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В сборник вошли доклады студентов, магистрантов, докторантов и молодых ученых по актуальным вопросам естественно-технических и гуманитарных наук.

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$$\eta^{[1]} = \frac{\left[\left|\omega - \left(t_0^{[1]}\right)_{11}\right|^2 - \left|\left(t_0^{[1]}\right)_{12}\right|^2\right]\eta + p\left(t_0^{[1]}\right)_{12}^{*}\left[\omega - \left(t_0^{[1]}\right)_{11}\right] + p^{*}\left(t_0^{[1]}\right)_{12}\left[\omega - \left(t_0^{[1]}\right)_{11}^{*}\right]}{\nabla}, \quad (32)$$

$$p^{[1]} = \frac{p\left[\omega - \left(t_0^{[1]}\right)_{11}\right]^2 + p^{*}\left(t_0^{[1]}\right)_{12}^{2} - 2\eta\left(t_0^{[1]}\right)_{12}\left[\omega - \left(t_0^{[1]}\right)_{11}\right]}{\nabla}, \quad (33)$$

$$\nabla = \det(M + \omega I) = \omega^2 - \omega\left[\left(t_0^{[1]}\right)_{11} + \left(t_0^{[1]}\right)_{11}^{*}\right] + \left|\left(t_0^{[1]}\right)_{11}\right|^2 + \left|\left(t_0^{[1]}\right)_{12}\right|^2. \quad (34)$$

In conclusion, application of Darboux transformation to solve explicit solutions for the (2+1)-dimensional Hirota-Maxwell-Bloch equation (1). In this work Darboux transformation was used for nonlinear differential equation and taken solutions [1-6]..

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BOUNDEDNESS OF THE FRACTIONAL MAXIMAL OPERATOR IN LOCAL MORREY-TYPE SPACES

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Spaces $L_p(\Omega)$, $0 < p < \infty$

Let $0 < p < \infty$ and Ω would be a measurable set in R^n, and a function $f : \Omega \to C$. The function $f \in L_p(\Omega)$ if f is measurable on Ω and

$$\|f\|_{L_p(\Omega)} := \left(\int_{\Omega} |f|^p dx\right)^{\frac{1}{p}} < \infty$$

Note that if meas Ω > 0, then the conditions $f \in L_p(\Omega)$ and $\|f\|_{L_p(\Omega)} < \infty$ are not equivalent. If, for example, f:= 1 on a non-measurable subset G of the set $\Omega \cap B_r$ where r>0 is such that meas($\Omega \cap B_r$)>0, f:=−1 on($\Omega \cap B_r$)\G, and f := 0 on Ω \ B_r, then f is not measurable on Ω, hence does not belong to $L_p(\Omega)$ for any 0<p<∞, but $\|f\|_{L_p(\Omega)} < \infty$.

Next, Let (X, μ) be a measurable space, for 0 < p < ∞, we consider

$WeakL_p := \left\{ f : \mu\left(\left(x \in X : |f(x)| > \lambda\right)\right) \le \left(\frac{c}{\lambda}\right)^p \right\}$, for some C > 0.

Let $f \in L_1^{loc}(R^n)$. The fractional maximal operator M_α is defined by

$M_\alpha f(x) = \sup_{t>0} |B(x,t)|^{-1+\frac{\alpha}{n}} \int_{B(x,t)} |f(y)|dy, 0 \le \alpha \le n$, where |B(x, t)| is the Lebesgue measure of the ball B(x,t). If α = 0, then M ≡ M_0 is the Hardy-Littlewood maximal operator.

$$Mf(x) = \sup_{t>0} |B(x,t)|^{-1} \int_{B(x,t)} |f(y)|dy$$

The operator M_α plays a special and very important role in real and harmonic analysis.

Also Morrey spaces $M_{p,\lambda}$ play an important role. They were introduced by C. Morrey in 1938 and defined as follows: for 1 ≤ p≤∞, 0≤λ≤n, a function f ∈M if $f \in L_p^{loc}(R^n)$ and

$$\|f\|_{M_{p,\lambda}} \equiv \|f\|_{M_{p,\lambda}(R^n)} = \sup_{x \in R^n, r>0} r^{-\lambda/p} \|f\|_{L_p(B(x,r))} < \infty$$

Definition of Morrey-type spaces:

Let $0 < p, \theta \le \infty$ and let w be a non-negative measurable function $(0,\infty)$. We denote by $LM_{p\theta,w}, GM_{p\theta,w}$, the local Morrey-type spaces, the global Morrey-type spaces respectively, the spaces of all functions $f \in L_p^{loc}(R^n)$ with finite quasinorms

$$\|f\|_{LM_{p\theta,w}} \equiv \|f\|_{LM_{p\theta,w}(R^n)} = \left\| w(r) \|f\|_{L_p(B(0,r))} \right\|_{L_\theta(0,\infty)}$$

and $\|f\|_{GM_{p\theta,w}} = \sup_{x \in R^n} \|f(x+\cdot)\|_{LM_{p\theta,w}}$ respectively.

