectiveness of direct training t1b0; g2 = 0.9, c = 3.2.

Fig. 16. The dependence of the intensity of the cumulative informational influence G on the reduced effectiveness of direct training t1b0; g2 = 0.9, c = 1.2.

Fig. 17. The dependence of the reduced population levels of the system $\frac{N_0}{c}$ (1); $\frac{N_1}{c}$ (2); $\frac{N_2}{c}$ (3) of the reduced effectiveness of direct training t1b0; g2 = 0.9, c = 1.2.

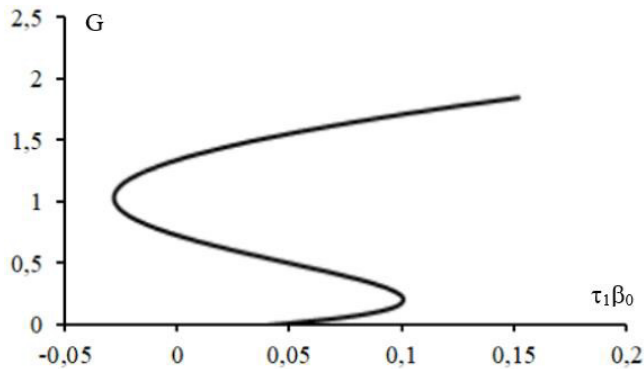**Fig. 18.** The dependence of the intensity of the cumulative informational influence G on the reduced effectiveness of direct training t1b0; g2 = 0.9, c = 3.3.

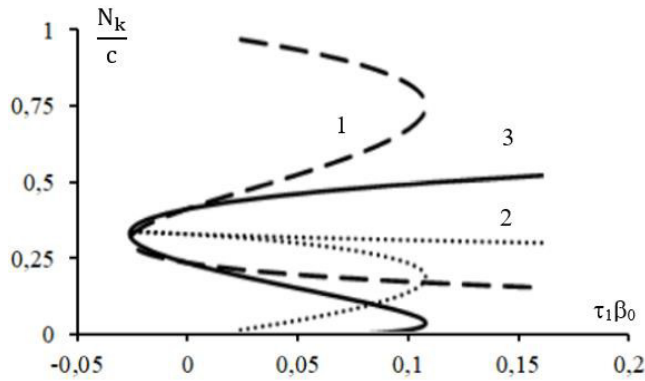**Fig. 19.** The dependence of the reduced population levels of the system the reduced effectiveness of direct training t1b0; g2 = 0.9, c = 3.3.
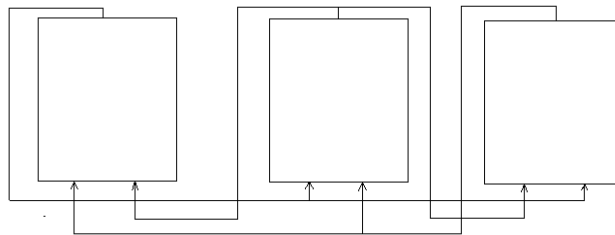

Fig. 20. Feedback Scheme in Hopfield Neuroprocessor.

3 Conclusion

Thus, the work shows that in conditions where the effectiveness of direct education is very low, which is realized, for example, in the conditions of the Republic of Kazakhstan, special attention should be paid to measures that encourage horizontal learning.

Due to such measures, it is possible to significantly increase the quality of education and, moreover, in this case, the transition from one state to another can occur abruptly. At the same time, in conditions when the quality of education is provided mainly by peer education, it turns out that the system of training subjects is converted into an analogue of a neural network. This, among other things, suggests that when implementing an approach based on peer education, it is necessary to carefully monitor the mutual influence of training in various subjects.

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