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The Evaluation of Creditworthiness of Trade and Enterprises of Service Using the Method Based on Fuzzy Logic

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Abstract: This article considered the problem of determining the creditworthiness of an enterprise operating in the field of trade and services. The assessment of the creditworthiness of borrowers, particularly small businesses, needs to be more careful: the level of development of small enterprises and their specific activities must be considered, as well as the uncertainty in obtaining any financial result. A method for assessing the creditworthiness of enterprises (trade and services) is proposed, based on the use of the mathematical apparatus of the theory of fuzzy sets. This article analyzes the indicators of industry and regional specifics, indicators of the activity of a small enterprise, and financial and economic indicators typical for the service sector and trade. The rules on the basis of which decisions are made are formed in the form of logical formulas containing parameters. In its most general form, one parameter is predicted, called the creditworthiness index, which varies from 0 to 1 and has a natural interpretation. On the basis of the proposed method, examples of calculating the assessment of the creditworthiness of enterprises operating in the field of trade and services are given. The proposed scientific approach can be used as a basis for creating expert decision support systems for lending to small businesses.

Keywords: creditworthiness; lending to trade and service enterprises; data unification; fuzzy logic; linguistic variables; logical rules; decision making

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1. Introduction

Small and medium-sized businesses play a significant role in the economic development of most countries. In the conditions of a market economic system, one of the most important issues in the functioning and development of small businesses is the search for funding sources. For banking organizations, this direction of lending is the most dynamic and complex process.

The assessment of the creditworthiness of small business borrowers should be more thorough. Usually, a standardized approach to clients is applied, which does not consider the individual characteristics of the entrepreneur. The credit assessment methods can be divided into two groups: expert and scoring methods. Experts use information about various characteristics of the borrower, such as reputation, capital (leverage), profit volatility, terms (purpose of the loan) and collateral when deciding whether to lend to this organization. One of the limitations of expert judgment is the difficulty of making a compromise decision between the strengths and weaknesses of a borrower, where an unsatisfactory level of one indicator is mitigated by the strength of some other.

In models based on fuzzy set theory, the scoring system is based on a fuzzy inference procedure, which includes several steps. The initial factors and the resulting attributes are defined using the membership functions and are linked together by a complete system of logical rules [1–3].

Neural networks for scoring models are a set of neurons grouped into layers. The input layer of the network consists of all of the characteristics used for predicting creditworthiness, and the output layer represents the solvency of the client [3–6].

The analysis of scoring systems showed that most of the mentioned research was conducted on a global scale and focused on large multinational companies whose shares are quoted on stock exchanges, and does not consider the specific features of small business.

There is a need for a tool to assess the creditworthiness of small and medium-sized businesses, and to achieve greater efficiency, it is necessary to consider the level of development of small enterprises and the specifics of their activity, as well as considering the factor of uncertainty in obtaining a particular financial result.

In current economic research, the toolkit of fuzzy logic is widely applied. In [7], a comprehensive review is presented of the work conducted from 1968 to 2005 on the application of statistical and intelligent methods for solving the problem of bankruptcy prediction, which is an issue faced by banks and firms.

In [8], a fuzzy short-term trading system is presented that uses a new trading strategy and a "mix" between modified, commonly used technical indicators and seldom-used ones to help investors to manage their portfolios.

The authors in [9] studied the applicability of fuzzy cognitive maps in models of socioeconomic systems, while the researchers in [10,11] created a cognitive model to estimate the socioeconomic impact of oil and gas production in Cyprus.

From the point of view of continuous model theory, L.A. Zadeh's logic is a very special case. However, it is interesting because it has found numerous applications in technical, natural sciences and humanities [12].

L.A. Zadeh's logic is one of the more simple and natural non-classical logics. As mentioned above, the values of L.A. Zadeh's logic formulas lie in the interval. The book by G.J. Keisler and Cheng Chen-Chun, *Continuous Model Theory* [13], considered a more general situation, namely, when the values of formulas lie in topological space. The main case studied is when the topological space is compact and Hausdorff and the logical bundles and quantizers are continuous functions. Interestingly, many constructions and theorems of classical logic (more precisely, the theory of moles) generalize to this case.

This article considers the peculiarities of lending to small businesses operating in trade and services. To define the expediency of small business crediting, we considered the following set of evaluation indexes: branch and regional specifics, the activity of the small business, and financial–economic indexes specific to the service and trade sphere.

A mixed research design requires both qualitative and quantitative research methods [14]. Currently, there are new knowledge extraction methods that use different algebraic structures to eliminate uncertainty [15]. To solve this problem, alternative methods are proposed for assessing the creditworthiness of small businesses, based on the fuzzy logic of Zadeh.

An approach to assessing creditworthiness using fuzzy logic mathematical apparatus is proposed, which allows the consideration of approximate qualitative information about the characteristics of the borrower. The theory of fuzzy sets and fuzzy logic are effective tools for formalizing qualitative and approximate concepts based on linguistic models and representing knowledge in the form of production rules such as "If ... then ... ". In this case, knowledge-based inference is based on fuzzy inference.

2. Materials and Methods

The following steps can be identified in solving the problem of creditworthiness assessment based on fuzzy modeling.