Pay attention that $\|f\|_{LM_{p\infty,1}} = \|f\|_{GM_{p\infty,1}} = \|f\|_{Lp}$.

Let $0 < p, \theta \le \infty$. We denote by $\mathbf{\Omega_\theta}$ the set of all function w which are non-negative, measurable on $(0,\infty)$, not equivalent to 0 and such that for some $t > 0$ $\|w(r)\|_{L_\theta(t,\infty)} < \infty$.

Moreover, we denote by $\mathbf{\Omega_{p,\theta}}$ the set of all functions w which are non-negative, measurable on $(0,\infty)$, not equivalent to 0 and such that for some $t_1, t_2 > 0$

$$\|w(r)\|_{L_\theta(t_1,\infty)} < \infty, \qquad \left\| w(r) r^{n/p} \right\|_{L_\theta(0,t_2)} < \infty$$

Theorem 1.3 from [1] in case of $\alpha = 0$:

1) If $1 \le p_1 \le \infty, 0 < p_2 \le \infty, 0 \le \alpha < n, 0 < \theta_1, \theta_2 \le \infty, w_1 \in \Omega_{\theta_2}$, and $W_2 \in \Omega_2$ then the condition $t^{\alpha - n/p_1 + \min\{n-\alpha, n/p_2\}} \left\| w_2(r) \frac{r^{n/p_2}}{(t+p)^{\min\{n-\alpha, n/p_2\}}} \right\|_{L_{\theta_2}(0,\infty)} < c\|w_1\|_{L_{\theta_1}(t,\infty)}$ for

all t>0, where c>0 is independent of t, is necessary for the boundedness of M_α from $LM_{P_1\theta_1,W_1}$ to $LM_{P_2\theta_2,W_2}$.

2) If $1 < p_1 < \infty, 0 < p_2 < \infty, 0 < \theta_1 \le \theta_2 \le \infty, \theta_1 \le p_1, n(1/p_1 - 1/p_2)_+ \le \alpha < n/p_1, w_1 \in \Omega_{\theta_1}$ and $w_2 \in \Omega_{\theta_2}$,
then the condition

$$\left\| w_2(r) \frac{r^{n/p_2}}{(t+p)^{n/p_1-\alpha}} \right\|_{L_{\theta_2}(0,\infty)} < c\|w_1\|_{L_{\theta_1}(t,\infty)}$$

for all t>0, where c>0 is independent of t, is sufficient for the boundedness of M_α from $LM_{P_1\theta_1,W_1}$ to $LM_{P_2\theta_2,W_2}$ and from $GM_{P_1\theta_1,W_1}$ to $GM_{P_2\theta_2,W_2}$.

3) In particular case, if
$1 < p_1 \le p_2 < \infty, 0 < \theta_1 \le \theta_2 \le \infty, \theta_1 \le p_1, \alpha = n(1/p_1 - 1/p_2)_+, w_1 \in \Omega_{\theta_1}$ and $w_2 \in \Omega_{\theta_2}$, then the condition

$$\left\| w_2(r) \left(\frac{r}{t+r} \right)^{n/p_2} \right\|_{L_{\theta_2}(0,\infty)} < c\|w_1\|_{L_{\theta_1}(t,\infty)}$$

for all *t>0*, where *c>0* is independent of t , is necessary and sufficient for the bounedness of M_α from $LM_{P_1\theta_1,W_1}$ to $LM_{P_2\theta_2,W_2}$.

This theorem was proved in [2]. Namely Theorem 9 from this source which is proved by a certain estimation for L_p-norms of $M_\alpha f$ over balls $B(x,r)$, which allowed to simplify the problem of boundedness of M_α in local Morrey-type spaces to the problem of boundedness of the Hardy operator on the cone of non-negative non-increasing functions.

