Fuzzy decision-making problem for controlling operating modes of technological systems and method for their solution

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Abstract. Statements of decision-making problems for managing fuzzy technological objects have been formalized and obtained. and methods for solving them are proposed. The object of study is a heating station for a "hot" main oil pipeline. Since such objects are often characterized by multi-criteria and often operate in conditions of unclear initial information, the tasks are formalized in the form of multi-criteria decision-making problems in a fuzzy environment. Based on the modification of various optimality principles, new mathematical formulations of the problems to be solved were obtained and interactive heuristic algorithms for solving them were developed. The novelty of the proposed approaches to solving formalized fuzzy problems from well-known methods for solving fuzzy problems lies in the fact that problems are posed and solved without first converting them to equivalent deterministic options, which does not reduce the loss of original fuzzy information and makes it possible to obtain more adequate and effective solutions. An example is given of the practical application of the proposed approach to solving decision-making problems by implementing one of the developed algorithms when solving the problem of choosing an effective operating mode for the oil heating station of the Uzen-Samara oil pipeline at the Atyrau point.

1 Introduction

In practice, decision-making problems often arise when managing the operating modes of technological objects, which are characterized by multi-criteria and unclear initial information. The complexity, large number of parameters and multicriteria of such technological objects and the vagueness of the initial information complicate the formalization, mathematical formulation and solution of such problems [1–3].

Recently, problems and approaches to solving multicriteria decision-making problems have been actively discussed in the scientific literature and publications [4,5], including in

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conditions of unclear initial information [6–8] . Methods for solving these problems in a fuzzy environment are based on the use of methods from fuzzy set theories [5,9,10]. The problems of setting and solving multicriteria problems of choosing optimal parameters and operating modes of technological units are considered in works [7,11].

The decision-making problems studied and solved in this work on the optimal control of the operating modes of technological units of the main oil pipeline are related to important scientific and practical issues of decision theory, the theory of fuzzy sets and multi-criteria optimization, which is a very relevant problem in the oil pumping industry. The purpose of this work is to study and solve issues of formalization and formulation of decision-making problems to control the operating modes of oil pipeline technological units in a fuzzy environment, as well as the development of algorithms for solving them . When formulating and solving a problem, the ideas of compromise decision-making schemes are used, modified and adapted to the conditions of vagueness of the initial information [5,11,12]. Since many technological objects are often characterized by multicriteria and vagueness of initial information and control criteria, the problems being studied and solved are an urgent task in theory and practice.

2 Materials and methods of research

The decision-making problem in the general case can be formalized as follows:

<Decision-making problem> = {given V, V_S, V_P, required to provide W},

where V – specified conditions; V_S – set of possible states (operating modes) of the object; V_P – set of possible operators that ensure the transition of an object from one state to another; W – desired mode of operation of the object. In this case, the solution to the decision-making problem consists in choosing a sequence of operators to transfer an object from the state at the current moment to the desired state. Thus, decision making is a process that consists of evaluating possible solutions (alternatives) and, taking into account given conditions, selecting the best solution according to given criteria.

In conditions of multi-criteria and unclear initial information, the problem of controlling the operating modes of technological objects can be formalized in the form of a multi-criteria decision-making problem in a fuzzy environment. The decision-making task is to evaluate possible decision options, which allows you to select the best one according to given economic and environmental criteria [13,14].

Let be $f(x) = f_1(x),...,f_m(x)$ a vector of criteria that evaluates the quality of the operation of the oil pipeline technological complex. For example, $f_1(x), f_2(x),...,f_k(x)$ – respectively, pumping volume, profit, etc.; $f_{k+1}(x), f_{k+2}(x),...,f_m(x)$ – local criteria for assessing environmental safety, for example, costs of environmental protection measures, damage from environmental pollution with oil, oil products and transportation waste, etc. Each of *the m* criteria depends on the vector of *n* parameters (control actions, operating parameters) $x = (x_1,...,x_n)$, for example: temperature and pressure; rheological properties of raw materials, reagent consumption, etc. In practice, there are always various restrictions (economic, technological, financial, environmental), which can be described by certain functions - restrictions $\varphi_q \geq b_q$, $q = \overline{1,L}$. Operating parameters also have their own change intervals, specified by the technological regulations of the unit, the requirements of environmental protection measures: $x_i \in \Omega = [x_i^{\min}, x_i^{\max}]$, where $x_i^{\min}$ is the lower, $x_i^{\max}$ is the upper limit of parameter change x_i. Constraints may be vague: $\tilde{>}$ – greater than or approximately equal, $\tilde{<}$ – less than or approximately equal, $\cong$ – approximately equal [15].