Let us assume that the borrower has a certain set of characteristics (indicators) that are subject to evaluation when making a decision on lending, represented as: $X = \{ x_i | i = \overline{1, N} \}$, where X is the set of borrower indicators, x_i is the i-th characteristic of the borrower, and N is the number of characteristics of the borrower.

The characteristics are assessed based on a set of criteria, $C = \{ c_j | j = \overline{1, M} \}$, where C is the set of criteria for assessing the borrower, c_j is the j-th criterion for assessing the borrower, and M is the number of criteria for assessing the feasibility of granting a loan to the borrower.

Next, a set of possible outcomes (alternatives) is determined, $A = \{ a_t | t = \overline{1, T} \}$, where A is the set of alternatives, T is the number of alternatives, and a_t is the classification level. The following values are used: a_1—low (L), a_2—below average (LM), a_3—average (M), a_4—above average (HM), and a_5—high (H).

The next step is to build a set of rules for fuzzy inference in the form "If . . . , then . . . " $\rightarrow P = \{ p_l | l = \overline{1, L} \}$, where P is the set of rules for fuzzy inference and L is the number of rules.

Then, the left part of the rules is collapsed according to the schemes that will be described in more detail below.

The choice of one or another alternative is also a fuzzy measure that determines characteristics of the loan such as the maximum loan amount, interest rate, or loan period.

The determination of a set of indicators and logical rules in the result is the choice of a lending scheme, $S = \{ s_k | k = \overline{1, K} \}$, where S is a set of credit schemes, s_k is a specific credit scheme, and K is the number of credit schemes.

Next, consider the indicators for assessing the creditworthiness of a small or medium-sized business. The initial indicators are displayed on the interval [0, 1]. This set of parameters is represented as a set of linguistic variables. In general, the issue of how exactly to display this requires serious research. More useful information is contained in the literature [16,17].

In the general case, we assume that any parameter takes values on a certain interval of real numbers [18]. The simplest case is when each parameter is associated with one linguistic one, which can be called the "degree of favorableness of the indicator". In explicit form, it is necessary to specify the mapping of the interval of real numbers that this parameter can take to the interval [0, 1]. This process is called the process of standardizing indicators.

Taking the composition of the "degree of auspiciousness" with the functions presented below, we accordingly obtain new linguistic variables: "the value of the parameter is low" (L); "average parameter value" (M); "high parameter value" (H). Note that these kinds of linguistic variables are often used in technical systems. An even greater number of gradations are also often considered: below average, above average, close to zero, etc.

Each linguistic variable has its own membership function. One of the possible ways to define the membership functions was proposed in [19]. It examines the management of a technical system, namely, a steam boiler. In principle, in our case, the situation is very similar.

In the result, the choice of the type of membership functions, both of the alternatives themselves and of the evaluated criteria, is determined based on expert preferences.

It is obvious that the expert's uncertainty in the assessment grows with an increase in the deviation of the value of the estimated parameter from the optimal value. Moreover, in most cases, this uncertainty does not grow linearly. However, the use of nonlinear membership functions entails a significant number of complicated mathematical calculations and graphical constructions.

For these reasons, in this paper, triangular and trapezoidal membership functions are used as the initial membership functions, which is primarily due to the simplicity of the subsequent calculations and graphical constructions. Wang presented a ranking method for triangular and trapezoidal fuzzy numbers based on the relative preference relation [20]. We can say that membership functions are piecewise linear.

Determining the degree of compliance of the selected set of assessed criteria with one or another alternative is a key factor in the subsequent selection of the most suitable lending scheme that has an optimally selected set of conditions for subsequent lending.

The set of estimated parameters was determined earlier in [21] and is given below (Table 1).

Table 1. Financial and general economic indicators.

№	Initial Indicators
1	Dynamics of development of the industry
2	Prospects for the development of the industry
3	Needs of the market (industry) for such products (work, services)
4	Dynamics of the development of the economy of the region
5	Prospects for the development of the economy of the region
6	Needs of the market (region) for such products (work, services)
7	Current liquidity ratio
8	Financial independence ratio
9	Working capital financed by equity to total assets ratio
10	Debt service coverage ratio
11	Accounts receivable turnover ratio
12	Accounts payable turnover ratio
13	Inventory turnover ratio
14	Debt-to-equity ratio
15	Profit margin ratio
16	Evaluation of the professional level of staff
17	Evaluation of the moral and psychological atmosphere in the enterprise
18	Sufficiency of the period of stay of the enterprise in the market
19	Economic policy of the enterprise
20	Technical policy of the enterprise
21	Personnel policy of the enterprise
22	Credit history of the borrower (if not absent)

3. Results

3.1. Indicator Unification Process

This section describes in detail how the given indicators are mapped onto the interval [0, 1], i.e., how the indicators are unified. The case is considered when one linguistic variable "degree of favorableness of a given indicator" is associated with each parameter (indicator) [22].

K_1—dynamics of the development of the industry. The average profit growth over a certain period is analyzed based on industry data and/or data (regional, provincial) regarding departments and services.

Let us denote i as the current year, N as the length of the analyzed period, and p_j as the profit earned in the industry in the year j.

$$K_1 = K_1(i) = \frac{\sum_{j=i-N+1}^{i} g(j)}{N}. \quad (1)$$

where $g(j) = \frac{p_j - p_{j-1}}{p_{j-1}} * 100\%$.

