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The proceedings are the papers of students, undergraduates, doctoral students and young researchers on topical issues of natural and technical sciences and humanities.

В сборник вошли доклады студентов, магистрантов, докторантов и молодых ученых по актуальным вопросам естественно-технических и гуманитарных наук.

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В этой статье исследуется осцилляторное поведение решений дифференциального уравнения Эмдем-Фаулера

$$\left(r(t)\left|u'(t)\right|^{\alpha-1}u'(t)\right)' + p(t)\left|u(t)\right|^{\beta-1}u(t)=0,\ t\ge t_0\ge 0 \qquad (1)$$

Предположим, что для уравнения (1)выполнены условия:

1. $\alpha\ne\beta, \alpha>0, \beta>0, \alpha\equiv\beta$
2. $r\in C,\ r(t)>0, R(t)=\int_{t_0}^{t} r^{-\frac{1}{\alpha}}(s)ds\to\infty,\ t\to\infty$
3. $p\in C_{[t_0,\infty]},\ p(t)>0$
4. Сущесивует такое $k>0$, что $u^{\frac{\beta}{\alpha}-1}\ge k$

Решением уравнение (1) назовем такую функцию $u(t)\in C^1_{[T,\infty]}, T>t_0$, которая имеет свойство $r(t)\left|u'(t)\right|^{\alpha-1}u'(t)\in C^1_{[T,\infty]}$ и удовлетворяет уравнению (1) на $[T,\infty]$.

Нетривиальное решение уравнения (1) называется осцилляторным, если оно имеет бесконечно много нулей, стремящихся к бесконечности. Уравнение называется осцилляторным,если его решения осцилляторные.

Основными методами [1,2]исследования этих вопросов являются вариационный метод и метод, называемый “Техника Риккати”.

В настоящей работе с помощью второго метода получен достаточный признак осцилляторности уравнения (1).

Теорема. Если выполнено условие

$$\lim_{t\to\infty}\int_{t_0}^{t}\left(\frac{1}{r(s)}\int_{s}^{\infty}p(z)dz\right)^{\frac{1}{\alpha}}ds=\infty \qquad (2)$$

и найдется такое число μ и дифференцируемая функция $\rho[t_0,\infty)\to(0,\infty)$,которые удовлетворяют соотношениям

$\rho'(s)\ge 0, \lim_{t\to\infty}\sup\int_{t_0}^{t}\left\{p(s)\rho^{\alpha}(s)-\mu\cdot\frac{r(s)\rho'^{\alpha+1}(s)}{\rho(s)}\right\}ds=\infty$ тогда уравнение (1) будет осцилляторным.

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EXPLICIT SOLUTION OF 2+1-DIMENSIONAL HIROTA-MAXWELL-BLOCH EQUATION

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We consider following nonlinear differential equation, namely the 2+1-dimensional Hirota-Maxwell-Bloch equation [1]

$$iq_t - q_{xy} - \frac{i}{2} q_{xxy} - vq + i(\omega q)_x - 2ip = 0, \qquad (1)$$

$$v_x - 2\left(|q|^2\right)_y + i\left(q^*_{xy} q - q^* q_{xy}\right) = 0, \qquad (2)$$

$$\omega_x + \left(|q|^2\right)_y = 0, \qquad (3)$$

$$p_x - 2i\omega p - 2\eta q = 0, \qquad (4)$$

$$\eta_x + \left(q^* p + p^* q\right) = 0, \qquad (5)$$

where $q = q(x, y, t),\ p = p(x, y, t)$ are complex functions, $v = v(x, y, t),\ \omega = \omega(x, y, t), \eta = \eta(x, y, t)$ are real functions, * means complex conjugation. The corresponding linear problem of the (2+1)-dimensional Hirota-Maxwell-Bloch equation takes the form [2-3]