It is required to make decisions on the selection of the most effective (optimal) solution - the optimal operating mode of the technological complex of the main oil pipeline, ensuring the extreme value of the vector of criteria when fulfilling the specified restrictions and taking into account the preferences of the decision maker (DM). In our cases, decision makers are operators for managing oil pumping modes through pipelines; they control and select the operating mode of an oil pipeline technological facility, for example, oil heating stations, oil pumping stations, providing optimal values of local control criteria: pumping volume, safety and reliability of the mode, etc.

3 Results

Let us formulate a mathematical formulation of a formalized decision-making problem to control the operating modes of technological objects in conditions of multicriteria and vagueness of the initial information.

Let there be a normalized vector of criteria of the form – $\mu_0(x) = (\mu_0^1,...,\mu_0^m))$ and L restrictions with unclear instructions – $\varphi_q(x) \gtrsim b_q$, $q = \overline{1,L}$. Let us assume that the constraint fulfillment membership functions $\mu_q(x)$, $q = \overline{1,L}$ for each constraint are constructed as a result of expert procedures and dialogue with specialist experts. Let the weight vector be known, reflecting the mutual importance of the criteria $\gamma = (\gamma_1,...,\gamma_m)$ and restrictions $\beta = (\beta_1,...,\beta_L)$ at the time of setting the problem [16].

Then the problem of choosing optimal operating modes of technological objects according to economic and environmental criteria can be written as the following decision-making problem in a fuzzy environment:

$$\max_{x \in X} \mu_0^i(x),\ i = \overline{1,m}$$

$$X = \left\{x : \arg\max_{x\in\Omega} \mu_q(x),\ q = \overline{1,L}\right\}$$

Based on the idea *of the main criterion method and the Pareto principle of optimality,* the given decision-making problem with a vector of criteria and restrictions [17–20] can be written in the following formulation:

$$\max_{x \in X} \mu_0^1(x), \tag{1}$$

$$X = \left\{x : x \in \Omega \wedge \arg\left(\mu_0^i(x) \geq \mu_r^i\right) \wedge \arg\max_{x\in\Omega} \sum_{q=1}^{L} \beta_q \mu_q(x) \wedge \sum_{q=1}^{L} \beta_q = 1 \wedge \beta_q \geq 0, i = \overline{2,m},\ q = \overline{1,L}\right\} \tag{2}$$

where $\wedge$ is the logical sign “and”, which requires that all statements associated with it be true, μ_r^i are the boundary values for local criteria $\mu_0^i(x), i = \overline{2,m}$ set by the decision maker.

According to the main criterion method, the main (in terms of importance) criterion is optimized, and the remaining local criteria are included in the constraints. According to the Pareto principle of optimality, the decision maker selects the optimal solution from an effective set, in which the improvement of one of them leads to the deterioration of the other.

By changing μ_r^i the constraint importance vector $\beta = (\beta_1,...,\beta_L)$, we can obtain a family of solutions to problem (1)–(2): $x^*(\mu_r,\beta)$. The choice of the best solution is carried out on the basis of dialogue and taking into account the preferences of the decision maker. To solve the multicriteria decision-making problem of determining the desired operating mode of an object in formulation (1)–(2), you can apply an algorithm based on a modification of the principles of the main criterion and Pareto optimality for working in a fuzzy environment and their combination.

By modifying various compromise decision-making schemes for the case of vagueness, it is possible to obtain other formulations of multi-criteria decision-making problems in a fuzzy environment and propose algorithms for solving them.