We assume $\mu_1(x) = \begin{cases} \frac{x}{\alpha}, & if\ x \leq \alpha, \\ 1, & if\ x > \alpha. \end{cases}$

We consider it positive if the average profit growth $\geq \alpha$. The α value is determined by the expert.

K_2—prospects for the development of the industry. Here, K_2 is calculated in the same way, only the predicted data for the years M are taken.

$$K_2 = K_2(i) = \frac{\sum_{j=i+1}^{j=i+M} h(j)}{M}. \quad (2)$$

where $h(j) = \frac{v_j - v_{j-1}}{v_{j-1}} * 100\%$, v_j—projected profit in j-th year.

Similarly, we assume $\mu_2(x) = \begin{cases} \frac{x}{\beta}, & if\ x \leq \beta, \\ 1, & if\ x > \beta. \end{cases}$

We consider it positive if the average projected profit growth is $\geq \beta$. In a general case, $\alpha \neq \beta$.

K_3—the needs of the market (industry) for such products (work, services). The range, demand, and supply of products in the industry are analyzed with the help of an assessment: fulfilling the assortment plan; level of average realized prices in the industry; indicators of the impact of product quality on its average price.

Certain factors may be considered such as: the proportion of new products in the total output; the proportion of certified products in the total output; the proportion of products that meet international standards in the total output.

K_4—the dynamics of the development of the economy of the region.

K_5—prospects for the development of the economy of the region.

K_6—the needs of the market (region) for such products (work, services).

The indicators of the second group, K_4, K_5, K_6, are assessed similarly, but on a regional scale, based on statistical information from regional departments and regional statistical services.

K_7—current liquidity ratio:

$$K_7 = \frac{CA}{StL}, \tag{3}$$

$$\mu_7(x) = \begin{cases} 1, & if\ x \geq 2, \\ \frac{x}{2}, & if\ x < 2. \end{cases}$$

where CA—value of current assets and StL —short-term liability. The recommended value is ≥ 2.

The current liquidity ratio characterizes the security of the short-term liabilities of the enterprise with all of its current assets. The current liquidity ratio gives an overall assessment of the liquidity of assets, showing how many KZT of current assets are accounted for in the KZT of the current liabilities. The logic of calculating this indicator is that the company mainly repays short-term liabilities due to current assets; therefore, if the current assets exceed the current liabilities, an enterprise can be considered to be functioning successfully. The value of the indicator can vary by industry and type of activity, and its reasonable growth in dynamics is usually regarded as a favorable trend.

K_8 —ratio of financial independence.

$$K_8 = \frac{TBs}{EC}, \tag{4}$$

$$\mu_8(x) = \begin{cases} 1, & if\ x \geq 50\%, \\ \frac{x}{50\%}, & if\ x < 50\%. \end{cases}$$

where TBs—total balance sheet and EC—equity capital. The recommended value is $\geq 50\%$.

This ratio shows the share of the business's own sources of funds in the capital of the enterprise.

K_9—the working capital financed by equity to total assets ratio:

$$K_9 = \frac{WC}{Res} * 100\%, \tag{5}$$

$$\mu_9(x) = \begin{cases} 1, & if\ x \geq \beta, \\ \frac{x}{\beta}, & if\ x < \beta. \end{cases}$$

where WC —working capital and Res—reserves. $\beta \approx 60$–80% is determined by the expert.

This indicator shows (evaluates) the financial potential of the enterprise's own sources of financing and ability to settle obligations. Its negative value indicates the lack of equity

in the turnover of the enterprise and the financing of the non-current part of its assets from borrowed funds. The coefficient shows the extent to which stocks are formed at the expense of the enterprise's own funds.

K_{10}—debt service coverage ratio.

$$K_{10} = \frac{NOI}{TDS}, \tag{6}$$

$$\mu_{10}(x) = \begin{cases} 1, & if\ 1.5 \leq x \leq 2, \\ \frac{x}{2}, & if\ x < 1.5. \end{cases}$$

where NOI —net operating income and TDS —total debt service.

The debt service coverage ratio is the ratio of the net cash flow generated during the payment period (once a month, once a quarter, etc.) by the company implementing the innovative project (net profit before interest is paid out depreciation ± change in the need for net working capital), which, in accordance with the forecast, can be used to pay off the main debt and interest to the planned amount of debt servicing for a given period under all loan agreements of the enterprise.

K_{11} —accounts receivable turnover ratio:

$$K_{11} = \frac{R}{\overline{AAR}}, \tag{7}$$

$$\mu_{11}(x) = \begin{cases} 1, & if\ x \geq 12, \\ \frac{x}{12}, & if\ x < 12. \end{cases}$$

where $\overline{AAR}$—average accounts receivables calculated according to the arithmetic mean formula and R—revenue.

This ratio shows the number of revolutions made by receivables for the period. The share of receivables in the current assets shows the proportion of receivables in the current assets of the organization. An increase in the value of this indicator, as a rule, is regarded as a negative phenomenon, since it indicates that an increasing share of the current assets is temporarily diverted from circulation and is not involved in the organization's current activities. However, an extremely low value of the indicator may be a sign of a rather tight credit policy, leading to a decrease in sales.

