ЕУРАЗИЯ ҰЛТТЫҚ УНИВЕРСИТЕТІ

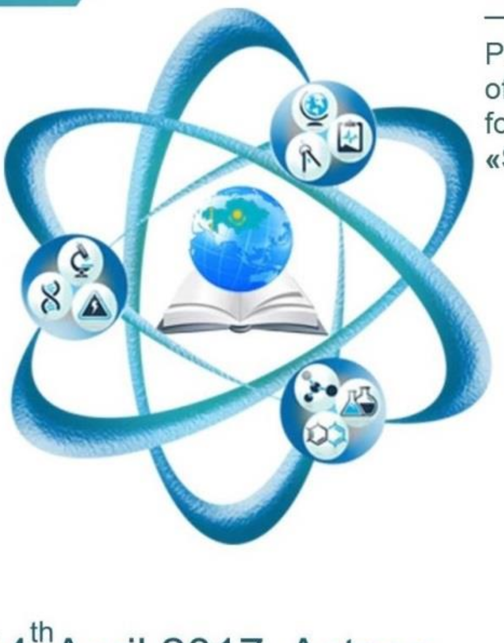
ҚАЗАҚСТАН РЕСПУБЛИКАСЫ ТҰҢҒЫШ ПРЕЗИДЕНТІ - ЕЛБАСЫНЫҢ ҚОРЫ

«ҒЫЛЫМ ЖӘНЕ БІЛІМ – 2017»

студенттер мен жас ғалымдардың
XII Халықаралық ғылыми конференциясының
БАЯНДАМАЛАР ЖИНАҒЫ

СБОРНИК МАТЕРИАЛОВ
XII Международной научной конференции
студентов и молодых ученых
«НАУКА И ОБРАЗОВАНИЕ – 2017»

PROCEEDINGS
of the XII International Scientific Conference
for students and young scholars
«SCIENCE AND EDUCATION - 2017»



14th April 2017, Astana

ҚАЗАҚСТАН РЕСПУБЛИКАСЫ БІЛІМ ЖӘНЕ ҒЫЛЫМ МИНИСТРЛІГІ
Л.Н. ГУМИЛЕВ АТЫНДАҒЫ ЕУРАЗИЯ ҰЛТТЫҚ УНИВЕРСИТЕТІ

«Ғылым және білім - 2017» студенттер мен жас ғалымдардың XII Халықаралық ғылыми конференциясының БАЯНДАМАЛАР ЖИНАҒЫ

СБОРНИК МАТЕРИАЛОВ XII Международной научной конференции студентов и молодых ученых «Наука и образование - 2017»

PROCEEDINGS of the XII International Scientific Conference for students and young scholars «Science and education - 2017»

2017 жыл 14 сәуір

Астана

УДК 378

ББК 74.58
Ғ 96

Ғ 96
«Ғылым және білім – 2017» студенттер мен жас ғалымдардың XII Халықаралық ғылыми конференциясы = The XII International Scientific Conference for students and young scholars «Science and education - 2017» = XII Международная научная конференция студентов и молодых ученых «Наука и образование - 2017». – Астана: http://www.enu.kz/ru/nauka/nauka-i-obrazovanie/, 2017. – 7466 стр. (қазақша, орысша, ағылшынша).

ISBN 978-9965-31-827-6

Жинаққа студенттердің, магистранттардың, докоранттардың және жас ғалымдардың жаратылыстану-техникалық және гуманитарлық ғылымдардың өзекті мәселелері бойынша баяндамалары енгізілген.

The proceedings are the papers of students, undergraduates, doctoral students and young researchers on topical issues of natural and technical sciences and humanities.

В сборник вошли доклады студентов, магистрантов, докторантов и молодых ученых по актуальным вопросам естественно-технических и гуманитарных наук.

УДК 378

ББК 74.58

ISBN 978-9965-31-827-6

УДК 519.6

PRACTICAL TRACKING FOR NONLINEAR HIGH ORDER TIME DELAY SYSTEMS BY STATE FEEDBACK

Azbergenova Baldyrgan
Postgraduate of ENU after named L.N.Gumyliev, Astana, Kazakhstan
Supervisor – Keylan Alimhan

Tracking control of nonlinear systems is a significant problem in control theory which is widely explored for many times. We may explain it by the importance of the solution of this problem in practice. There are many applications of tracking. For example, tracking had been widely employed for military applications and civil applications as in Air Traffic Control in an airport. In aerospace tracking is applied tracking of a satellite in orbit. There are many examples to list which prove how widely may be used tracking applications in practice.