Theorem 9. If $1 \le p_1 \le \infty, 0 \le p_2 \le \infty, \alpha = 0, 0 < \theta_2 \le \infty, w_2 \in \Omega_{\theta_2}$, then the condition $\sup_{t>0} t^{-n/p_1 + \min\{n, n/p_2\}} \left\| w_2(r) \frac{r^{n/p_2}}{(t+p)^{\min\{n,n/p_2\}}} \right\|_{L_{\theta_2}(0,\infty)} < \infty$ is necessary for the boundedness of M_α from L_{p_1} to $LM_{P_2\theta_2,W_2}$.

2. If $1 < p_1 < \infty, 0 < p_2 < \infty, n(1/p_1 - 1/p_2)_+ \le \alpha \le n/p_1, w_2 \in \Omega_{\theta_2}$, $\alpha = 0$
then the condition $w_2(r) r^{-n(1/p_1 - 1/p_2)} \in L_{\theta_2}(0,\infty)$ and assume that $p_1 = p_2$, is sufficient for the boundedness of M_α from L_{p_1} to $LM_{P_2\theta_2,W_2}$.

3. In particular case, if $1 < p_1 \le p_2 < \infty, 0 < \theta_2 \le \infty, \alpha = n(1/p_1 - 1/p_2)$ and $w_2 \in \Omega_{\theta_2}$, or
$1 < p_1 \le p_2 < \infty, 0 < \theta_2 \le \infty, \alpha = n(1/p_1 - 1/p_2)$ and $w_2 \in \Omega_{\theta_2}$, or
$1 < p_1 < \infty, 0 < p_2 < \infty, n(1/p_1 - 1/p_2)_+ \le \alpha \le n/p_1, \theta_2 = \infty$ and $w_2 \in \Omega_{\theta_2}$, then condition from 2 is necessary and sufficient for the boundedness of M_α from L_{p_1} to $LM_{P_2\theta_2,W_2}$.
Later we will assume that $\alpha = 0$.

Proof. First consider the case of spaces $LM_{p_2\theta_2,w_2}$.

Sufficiency : By Corollary (Let $1 < p_1 < \infty, 0 < p_2 < \infty, n(1/p_1 - 1/p_2)_+ \le \alpha < n/p_1$.

Then there exists C>0 such that $\|M_\alpha f\|_{L_{p_2}(B(0,r)} \le Cr^{\alpha - n(1/p_1 - 1/p_2)}\|f\|_{L_{p_1}}$ for all r>0 and for all $f \in L_{p_1}$.) applied to $LM_{p_2\theta_2,w_2}$

$$\|M_\alpha f\|_{LM_{p_2\theta_2,w_2}} = \left\| w_2(r)\|M_\alpha f\|_{L_{p_2}(B(0,r))} \right\|_{L_{\theta_2}(0,\infty)} \le C\left\| w_2(r) r^{\alpha - n(1/p_1 - 1/p_2)} \right\|_{L_{\theta_2}(0,\infty)} \|f\|_{L_{p_1}}.$$

In our case:

$$\|Mf\|_{LM_{p_2\theta_2,w_2}} = \left\| w_2(r)\|M_\alpha f\|_{L_{p_2}(B(0,r))} \right\|_{L_{\theta_2}(0,\infty)} \le C\left\| w_2(r) r^{-n(1/p_1 - 1/p_2)} \right\|_{L_{\theta_2}(0,\infty)} \|f\|_{L_{p_1}}$$

Remark. Another approach to to the problem овуf finding sufficient conditions for the boundedness of M_α in local Morrey-type spaces may be based on the inequality established by Sawer «Two Weight Norm Inequalities for Certain Maximal and Integral Operators».

$\int\limits_{B(0,r)} (M_\alpha f)(x)^p dx \le C \int\limits_{R^n} |f(x)|^p (M_{\alpha p \chi_{B(0,r)}})(x) dx$, where $1 < p < \infty$ and $C > 0$ is independent of $f \in L_{p_1,v_1}$ and r, which is generalization of the weighted inequality of Fefferman and Stein for $\alpha = 0$. However if one follows the argument which worked for $\alpha = 0$, this will lead in the case $\alpha = n(1/p_1 - 1/p_2)$ only to a sufficient condition

$$\left\| w_2(r)\left(\frac{r}{t+r}\right)^\gamma \right\|_{L_{\theta_2}(0,\infty)} \le C(\gamma)\|w_1\|_{L_{\theta_1}(t,\infty)}$$

where γ any positive number less than n/p_2 .

The case of the assumption without $\theta_1 \le p_1$ is still an open question.

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КОМПАКТНОСТЬ ОПЕРАТОРА ДРОБНОГО ИНТЕГРИРОВАНИЯ ТИПА ХОЛЬМГРЕНА

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