K_{12} —accounts payable turnover ratio:

$$K_{12} = \frac{R}{\overline{AAP}}, \tag{8}$$

$$\mu_{12}(x) = \begin{cases} 1, & if\ x \geq 1, \\ x, & if\ x < 1. \end{cases}$$

where $\overline{AAP}$—average accounts payable. This ratio shows the number of turns made by the accounts payable for the period. The share of accounts payable in current liabilities shows the share of accounts payable in current liabilities. Its increase, as a rule, indicates a certain decrease in the financial stability of the organization.

K_{13} —inventory turnover ratio.

Inventory turnover ratio shows how many times during the analyzed period the organization used the average available inventory balance. This indicator characterizes the quality of reserves and the effectiveness of their management. It allows us to identify the remaining unused, obsolete, or substandard reserves. The importance of the indicator lies in the fact that profit arises with each "turnover" of stock (i.e., use in production, the operating cycle). Please note that in this case, stock is also understood as inventory (stocks of finished products) and production stock (stocks of raw materials).

Stock turnover can be calculated as a ratio: the cost of sales to the average inventory.

$$K_{13} = \frac{CS}{\overline{ASB}}, \tag{9a}$$

$$\mu_{13}(x) = \begin{cases} 1, & if\ x \geq \alpha, \\ \frac{x}{\alpha}, & if\ x < \alpha. \end{cases}$$

where CS —cost of sales, $\overline{ASB}$ —average annual stock balance.

Along with the turnover ratio, the turnover ratio in days (trade) is often calculated. In this case, this means how many days the company has enough stock.

$$K_{13} = \frac{\overline{ASB}}{CS} * 365. \tag{9b}$$

There are no generally accepted standards for the turnover indicators; they should be analyzed within the framework of one industry and, even better, in the dynamics for a particular enterprise. The decrease in the inventory turnover ratio may reflect the accumulation of excess reserves, inefficient warehouse management, and the accumulation of unusable materials. However, high turnover is not always a positive indicator, because it can indicate the depletion of stock, which can lead to interruptions in the production process.

In addition, inventory turnover depends on the organization's marketing policy. For organizations with a high sales profitability, a lower turnover is more characteristic than for companies with a low rate of return.

K_{14}—debt-to-equity ratio.

$$K_{14} = \frac{LC}{\overline{EC}}, \tag{10}$$

$$\mu_{14}(x) = \begin{cases} 1, & if\ x < \lambda, \\ 1 - \lambda, & if\ x \geq \lambda. \end{cases}$$

where LC—loan capital and EC—equity capital. The recommended value is ≤ 1. This means that the debt ratio shows how much of the borrowed funds is in the capital of the enterprise.

K_{15} —profit margin ratio.

$$K_{15} = \frac{PoS}{\overline{R}}, \tag{11}$$

$$\mu_{15}(x) = \begin{cases} 1, & if\ x \geq \delta, \\ \frac{x}{\delta}, & if\ x < \delta. \end{cases}$$

where PoS—profit on sales and $\overline{R}$—revenue. Return on sales characterizes not only the cost effectiveness of the production and sale of products (goods, work, or services), but also pricing, and shows the share of sales profit in revenue.

K_{16}—the evaluation of the professional level of staff. It is assumed that several types of workers are needed at the enterprise (Table 2). The state with frames can be represented in the form of a table.

Table 2. Types of employees.

№	Types of Workers	Type 1	Type 2	...	Type k
1	Number of existing employees	n_1	n_2	...	n_k
2	Required number of workers	m_1	m_2	...	m_k

Next, we consider the vector $\delta' = \delta_1 \ldots \delta_k$, where $\delta_i = n_i - m_i$, $(1 \leq i \leq K)$.
The value $K_{16} = E(\delta') \in [0, 1]$ is determined by experts based on their preference.
K_{17}—the evaluation of the moral and psychological atmosphere in the enterprise.
We can assume that this value is also evaluated by an expert.

K_{18} —sufficiency of the period of stay of the enterprise in the market.

We assume $\mu_{18}(x) = \begin{cases} 1, & if\ x \geq \gamma, \\ \frac{x}{\gamma}, & if\ x < \gamma. \end{cases}$

Thereby, γ is the minimum period that, in the opinion of the expert, the enterprise should remain in the market [23].

The following parameters are also usually evaluated by an expert. The second option is when these parameters, or some of them, are not considered at all.

K_{19} —the economic policy of the enterprise. This parameter characterizes a set of organizational and managerial measures of economic development, developed and approved to achieve goals and objectives at various levels of management, from the enterprise (increasing the competitiveness of production and goods) to the government level (tax and investment policies, etc.).

K_{20}—the technical policy of the enterprise. This is generally a system of strategic measures carried out by the enterprise's management in the field of strategies to improve product quality, resource conservation, and the organizational and technical development of production as components of the target subsystem of the management system.