Time delay phenomena exists in many practical systems such as electrical networks, microwave oscillators, and hydraulic systems. They may emerge for many reasons. Many dynamical systems cannot be properly described by differential equation because the condition of the state variables $x(t)$ in the future time depends not only on their current value, but also on their past values. Such system is defined as a time-delay system. Time delay may be occurred due to the delayed measurements and delayed control. In both occasions, the delay considered to be undesirable because they may cause the deterioration of the system performance and even destabilize the system. By using time delay systems we can describe biological, chemical and physical and other phenomena. It is familiar that time delay phenomena started to take our interest from the beginning of the 21th century and in its turn lead to numerous important results. Due to these results many problems in different fields of life found their solution [1]. For the time systems without delay Lyapunov method is an effective way for solving stabilization or tracking problem. To say exactly, it is required to construct a Lyapunov function. But in the case of system with time-delay Lyapunov function considered to be a functional. We call it Lyapunov-Krasovskii functional. This functional is used first in the work of Krasovskii [2].

Tracking problem for time delay systems is an issue related to control theory, which we are going to investigate in this paper. It has the following form:

$$\begin{aligned} \dot{x}_i &= x_{i+1}^{p_i} + f_i(x_1,...,x_i,x_1(t-d),...,x_i(t-d)) \\ \dot{x}_n &= u^{p_n} + f_n(x_1,...,x_n,x_1(t-d),...,x_n(t-d)) \quad (1) \\ y &= x_1 \end{aligned}$$

where $x(t)=(x_1(t),x_2(t),...,x_n(t))\in R_n$ is the state and u is the control input of the system; $d\in R^+$ is the time delay of the state; $f_i, i=1,...,n$ are unknown continuous functions; $p_i \in R_{odd}^{\geq 1} := \{\frac{p}{q} \mid \text{p and q are positive integers, and } p\geq q\}$.

The purpose of this paper is to show that tracking problem of high-order time delay non-linear systems is solvable by state feedback. Tracking problem when $d=0$ is investigated from the last decades and we get results which gave us opportunity to solve many problems. Now we study tracking problem when $d\neq 0$ and design a controller for high-order non-linear time delay systems.

Now we present several important lemmas and definition what are useful for obtaining the main result.

Definition [3]. For fixed coordinates $x(t)=(x_1(t),x_2(t),...,x_n(t))\in R_n$ and real numbers $r_i>0,\ i=1,...,n$ consider the following:

1. The dilation $\Delta_\varepsilon(x)$ is defined by $\Delta_\varepsilon(x)=(\varepsilon^{r_1}x_1,...,\varepsilon^{r_n}x_n)$ for any ε>0, where $r_i$is called the weights of the coordinates. For simplicity, we define dilation weight $\Delta=(r_1,...,r_n)$.

2. A function $V\in(R^n,R)$ is said to be homogeneous of degree τ if there is a real number $\tau\in R$ such that $\Delta_\varepsilon(x)=\varepsilon^{\tau}V(x_1(t),...,x_n(t))$ for any $x\in R^n\setminus\{0\},\varepsilon>0.$

3. A vector field $f\in(R^n,R^n)$ is said to be homogeneous of degree τ if there is a real number $\tau\in R$ such that $f_i(\Delta_\varepsilon(x))=\varepsilon^{\tau+r_i}f_i(x)$, for any $x\in R^n\setminus\{0\},\ \varepsilon>0, i=1,...,n.$

Lemma 1 [3]. Given a dilation weight $\Delta=(r_1,...,r_n)$, suppose $V_1(x)$ and $V_2(x)$ are homogeneous functions of degree τ_1 and τ_2, respectively. Then $V_1(x)V_2(x)$ is also homogeneous with respect to the same dilation weight Δ. Moreover, the homogeneous degree of $V_1(x)V_2(x)$ is $\tau_1+\tau_2$.