Using the ideas of *the main criterion and ideal point methods* and modifying them for the case of vagueness, the multicriteria decision-making problem when the initial information is unclear can be formulated as:

$$\max_{x \in X} \mu_0^1(x), \tag{3}$$

$$X = \left\{x : x \in \Omega \wedge \arg\left(\max_{x \in \Omega} \max \mu_0^i(x) \geq \mu_r^i\right) \wedge \arg \mu_q(x) \geq \min \left\|\mu(x) - \mu^u\right\|_D), i = \overline{2,m}, q = \overline{1,L}\right\} \tag{4}$$

where is the metric $\|\mu(x) - \mu^u\|_D$ D used , the components $\mu(x)$ and coordinates of the ideal point μ^u are defined as follows $\mu(x) = (\mu_1(x),...,\mu_L(x))$, $\mu^u = (\max \mu_1(x),..., \max \mu_1(x)$. It is possible to use units $\mu^u = (1,...,1)$ as coordinates of the ideal point: μ^u .

The essence of the main criterion method is disclosed above. The ideal point method allows you to find the optimal solution based on minimizing the measure (distance) of the current solution from the ideal solution (point).

To solve the multicriteria decision-making problem (3)–(4), this paper proposes a method developed based on modification of compromise schemes of the *main criterion and ideal point methods* .

Based on the application of the idea of the main criterion method to a vector of local criteria, and the idea of an ideal point to constraints, modifying them for the case of fuzziness, we propose the following algorithm for solving the multicriteria decision-making problem (3)–(4) with fuzziness of the initial information:

***GK-IT algorithm*:**

Sets a number of priorities for local criteria $I_k = \{1,...,m\}$ (the main criterion should have priority 1).

Based on information received from decision makers and specialist experts, a term set of fuzzy parameters is determined $T(X,Y)$ and for each constraint, membership functions for fulfilling the constraints are constructed $\mu_q(x)$, $q = \overline{1,L}$.

The decision maker is assigned boundary values of local criteria $\mu_r^i(x), i = \overline{2,m}$.

The coordinates of the ideal point are determined. The maximum values of the membership function can e used as the coordinates of these points $\mu^u = (\max \mu_1(x),..., \max \mu_L(x))$ or unity $\mu^u = (1,...,1)$ (if the membership functions are normal).

The type of metric is selected $\|\mu(x) - \mu^u\|_D$ that determines the distance of the current solution x^* from the ideal point – μ^u .

Problem (3)–(4) is solved and the current solution is determined: $x(\mu_r^i, \|\mu(x) - \mu^u\|_D)$ – the value of the vector of control parameters, $\mu_0^1(x(\mu_r^i, \|\mu(x) - \mu^u\|_D))$, $\mu_0^2(x(\mu_r^i, \|\mu(x) - \mu^u\|_D))$, …, $\mu_0^m(x(\mu_r^i, \|\mu(x) - \mu^u\|_D))$ – the values of local criteria and

$\mu_1(x(\mu_r^i,\|\mu(x)-\mu^u\|_D))$, $\mu_2(x(\mu_r^i,\|\mu(x)-\mu^u\|_D))$, $\cdots$, $\mu_L(x(\mu_r^i,\|\mu(x)-\mu^u\|_D))$, $i=\overline{2,m}$ – values of the membership function of fulfilling the constraint.

The decision maker is presented with the resulting current solution. If the current results do not satisfy the decision maker, then new values are assigned to them $\mu_r^i(x)$, and (or) a new type of metric is selected $\|\mu(x)-\mu^u\|_D$ and the search for an acceptable solution is repeated, i.e. a return to the previous item is carried out, otherwise a transition to the next 8 item is carried out.

A final solution is derived that satisfies the decision maker: the values of control and operating parameters $x^*(\mu_r^i,\|\mu(x)-\mu^u\|_D)$, that provide optimal values of local criteria $\mu_0^1(x^*(\mu_r^i,\|\mu(x)-\mu^u\|_D))$, $\mu_0^2(x^*(\mu_r^i,\|\mu(x)-\mu^u\|_D))$, $\cdots$, $\mu_0^m(x^*(\mu_r^i,\|\mu(x)-\mu^u\|_D))$ and the maximum values of the membership functions of the execution constraint $\mu_1(x^*(\mu_r^i,\|\mu(x)-\mu^u\|_D))$, $\mu_2(x^*(\mu_r^i,\|\mu(x)-\mu^u\|_D))$, $\cdots$, $\mu_L(x^*(\mu_r^i,\|\mu(x)-\mu^u\|_D))$.