$$\Psi_x = A\Psi, \qquad (6)$$

$$\Psi_t = -2\left(\lambda + \lambda^2\right)\Psi_y + B\Psi, \qquad (7)$$

where

$$\Psi = \Psi(x, y, t, \lambda) = \begin{pmatrix} \psi_1(x, y, t, \lambda) \\ \psi_2(x, y, t, \lambda) \end{pmatrix}, \qquad (8)$$

$$A = -i\lambda \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 0 & q \\ -r & 0 \end{pmatrix}, \qquad (9)$$

$$B = \lambda \begin{pmatrix} i\omega & q_y \\ r_y & -i\omega \end{pmatrix} + \begin{pmatrix} -0.5iv & -iq_y + 0{,}5q_{xy} - \omega q \\ -ir_y - 0{,}5_2 r_{xy} + \omega r & 0.5iv \end{pmatrix} + \frac{i}{\lambda + \omega} \begin{pmatrix} \eta & -p \\ -\kappa & -\eta \end{pmatrix}.$$

λ is the complex eigenvalue parameter.

The inerrability condition of the system (6)-(7)

$$A_t + 2(\lambda + \lambda^2) A_y - B_x + [A, B] = 0, \qquad (10)$$

where $[A, B]$ is commuter.

Here we use Darboux transformation for nonlinear differential equation that was described [4].The Darboux transformation on the solution of the Lax representation (6)-(7) if defined by [5].

$$\Psi^{[1]}(x, y, t) = T(x, y, t)\Psi(x, y, t), \qquad (11)$$

where $T(x, y, t)$ is Darboux 2×2 matrix. This matrix has form

$$T(x, y, t) = \lambda I - M(x, t), \qquad (12)$$

where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$ here m_{ij} are functions of x, y and t.

The new function $\Psi^{[1]}$ satisfies the following Lax representation

$$\Psi^{[1]}{}_x = A^{[1]}\Psi^{[1]}, \qquad (13)$$

$$\Psi^{[1]}{}_t = -2\left(\lambda + \lambda^2\right)\Psi^{[1]}{}_y + B^{[1]}\Psi^{[1]}. \qquad (14)$$

Then Darboux matrix T must satisfy following identities

$$T_x + TA = A^{[1]}T, \tag{15}$$

$$T_t + TB = -2(\lambda + \lambda^2)T_y + B^{[1]}T. \tag{16}$$

The relation between solutions $q,\ p,\ v,\ \omega,\ \eta$ and new solutions $q,\ p,\ v,\ \omega,\ \eta$ which called Darboux transformation can be got by equations (15)-(16) [6]. From system (15)-(16) get

$$A_0^{[1]} = A_0 + i[M, \sigma_3], \tag{17}$$

$$B_0^{[1]} = B_0 - MB_1 + (B_1 - 2M_y)M - 2M_y, \tag{18}$$

$$B_1^{[1]} = B_1 - 2M_y, \tag{19}$$

$$B_{-1}^{[1]} = (M + \omega I)B_{-1}(M + \omega I)^{-1}, \tag{20}$$

and $M(x,t)$ matrix should have a condition as $m_{21} = -m_{12}^*$.

We make the following choice of the matrix $M(x,t)$:

$$M = H\Lambda H^{-1}, \tag{21}$$

where $\det H \neq 0$,

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}, \tag{22}$$

where λ_1 and λ_2 are complex constants

$$H = \begin{pmatrix} \psi_1(\lambda_1, x, y, t) & \psi_1(\lambda_2, x, y, t) \\ \psi_2(\lambda_1, x, y, t) & \psi_2(\lambda_2, x, y, t) \end{pmatrix}. \tag{23}$$

The matrix $H(\lambda_1, \lambda_2, x, y, t)$ obey the liner system (6)-(7)

$$H_x = -\sigma_3 H\Lambda + A_0 H, \tag{24}$$

$$H_t = 2H_y\Lambda + B_0 H + A_{-1}H\Sigma, \tag{25}$$

where

$$\Sigma = \begin{pmatrix} \dfrac{i}{\lambda_1 + \omega} & 0 \\ 0 & \dfrac{i}{\lambda_2 + \omega} \end{pmatrix}. \tag{26}$$