Lemma 2 [3]. Suppose: $R_n\to R$ is a homogeneous function of degree τ with respect to the dilation weight Δ. Then the following holds:

1. $\dfrac{\partial V}{\partial x_i}$ is homogeneous of degree $\tau-r_i$ with r_i being the homogeneous weight of x_i;

2. There is a constant c such that $V(x)\le c\|x\|_\Delta^\tau$. Moreover, if $V(x)$ is positive definite; then $\bar{c}\|x\|_\Delta^\tau\le V(x)$, where $\bar{c}$ is a constant.

Lemma 3 [3]. For $x\in R, y\in R$ and $p\ge 1$ being a constant, the following inequalities hold:

$$|x+y|^p\le 2^{p-1}|x^p+y^p|$$

$$(|x|+|y|)^{\frac{1}{p}}\le|x|^{\frac{1}{p}}+|y|^{\frac{1}{p}}\le 2^{\frac{p-1}{p}}(|x+y|)^{\frac{1}{p}}.$$

If $p\ge 1$is odd, then

$$|x-y|^p\le 2^{p-1}|x^p-y^p|$$

$$\left|x^{\frac{1}{p}}-y^{\frac{1}{p}}\right|\le 2^{\frac{p-1}{p}}(|x-y|)^{\frac{1}{p}}.$$

Lemma 4 [3]. Let c,d be positive constants. Given any positive number $\gamma>0$, the following inequality holds:

$$|x|^c|y|^d\le\frac{c}{c+d}\gamma|x|^{c+d}+\frac{d}{c+d}\gamma^{-\frac{c}{d}}|y|^{c+d}$$

Assumption 1 [4].

1. For $i=1,...,n$ thereareconstants $a>0$ and $\tau>\dfrac{-1}{\sum_{l=1}^{n}p_1\cdots p_{l-1}}$ such that

$$f_i(x_1,...,x_i,x_1(t-d),...,x_i(t-d))\le a\sum_{j=1}^{i}(|x_j(t)|^{\frac{r_i+\tau}{r_j}}+|x_j(t-d)|^{\frac{r_i+\tau}{r_j}}),$$

where $r_1 = 1$ and $r_{i+1} = \dfrac{r_i + \tau}{p_i} > 0, i = 1,\ldots,n.$

2. The reference signal is bounded, that is, it is a function such that there is a known constant satisfying

$$|y_r(t)| \le D, |y_r(t)| \le D, \forall t \in [0,\infty)$$

The objective of this paper is to design a state feedback controller for system (1) under Assumption 1 and 2 such that closed-loop system tracks the reference signal.

Theorem (main result): Under Assumption 1-2 on system (1), the practical tracking problem stated in Definition is solvable by controller in the following form:

$$u^{p_n} = L^{k_n+1}\upsilon^{p_n}.$$

Example and simulation

Consider the nonlinear system

$$\dot{x}_1 = x_2^{\frac{5}{3}} + \frac{1}{7}x_1^{\frac{1}{3}}(t-d)$$

$$\dot{x}_2 = x_3^{\frac{5}{3}} + \frac{1}{7}x_2(t)$$

$$\dot{x}_3 = u^{\frac{7}{3}} + \frac{1}{7}x_3^{\frac{1}{3}}(t)$$

$$y = x_1$$

where $p_1 = \dfrac{5}{3}, p_2 = \dfrac{5}{3}, p_3 = \dfrac{7}{3}$ and $y_r = \sin(t)$. The controller we designed by state feedback will be in the following form:

$$u^{\frac{7}{3}} = -2L^{\frac{49}{25}}(L^{-1}x_3 + 2(L^{-\frac{3}{5}}x_2 + 2(x_1 - y_r)))^{\frac{5}{3}}.$$

The result we obtained is presented in the Figs 1-2.