The given formulations of new multicriteria selection problems and the developed algorithms for solving them are based on the use of modified deterministic methods of multicriteria optimization and compromise decision-making schemes. The results obtained are a generalization and development of these methods in the case of vagueness of the initial information.

4 Practical application, comparison and discussion of results

As an example of the implementation of the proposed approach to optimization of technological objects, let us consider the formalization of the formulation and solution of the problem of adopting an effective operating mode for the oil heating station (OSH) of the Uzen-Samara oil pipeline at the Atyrau point. The main task of the SPN is to ensure emergency-free and uninterrupted operation of heating furnaces and structures attached to them, ensuring optimal technological operating conditions for the "hot" oil pipeline. In this case, the following types of decision-making problems for optimizing criteria are solved: minimizing the cost of heating and pumping oil; minimizing fuel and operating costs; increasing the degree of reliability of mechanisms and devices; increasing the environmental safety of the oil pipeline.

The volume of pumped oil can be determined by the indicators of various devices (flow meters, etc.). In our case, the volume of pumped oil is measured in units of t/hour. As for assessing quality and environmental safety, the situation here is much more complicated. It is very difficult and not always possible to evaluate the quality of work of the technological and production complex of an oil pipeline, the environmental safety of the facility's operation in one number. Often these indicators are difficult to measure quantitatively and are characterized by uncertainty and vagueness of the initial information. In fact, quality indicators and environmental safety indicators are often characterized by restrictions such as "no more" and "about", i.e. are unclear.

In practice, you want economic criteria (productivity, profit, pumping volume, etc.) and quality indicators to be maximum, and environmental impacts to be minimal. But, as is known, these criteria are often contradictory and it is often not possible to improve them at the same time. The challenge is to find the optimal trade-off solution that depends on the production situation and plan, and also satisfies the decision maker.

Thus, using the above problem statements, we pose the decision-making problem when managing the modes of the oil transportation process through main oil pipelines as follows:

Let $f(x) = F(f(x)) = \mu_0^i(x), i = \overline{1,3}$ – normalized local criteria that evaluate the volume of oil pumping ($\mu_0^1(x)$) , temperature ($\mu_0^2(x)$) and pressure ($\mu_0^3(x)$) at the outlet of the SPN. Let us assume that for each fuzzy constraint describing environmental indicators $\varphi_q(x) \gtrsim b_q, q = 1,2$, the membership functions of its implementation $\mu_q(x)$, are constructed $q = 1,2$. The path is known or we will define a number of priorities for local criteria $I_k = \{1,2,3\}$ and a weight vector reflecting the mutual importance of these restrictions $\beta = (\beta_1, \beta_2)$.

The criterion and restrictions depend on the parameter vector $x_i, i = \overline{1,4}$ (x1– temperature, x2 – pressure, x3 – fuel consumption, x4 – oil consumption at the furnace inlet). These dependencies are determined on the basis of mathematical models developed in [21].

A formalized problem, in conditions of vagueness of some part of the initial information, can be written similarly to (3)–(4) in the form of the following multicriteria decision-making problem in a fuzzy environment:

$$\max_{x \in X} \mu_0^1(x), \tag{5}$$

$$X = \left\{x : x \in \Omega \wedge (f_i(x) \geq b_i) \wedge \arg\left(\mu_q(x) \geq \min_{x \in \Omega} \left\| \mu(x) - \mu^u \right\|_D\right) i = 2,3, q = 1,2\right\} \tag{6}$$

where $f_i(x), i = 2,3$ are the functions of restrictions for temperature and pressure at the outlet of oil heating stations, $\left\| \mu(x) - \mu^u \right\|_D$ is the metric used D , $\mu(x) = (\mu_1(x), \mu_2(x))$, $\mu^u = (\max\, \mu_1(x), \max\, \mu_2(x))$ or $\mu^u = (1,.1)$.

The solution to this problem is the value of the vector of optimized operating parameters $x^* = (x_1^*, x_2^*, x_3^*, x_4^*)$, which provides an extreme value of the criterion when the specified restrictions are met, taking into account the preferences of the decision maker and satisfying it.

To solve the problem (5)–(6), we use a modification of the GK-IT algorithm.

A number of priorities are set for local criteria $I_k = \{1,2,3\}$ (the main criterion is the volume of oil pumping of the oil pipeline, which has priority 1, priority 2 is assigned to the temperature at the furnace outlet, pressure at the outlet of the furnaces has priority 3).