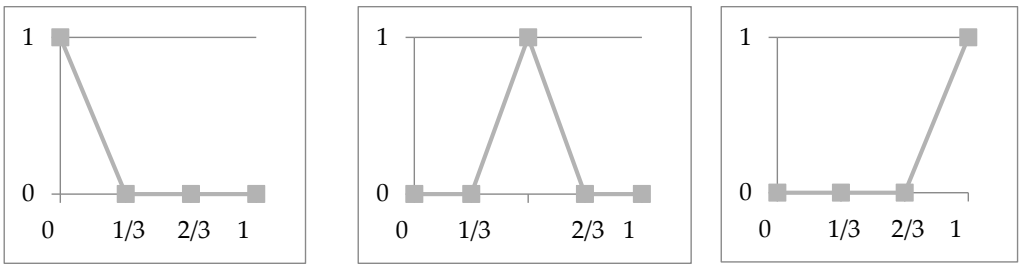
K_{21} —the personnel policy of the enterprise. A system of rules and norms (which must be understood and formulated in a certain way) that brings human resources into line with the firm's strategy (it follows that all HR activities—selection, staffing, certification, training, and promotion—are planned and consistent with a common understanding of the goals and objectives of the organization).

K_{22}—the credit history of the borrower (if not absent). This involves data showing how well the borrower complies with the rules for borrowing and fulfilling obligations arising from the terms of credit transactions.

3.2. Decision-Making Process

It was noted above that parameters (indicators) take rather arbitrary values. More precisely, each parameter varies in a certain inherent interval. Further, we assume that the parameters are unified, i.e., the corresponding intervals are mapped to the interval [0, 1].

Each i-th indicator can be associated with one "universal" predicate $P^i(x)$ or three single-place predicates $P^i_L(x)$, $P^i_M(x)$, $P^i_H(x)$, which naturally arise through composition with the functions below (Figure 1a–c).



(**a.** Low) (**b.** Average) (**c.** High)

Figure 1. Membership functions.

For simplicity, formula-definable predicates are also usually introduced:

$$P^i_{LM}(x) = P^i_L(x) \cup P^i_M(x), \quad P^i_{HM}(x) = P^i_M \cup P^i_H$$

Similarly, with each financial indicator, i.e., characteristic of the loan, we associate the predicate Q^i_D, $D \in \{L,\ M,\ H,\ LM,\ HM\}$. The rules based on which decisions are made have the form $\varphi(x_1, \ldots,\ x_n) \to \psi(y_1,\ \ldots,\ y_m)$.

Usually, $\varphi(x_1, \ldots, x_n) = \Lambda_{j=1}^{n} Q_j(x_j)$, i.e., we have a conjunction of unary predicates of the above signature. Moreover, each Q_j has the form P_D^i, $D \in \{L, M, H, LM, HM\}$. As a result, we obtain a rule of the following form [24]:

$$\begin{aligned} P_{LM}^1(x_1) \cap P_{LM}^2(x_2) \cap P_M^3(x_3) \cap P_M^4(x_4) \cap P_{HM}^5(x_5) \cap P_M^6(x_6) \cap P_{LM}^7(x_7) \cap P_M^8(x_8) \\ \cap P_L^9(x_9) \cap P_L^{10}(x_{10}) \cap P_H^{11}(x_{11}) \cap P_M^{12}(x_{12}) \cap P_L^{13}(x_{13}) \\ \cap P_{LM}^{14}(x_{14}) \cap P_L^{15}(x_{15}) \cap P_{LM}^{16}(x_{16}) \cap P_M^{17}(x_{17}) \cap P_{HM}^{18}(x_{18}) \\ \cap P_M^{19}(x_{19}) \cap P_{LM}^{20}(x_{20}) \cap P_{LM}^{21}(x_{21}) \cap P_L^{22}(x_{22}) \rightarrow Q_M^1(y_1). \end{aligned} \tag{12}$$

Several rules may be used $R_i : \varphi_i(x_1, \ldots, x_n) \rightarrow \psi_i(y_1, \ldots, y_n)$, $i = 1, ..N$.

For brevity, we write $R_i = R_i(x_1, \ldots, x_n, y_1, \ldots, y_m)$.

In most cases, it is natural to consider their disjunction $R = \bigvee_i R_i$.

Parameters $x_1, \ldots, x_n$ are obtained because of the analysis of the enterprise. These are the indicators mentioned in Section 1. Parameters $y_1, \ldots, y_m$ are predicted. These are indicators such as loan amount, interest rate, and loan duration. In its most general form, one parameter is predicted called the credit index, varying from 0 to 1 and having a natural interpretation. Preference is given to enterprises with a higher credit rating.

3.3. Examples of Lending to Enterprises

Let us consider an algorithm for constructing assessments of the creditworthiness of enterprises using examples. The proposed method has been approved by the banks of the Akmola region of Kazakhstan. Loans were issued, which are currently being successfully repaid.

The enterprises specialize in the provision of services and trade. As input variables, we will use indicators of industry and regional specifics, financial and economic indicators, and indicators of a small enterprise to assess the creditworthiness of the borrower. The output variable "enterprise solvency" plays the role of an indicator and takes values from 0 to 1. On its basis, a decision on the loan is made.