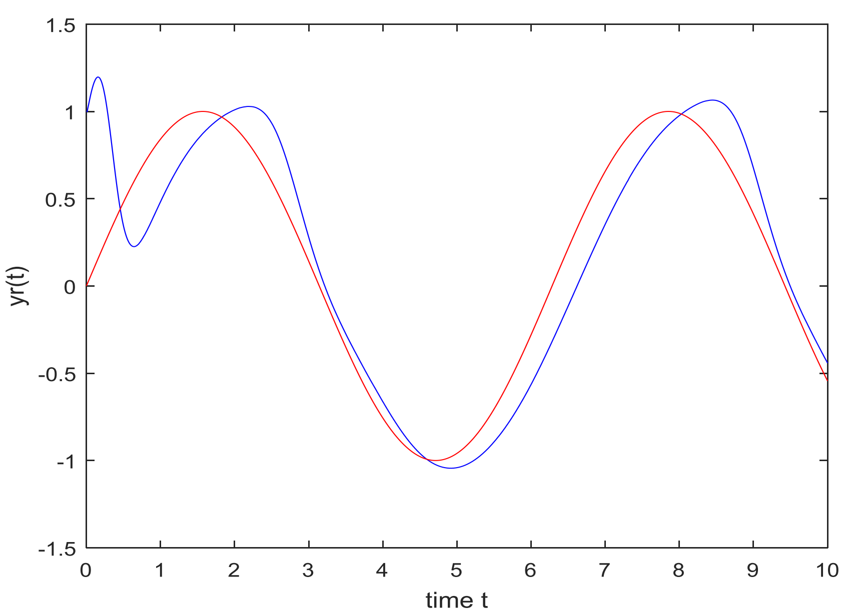


Fig.1.The trajectories of the first state and reference signal.

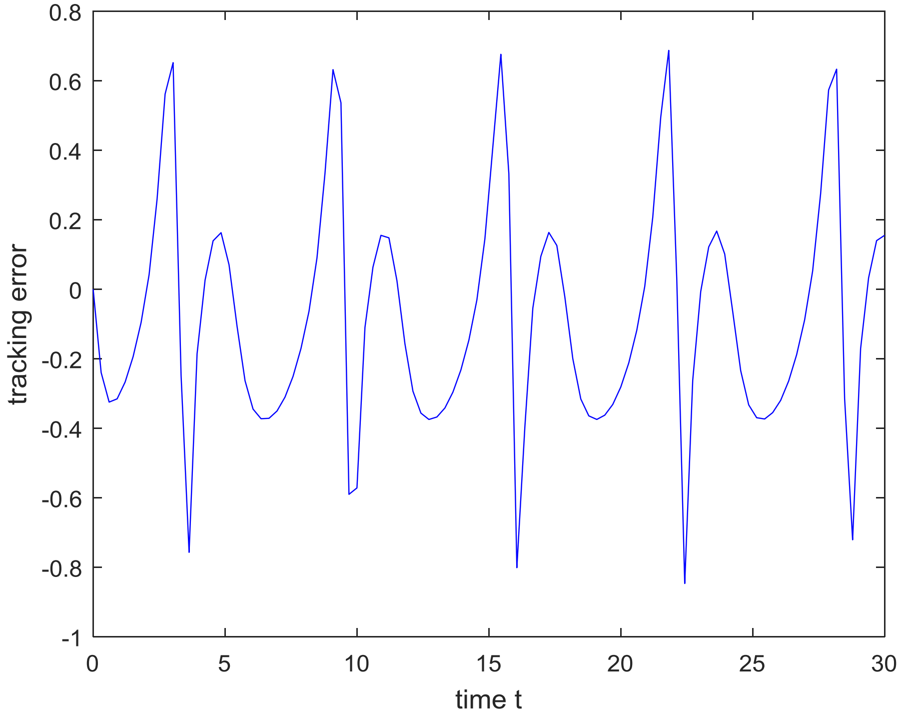


Fig.2. Tracking error between the first state and reference signal.

Conclusion

In this paper we investigated tracking problem for nonlinear high-order time delay systems. Using Lyapunov function and homogeneous theory we designed controller that allow to track a given reference signal. In Fig.1 we can see that the controller we obtained satisfy the conditions that we need. Fig.2 shows that tracking error changes in the interval [-0.8, 0.6]. It shows that our controller needs to be improved to make the tracking error small.

Literature

1. KeqinGu, Vladimir L. Kharitonov and Jie Chen Stability of Time-Delay Systems
2. N.N.Krasovskii Stability of motion
3. Nengwei Zhang, Enbin Zhang and Fangzheng Gao Global Stabilization of High-Order Time-Delay Nonlinear Systems under a Weaker Condition // Abstract and Applied Analysis 2014. P.2
4. KeylanAlimhan, Hiroshi InabaRobust practical output tracking by output compensator for a class of uncertain inherently non-linear systems // Int. J. Modelling, Identification and Control 2008. №4P. 307