Based on the information received from decision makers and expert specialists, a term set of fuzzy parameters is determined and for each constraint membership functions of constraint fulfillment $\mu_q(x),$ are constructed $q = 1,2$.

Based on the results of expert assessments and studies, the following membership functions of constraint fulfillment were constructed:

$$\mu_1(x) = \exp(0.20\,|\,a_1 - 50.0\,|\cdot 0.5); \quad \mu_2(x) = \exp(0.10\,|\,a_2 - 80.0\,|\cdot 0.7);$$

where a_1, a_2 are the average numerical values of fuzzy parameters, respectively: temperature and pressure of the furnace (oil heating station) at the outlet.

The type of constraint function is determined $f_i(x), i = 2,3$ and the values are specified $b_i, i = 2,3$. Based on the research results, it was determined:

$$f_1(x) = 7 + 1,2 \cdot x_1 - 0.25 \cdot x_2 + 5.7 \cdot x_3 - 1.3 \cdot x_4 + 1.8 \cdot x_1^2 + 8.3 \cdot x_3^2; b_1 = 55,$$

$$f_2(x) = 0.25 - 1.31 \cdot x_1 + 7.35 \cdot x_2 - 3.1 \cdot x_3 + 2.25 \cdot x_4 + 9.85 \cdot x_2^2 + 8.7 \cdot x_3^2; b_2 = 8.5$$

The coordinates of the ideal point are determined. The maximum values of the membership function can be used as the coordinates of these points. In our case, the membership functions are normal, therefore $\mu^u = (1,1)$.

The type of metric $\|\mu(x) - \mu^u\|_D$ that determines the distance of the current solution ($\mu(x)$) from the ideal point (μu) is selected. In our case, the type of metric is defined as follows:

$$\|\mu(x) - \mu^u\|_E^2 = \sum_{q=1}^{2} (\beta_q (\max_{x \in \Omega} \mu_q(x) - \mu_q(x)))^2,$$

where β_q is the weighting coefficient of the qth fuzzy constraint.

Problem (5)–(6) is solved (in our case, mathematical programming methods are used) and the current solution is determined:

$x(b_i, \sum_{q=1}^{2} (\beta_q (\max_{x \in \Omega} \mu_q(x) - \mu_q(x)))^2)$, $i = 2,3$ – value of the vector of control parameters;

$\mu_0^1(x(b_i, \sum_{q=1}^{2} (\beta_q (\max_{x \in \Omega} \mu_q(x) - \mu_q(x)))^2))$, $\mu_0^2(x(b_i, \sum_{q=1}^{2} (\beta_q (\max_{x \in \Omega} \mu_q(x) - \mu_q(x)))^2))$,

$\mu_0^3(x(b_i, \sum_{q=1}^{2} (\beta_q (\max_{x \in \Omega} \mu_q(x) - \mu_q(x)))^2))$, $i = 2,3$ – values of local criteria;

$\mu_1(x(b_i, \sum_{q=1}^{2} (\beta_q (\max_{x \in \Omega} \mu_q(x) - \mu_q(x)))^2))$, $\mu_2(x(b_i, \sum_{q=1}^{2} (\beta_q (\max_{x \in \Omega} \mu_q(x) - \mu_q(x)))^2))$,

$i = 2,3$ – values of the membership function of fulfilling the constraint.

The decision maker presents the current solution obtained. If the current results do not satisfy the decision maker, then new boundary values are assigned to them $\mu_r^1(x)$, $\mu_r^2(x)$ and (or) a new type of metric is selected $\|\mu(x) - \mu^u\|_D$ and the search for an acceptable solution is repeated, i.e. a return to the previous item is carried out, otherwise a transition to the next 8 item is carried out.