Enterprise-1 (E-1) is engaged in the trade of clothing and footwear (women, men, and children). The company specializes in imported goods, which are purchased as needed rather than as much as possible. There are three warehouses for storing the autumn, spring, summer, and winter collections. The company employs three vendors, one cleaner, and a manager with over 10 years of experience in the trade. The purpose of the loan is to replenish working capital. There are three more similar shops in the area. The enterprise has been on the market for 8 years. The borrower's credit history is missing.

Enterprise-2 (E-2) provides taxi services. The taxi company employs four dispatchers, 20 drivers, and a manager with 6 years of experience in the provision of services. There are nine cars and new equipment (walkie-talkies and a dispatcher's console). In terms of real estate, the enterprise rents a box measuring 300 square meters, three containers, an office, and a second box for parking. The client is not on the register of debtors and has no fines. The enterprise has been on the market for 6 years.

Enterprise-3 (E-3) provides catering services. The cafe is located along the Astana–Petropavlovsk highway. The premises are rented with subsequent purchase. The company employs two cooks, three assistant cooks, three bartenders, three people preparing sushi, two people preparing barbecue, and a leader. The enterprise has been on the market for 5 years.

Enterprise-4 (E-4) specializes in trade and services. The company sells computers, office equipment, satellite dishes, mobile phones, and greenhouse structures, and also repairs computers, refills cartridges, and provides accounting services. The company employs 25 workers, an accountant, and a manager. The property has a trading house in the center of the district. The equipment includes 20 computers, furniture, six printers, a greenhouse, and three vehicles. The enterprise has been on the market for 10 years.

To represent the terms of the input linguistics, triangular fuzzy numbers were used [25]. A triangular fuzzy number (a, l, r) is determined by the following parameters: a is the modal value, and l, r are the left and right uncertainty coefficients, respectively. When defining fuzzy intervals, the modal value is replaced by the interval $[a, b]$ where the membership function takes the value 1.

Let us analyze a given set of quality indicators. The indicators of industry and regional specifics were obtained by us based on data from the Committee on Statistics of the Ministry of National Economy of the Republic of Kazakhstan and the Department of Statistics of the Akmola region (Table 3).

Table 3. Input variables—indicators of industry and regional specifics.

	Indicators	E-1	E-2	E-3	E-4
K_1	Dynamics of development of the industry	M (0.75; 0.25)	M (0.75; 0.25)	M (0.75; 0.25)	HM (1; 0.5)
K_2	Prospects for the development of the industry	M (0.75; 0.25)	M (0.75; 0.25)	M (0.75; 0.25)	HM (1; 0.5)
K_3	Needs of the market (industry) for such products (work, services)	M (0.75; 0.25)	M (0.75; 0.25)	M (0.75; 0.25)	HM (1; 0.5)
K_4	Dynamics of the development of the economy of the region	HM (1; 0.5)	H (1; 0.75)	M (0.75; 0.25)	HM (1; 0.5)
K_5	Prospects for the development of the economy of the region	M (0.75; 0.25)	M (0.75; 0.25)	M (0.75; 0.25)	H (1; 0.75)
K_6	Needs of the market (region) for such products (work, services)	HM (1; 0.5)	HM (1; 0.5)	M (0.75; 0.25)	HM (1; 0.5)

We will calculate financial and economic indicators using the data from the balance sheet and the profit and loss statement of the enterprise for the current year (Table 4).

Table 4. Input variables—financial and economic indicators.

	Indicators	Recommended Value	E-1	E-2	E-3	E-4
K_7	Current liquidity ratio	$K_7 \geq 2$	$K_7 = 8$ H (1; 0.75)	$K_7 = 0.02$ L (0.25; 0)	$K_7 = 0.18$ L (0.25; 0)	$K_7 = 1.83$ HM (1; 0.5)
K_8	Ratio of financial independence	$K_8 \geq 50\%$	$K_8 = 93\%$ H (1; 0.75)	$K_8 = 91\%$ H (1; 0.75)	$K_8 = 60\%$ H (1; 0.75)	$K_8 = 84\%$ H(1; 0.75)
K_9	Working capital financed by equity to total assets ratio	$K_9 \geq 70\%$	$K_9 = 89\%$ H (1; 0.75)	Not recommended	Not recommended	$K_9 = 0.47$ HM (1; 0.5)
K_{10}	Debt service coverage ratio	$2 > K_{10} \geq 1.5$	$K_{10} = 2.21$ HM (1; 0.5)	$K_{10} = 1.5$ H (1; 0.75)	$K_{10} = 1.29$ HM (1; 0.5)	$K_{10} = 1.33$ HM (1; 0.5)
K_{11}	Accounts receivable turnover ratio	$K_{11} \geq 12$	$K_{11} = 15.77$ H (1; 0.75)	$K_{11} = 0$ L (0.25; 0)	$K_{11} = 19$ H (1; 0.75)	$K_{11} = 5.89$ M (0.75; 0.25)
K_{12}	Accounts payable turnover ratio	$K_{12} \geq 1$	$K_{12} = 1.83$ H (1; 0.75)	$K_{12} = 0.95$ HM (1; 0.5)	$K_{12} = 0.65$ M (0.75; 0.25)	$K_{12} = 0.9$ HM (1; 0.5)
K_{13}	Inventory turnover ratio	$18 > K_{13} \geq 8$	$K_{13} = 248$ L (0.25; 0)	Not recommended	$K_{13} = 7$ HM (1; 0.5)	Not recommended
K_{14}	Debt-to-equity ratio	$K_{14} \leq 1$	$K_{14} = 0.07$ H (1; 0.75)	$K_{14} = 0.09$ H (1; 0.75)	$K_{14} = 0.64$ H (1; 0.75)	$K_{14} = 0.19$ H (1; 0.75)
K_{15}	Profit margin ratio	$K_1 \geq 40\%$	$K_{15} = 50\%$ H (1; 0.75)	$K_{15} = 100\%$ H (1; 0.75)	$K_{15} = 57\%$ H (1; 0.75)	$K_{15} = 45\%$ H (1; 0.75)

The performance of a small business is determined based on the professional judgment of a loan officer (Table 5).