IN the order to satisfy the constraints of matrix $M(x,t)$ and B_{-1} we notes that $A_0^+ = -A_0$,

$$\lambda_2 = \lambda_1^*, \quad H = \begin{pmatrix} \psi_1(\lambda_1, x, y, t) & -\psi_2^*(\lambda_1, x, y, t) \\ \psi_2(\lambda_1, x, y, t) & \psi_1^*(\lambda_1, x, y, t) \end{pmatrix}. \tag{27}$$

Then, assume H matrix $\det H \neq 0$, find matrix H^{-1}

$$H^{-1} = \frac{1}{\Delta}\begin{pmatrix} \psi_1^* & \psi_2^*) \\ -\psi_2 & \psi_1 \end{pmatrix}, \quad \Delta = |\psi_1|^2 + |\psi_2|^2. \tag{28}$$

Formula (27) and (28) substituting in the equation (21), then we get the new seed solutions

$$q^{[1]} = q + 2i\left(t_0^{[1]}\right)_{12}, \tag{29}$$

$$v^{[1]} = v + 4i\left(t_0^{[1]}\right)_{11y} - 4\omega\left(t_0^{[1]}\right)_{11} + 2\left[\left(t_0^{[1]}\right)_{12} q_y^* - \left(t_0^{[1]}\right)_{12}^* q_y - 2i\left(t_0^{[1]}\right)_{11}\left(t_0^{[1]}\right)_{11y} + 4i\left(t_0^{[1]}\right)_{12}^*\left(t_0^{[1]}\right)_{12y}\right], \tag{30}$$

$$\omega^{[1]} = \omega - 2i\left(t_0^{[1]}\right)_{11y}, \tag{31}$$

$$\eta^{[1]} = \frac{\left[\left|\omega - \left(t_0^{[1]}\right)_{11}\right|^2 - \left|\left(t_0^{[1]}\right)_{12}\right|^2\right]\eta + p\left(t_0^{[1]}\right)_{12}^{*}\left[\omega - \left(t_0^{[1]}\right)_{11}\right] + p^{*}\left(t_0^{[1]}\right)_{12}\left[\omega - \left(t_0^{[1]}\right)_{11}^{*}\right]}{\nabla}, \quad (32)$$

$$p^{[1]} = \frac{p\left[\omega - \left(t_0^{[1]}\right)_{11}\right]^2 + p^{*}\left(t_0^{[1]}\right)_{12}^2 - 2\eta\left(t_0^{[1]}\right)_{12}\left[\omega - \left(t_0^{[1]}\right)_{11}\right]}{\nabla}, \quad (33)$$

$$\nabla = \det(M + \omega I) = \omega^2 - \omega\left[\left(t_0^{[1]}\right)_{11} + \left(t_0^{[1]}\right)_{11}^{*}\right] + \left|\left(t_0^{[1]}\right)_{11}\right|^2 + \left|\left(t_0^{[1]}\right)_{12}\right|^2. \quad (34)$$

In conclusion, application of Darboux transformation to solve explicit solutions for the (2+1)-dimensional Hirota-Maxwell-Bloch equation (1). In this work Darboux transformation was used for nonlinear differential equation and taken solutions [1-6]..

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BOUNDEDNESS OF THE FRACTIONAL MAXIMAL OPERATOR IN LOCAL MORREY-TYPE SPACES

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Spaces $L_p(\Omega)$, $0 < p < \infty$

Let $0 < p < \infty$ and Ω would be a measurable set in R^n, and a function $f : \Omega \to C$. The function $f \in L_p(\Omega)$ if f is measurable on Ω and

$$\|f\|_{L_p(\Omega)} := \left(\int_{\Omega} |f|^p \, dx\right)^{\frac{1}{p}} < \infty$$