A final solution is derived that satisfies the decision maker: the values of the control and operating parameters $x^*(b_i, \sum_{q=1}^{2} (\beta_q (\max_{x \in \Omega} \mu_q(x) - \mu_q(x)))^2)$ that provide the optimal values of the local criteria $\mu_0^1(x^*(b_i, \sum_{q=1}^{2} (\beta_q (\max_{x \in \Omega} \mu_q(x) - \mu_q(x)))^2))$, $\mu_0^3(x^*(b_i, \sum_{q=1}^{2} (\beta_q (\max_{x \in \Omega} \mu_q(x) - \mu_q(x)))^2))$ and the maximum values of the membership functions of the execution constraint $\mu_2(x(b_i, \sum_{q=1}^{2} (\beta_q (\max_{x \in \Omega} \mu_q(x) - \mu_q(x)))^2))$, $\mu_2(x^*(b_i, \sum_{q=1}^{2} (\beta_q (\max_{x \in \Omega} \mu_q(x) - \mu_q(x)))^2))$.

The results obtained in the work are shown in the table (see Table 1).

Table 1. Comparison of the results of solving a multicriteria decision-making problem for choosing an effective SPN mode using the proposed algorithm (GK-IT), using a deterministic method and experimental data

No. p/p	Criterion values and restrictions	Deterministic method [20] (literary data)	Proposed algorithm (GK-IT)	Experimental data (PS Atyrau)
1.	Oil pumping volume (productivity), tons/hour, ($\widetilde{y}_1$)	707	≈ 710	709
2.	Furnace outlet temperature, °C (y_2)	48	50	50
3.	Pressure at the furnace outlet, kgf/cm^2 (y_3)	8.5	8	8.1
	Constraint fulfillment membership function 1 $-\mu_1^*(x^*(\beta))$	-	1.0	-
4.	Constraint 2 fulfillment membership function $-\mu_2^*(x^*(\beta))$	-	0.98	-
5.	Optimal values of operating parameters $x^* = (x_1^*,...,x_4^*)$: x_1^* – temperature at the furnace inlet, ^{o}C;	35	33	34
6.	x_2^* – pressure at the furnace inlet, kgf/cm^2;	10.5	9.8	10
7.	x_3^* – fuel consumption, kg/hour;	27	25	26
8.	x_4^* – volume of raw materials (oil) at the furnace inlet, tons/hour.	710	710	710

Note: (-) means that the corresponding indicators are not determined by this method. The time it takes to find a solution in the compared methods is the same.

Comparison and analysis of the results given in Table 1 gives grounds to draw the following conclusions:

1) The proposed algorithm is more efficient than the deterministic method and more accurately matches the experimental data.

2) When solving optimization problems based on the proposed algorithms, the adequacy of solving the production problem increases, since additional fuzzy information (experience, knowledge) is taken into account, more fully describing the real situation without idealization.

3) The applied algorithm for solving a multi-criteria decision-making problem (MC-IT) allows us to determine the degree (function) of belonging to the fulfillment of a particular fuzzy constraint, i.e. degree of correctness of the solutions obtained.

The reliability of the results and conclusions obtained is confirmed by: the correctness of the research methods used, based on the scientific principles of the theory of decision making and optimization, theories of fuzzy sets, methods of expert assessments; sufficient convergence of calculation-model (theoretical) and experimental (pilot-industrial) research results.

5 Conclusion

In this work, based on a combination and modification of various optimality principles, new formulations of multi-criteria decision-making problems for selecting operating modes of technological units of a main oil pipeline are proposed, and interactive algorithms for solving the problems are developed. The developed algorithms are based on the use of the idea of various compromise schemes (various combinations of the main criterion method, the Pareto principles of optimality and the ideal point) for decision making, modified to work in a fuzzy environment. The results of the implementation of the proposed approach in practice for the selection of optimal operating modes for the oil heating station of the Uzen-Samara oil pipeline at the Atyrau point are presented based on a modification of the idea of the methods of the main criterion (for criteria) and the ideal point (for limitation).

The scientific novelty of the results of the work lies in the fact that problems are posed and solved in a fuzzy environment without prior conversion to deterministic problems. This ensures, through the full use of the collected fuzzy information, obtaining a more adequate solution to a complex production problem when the initial information is unclear.

The practical significance of the work is determined by the effective solution of complex production problems in conditions of multi-criteria and fuzziness, which cannot be solved or are difficult to solve by traditional deterministic or stochastic mathematical methods. The advantage of the proposed approach to solving the problem is also that, depending on the production situation and the availability of initial information of a different nature, the decision maker is given the opportunity to select a more suitable algorithm for solving the problem from the proposed set of algorithms.

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