Table 5. Input variables—indicators of activity of small enterprises.

	Indicators	Assessment Criteria	E-1	E-2	E-3	E-4
K_{16}	Evaluation of the professional level of staff	Number of specialists by level	LM (0.5; 0)	LM (0.5; 0)	LM (0.5; 0)	M (0.75; 0.25)
K_{17}	Evaluation of the moral and psychological atmosphere in the enterprise	Specializations: qualifications, education, and work experience	HM (1; 0.5)	M (0.75; 0.25)	M (0.75; 0.25)	M (0.75; 0.25)
K_{18}	Sufficiency of the period of stay of the enterprise in the market	These data were extracted from the survey	H (1; 0.75)	H (1; 0.75)	H (1; 0.75)	H (1; 0.75)
K_{19}	Economic policy of the enterprise	Enterprises on the market	LM (0.5; 0)	M (0.75; 0.25)	LM (0.5; 0)	HM (1; 0.5)
K_{20}	Technical policy of the enterprise	Compliance with submitted business plan	LM (0.5; 0)	HM (1; 0.5)	LM (0.5; 0)	HM (1; 0.5)
K_{21}	Personnel policy of the enterprise	Availability of real estate, transport, equipment, and their use within work at the enterprise	M (0.75; 0.25)	HM (1; 0.5)	M (0.75; 0.25)	M (0.75; 0.25)
K_{22}	Credit history of the borrower (if not absent)	Human resources management: selection, staffing, certification, training	L (0.25; 0)	H (1; 0.75)	H (1; 0.75)	H (1; 0.75)

The fuzzy production system for assessing the creditworthiness of a small business based on the theory of fuzzy sets is a functional mapping of the form:

$$X = \{x_i\} \to U = \{u_j\},\ i = 1, \ldots, n; j = 1, \ldots, s$$

$$U = \{u_j\} \to Q,\ j = 1, \ldots, s$$

where X is the vector of influencing indicators, U is the aggregated influencing indicators, and Q is the overall creditworthiness of a small business.

The set of groups of evaluation indicators, $U = \{u_j\},\ j = 1, \ldots, s$ by which it is proposed to determine the feasibility of lending to the borrower, must be divided into qualitative and quantitative:

- Indicators of industry and regional specifics;
- Financial and economic indicators;
- Small business performance indicators.

Decision making on lending (or refusing to lend) to a borrower is the construction of mappings of indicators x_i from set X according to criteria a_j of set A. As a result, for each indicator x_i, considering a set of criteria, it becomes possible to find its own value of creditworthiness. To obtain the final output Q, we summarize the values of the indicators, considering the influence of the weight:

$$Q_j = n_1 \sum_{i=1}^{6} \omega_i X_{ij} + n_2 \sum_{i=7}^{15} \omega_i X_{ij} + n_3 \sum_{i=16}^{22} \omega_i X_{ij}. \quad (13)$$

where $n_1,\ n_2,\ n_3$ is the weight of importance for aggregated indicators and ω_i is the weight of importance of the i-th indicator. The importance weights were found based on the Saaty hierarchy analysis method.

The weight depends on the information provided. According to the expert, various assessed indicators of the borrower may have a different impact on the final decision on the appropriateness of lending. In this work, methods of pairwise comparison of the criteria based on floating preference are used, which form the basis of the hierarchy analysis method. Paired comparisons are represented by a matrix, determined by a scoring scale developed by the mathematician T. Saaty.

In [26], a simplified approach to the calculation of criteria weights using the method of pairwise comparison of criteria based on floating preference is proposed in which, for

a strictly consistent matrix of pairwise comparison of criteria, the values of ω_j can be calculated using the following formula [26]:

$$\omega_j = \frac{1}{\sum_{i=1}^{n} k_{ij}}. \quad (14)$$

where k_{ij} —the enterprise parameters.

The value of Q allows you to make a decision to lend or refuse lending. Taking into account the variants of lending schemes available to the bank (set S) and based on the value of the final rating Q, the optimal lending scheme is selected.

As a result, we obtain the values of the estimates presented in Table 6.

Table 6. Output variable "Creditworthiness of the enterprise".

№	Name of Enterprises	Creditworthiness of the Enterprise
1	Enterprise-1	M (0.83; 0.42)
2	Enterprise-2	M (0.81; 0.41)
3	Enterprise-3	M (0.73; 0.31)
4	Enterprise-4	HM (0.96; 0.52)

4. Discussion

This article discusses the peculiarities of lending to small businesses operating in the trade and services sector. The modern small-scale enterprise is an open and dynamically developing socioeconomic and production-marketing system. The different economic, financial, social, and investment results received as the result of industrial and economic activity cannot be estimated by means of any single indicator. It is necessary to use a set of indicators. This is caused by the fact that none of the known indicators can reflect the diverse final economic, financial, and social results of the economic activity of small enterprises.

An approach to assess creditworthiness using the mathematical apparatus of fuzzy logic, which allows taking into account approximate qualitative information about the characteristics of the borrower, is proposed.

In order to determine the appropriateness of lending to a small enterprise, we considered a set of groups of evaluation indicators: industry and regional specifics, small business activities, and the financial and economic characteristics of the service and trade sectors.

The set of financial and economic indicators (Table 1) can be modified depending on the preferences of the loan officer or in accordance with the input information provided by the borrower, which can be financial statements or data from the simplified taxation system.

The following values are used to classify the level of indicator x_i: a_1 = L (low)—low; a_2 = LM—low medium; a_3 = M (medium)—medium; a_4 = HM—high medium; a_5= H—high.

Ultimately, the choice of the type of affiliation function, both of the alternatives themselves and of the criteria to be evaluated, is determined on the basis of expert preferences. Obviously, the expert uncertainty in an evaluation increases as the deviation of the value of the evaluated parameter from the optimal value increases. Moreover, in most cases, this uncertainty does not grow linearly. However, the use of nonlinear membership functions entails a significant number of complicated mathematical calculations and graphical constructions. For these reasons, in this paper, triangular and trapezoidal accessory functions are used as initial accessory functions, which is primarily due to the simplicity of performing subsequent calculations and graphical constructions. We can say that the membership functions are piecewise linear. Note that the parameters (indicators) take rather arbitrary values. More precisely, each parameter changes in a certain inherent interval. Further, the unification of the parameters is made, i.e., the corresponding intervals are mapped on the segment [0, 1]. The rules by which decisions are made are formulated as logical formulas containing parameters. In its most general form, one parameter is predicted, called the creditworthiness index, which varies from 0 to 1 and has a natural interpretation. Preference is given to enterprises with a higher creditworthiness index. The value

of the fuzzy set resulting from rules 12 and 13 can be used as a criterion for choosing the most appropriate lending scheme. Interval numbers can be used to evaluate expert opinions [27–29]. In addition, interval weights are widely used in various decision-making tasks [30–34]. Sengupta and Pal provided a comprehensive review of the methods for comparing interval numbers [35]. The idea of using probabilistic measures to quantify the intensity of a preference between two intervals is not entirely new [36–38].

The proposed method was tested in four small-scale enterprises of Akmola oblast, which were granted loans by commercial banks, and these loans have been successfully repaid by the enterprises. In order to justify the applicability of the methodology proposed in the paper, the creditworthiness of the borrowers was analyzed and the results obtained were compared with the expert opinions of three borrowers. To confirm the cost-effectiveness of the methodology, the decision was made to subject the existing borrowers to stress testing (artificially changing the initial values of the assessed characteristics) in order to determine the consistency of the newly obtained results with the new expert assessments, which would allow conclusions to be drawn about the validity of the results obtained in the testing.

5. Conclusions

Working with data and knowledge, the implementation of inference procedures, and the need to perform complex information and logical procedures in Information Systems often require the transformation and processing of information presented both in a clear form and with the help of fuzzy mathematics and fuzzy logic. As a rule, software tools do not achieve the required management efficiency. Therefore, at present, an important area of research is the creation of hardware that implements these procedures. Among the hardware support tools, an important class is technical tools for implementing decision-making procedures based on transformations and processing fuzzy data and knowledge. More and more attention is being paid to the creation of specialized technical methods focused on the processing of fuzzy information. This is due to the desire to bring existing Information Systems beyond the scope of simple symbolic conclusions and bring them closer to human thinking. The use of fuzzy sets and fuzzy logic to represent the knowledge of a specialist expert allows the capabilities of Information Systems to be expanded in relation to modeling reasoning and decision making.

The proposed method for assessing the creditworthiness of a small enterprise borrower, based on fuzzy logic with qualitative and quantitative indicators, allows the user to:

- Maintain the quality of the analysis of creditworthiness due to the participation of a loan officer in the decision-making process, while correctly operating with expert assessments through the use of the mathematical apparatus of fuzzy sets;
- Carry out an assessment of the creditworthiness of an enterprise as a dynamic process throughout the entire period of lending, which makes it possible for a commercial bank to organize a system for monitoring the state of the borrower, as well as to signal the period of the onset of credit risk;
- Replace the set of financial indicators depending on the preferences of the loan officer, the specifics of the borrower's activities, or in accordance with the financial information provided by the borrower, which can be taken from standard financial statements or data from the simplified taxation system;
- Form a single automated database based on indicators for assessing the dynamics of development by industry and the region as a whole, which will allow banks to avoid the recalculation of quality indicators when assessing the creditworthiness of a small enterprise.

The proposed scientific approach can be used as the basis for the creation of expert decision support systems in various areas of socioeconomic activity and process monitoring for the analysis of the financial and economic activities of an enterprise.

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