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METHODOLOGY FOR SOLVING A GEOMETRIC CONSTRUCTION PROBLEM

Beyisbai Nazerke Berikkizi
naz2_ktl@mail.ru
1st year Master student, Mathematics Education,
L. N. Gumilyov Eurasian National University
Scientific leader: **- Abutalipova Sh.U.**

The task of the construction is that it is required to build a certain figure in advance with the indicated tools if some other figure is given and some relations between the elements of the sought figure and the elements of the given figure are indicated.

Each figure that satisfies the conditions of the problem is called a solution to this problem.

To find a solution to the problem of construction means to reduce it to a finite number of basic constructions, that is, to indicate a finite sequence of basic constructions, after which the requiring figure will be considered constructed by virtue of the accepted axioms of constructive geometry. The list of permissible basic constructions, and, consequently, the course of solving the problem, essentially depends on what specific tools are used for the constructions.

The question of choosing a particular scheme for solving a constructive problem is a purely methodological issue.

The solution of a geometric construction problem is quite benign if it is carried out, for example, according to the following scheme:

1. A finite number of cases are established that exhaust all possibilities in the choice of data.
2. For each case, an answer is given to the question whether the problem has a solution and how much it has.
3. For each case where the problem has a solution, a method is given for finding each of the possible solutions (using these geometric tools) of each of the possible solutions, or that it cannot be obtained by these means.

When solving each somewhat complex construction problem, the question arises of how to argue in order to find a way to solve the problem, in order to obtain all the solutions of the problem, in order to clarify the conditions for the possibility of solving the problem, etc.

Therefore, in solving constructive tasks in training conditions, it is recommended to use a well-known solution scheme consisting of the following four steps:

1. Analysis
2. Construction
3. Proof
4. Research

Of course, this scheme is not absolutely necessary and unchanging; it is not always convenient and expedient to strictly separate its individual stages and precisely carry them out in the indicated order. But for the most part, this scheme greatly helps in solving constructive problems. Consider each stage of this scheme.

Analysis. This is the preparatory and at the same time the most important stage of solving the task of building, since it gives the key to solving the problem. The purpose of the analysis is to establish such dependencies between the elements of the desired figure and the elements of these figures, which would allow building the desired figure. This is achieved by constructing a draft drawing depicting the data and approximately sought for in the same location as required by the condition of the problem. This drawing can be performed “by hand.” Sometimes the construction of an auxiliary drawing is accompanied by the words: “Suppose that the problem has already been solved.”

In the auxiliary drawing should be highlighted these elements and the most important elements sought. In practice, it is often more convenient to begin the construction of an auxiliary drawing not from a given frame, but from an approximate image of the figure being sought, attaching data to it so that they are in the relations indicated in the statement of the problem.

Construction. This stage of the solution is to indicate a sequence of basic constructions (or previously solved problems) that are sufficient to produce for the required figure to be constructed.

The construction is usually accompanied by the graphic design of each of its steps using the tools accepted for the construction.

Proof. The proof is intended to establish that the constructed figure really satisfies all the conditions set in the problem.

Research. When building is usually limited to find one of any solutions, and it is assumed that all the steps of construction are really executable. For a complete solution of the problem, it is necessary to clarify the following questions: 1) is it always (i.e., with any choice of data) you can build in the chosen way; 2) is it possible and how to build the required figure, if the chosen method cannot be applied; 3) how many solutions does the problem have at every possible choice of data.

Consideration of all these issues and makes the study. Thus, the study aims to establish solvability conditions and determine the number of solutions.

As an example, consider the following task: With the help of a caliper and a ruler to build angles:

a) $30°$
b) $45°$
c) $60°$

1. First, we show how to build an angle$30°$:

First we take the segment PQ, and then we find its middle S. Then we draw circles with the centers P and S of radius equal to the length of the segment PS. Denote by T the intersection point of these two circles. Then again build a circle with center T and radius equal to the length of the segment PS. Denote by T the intersection point of the corresponding circles as in Figure 1. Note that the lengths of the segments PS, SR, TS, TR, PT are equal to each other, that is, the quadrilateral PTRS is a rhombus. The segment AS is equal to half of the segment TS, and since the diagonal of the diamond is perpendicular to each other, the triangle PAS will be rectangular. Given that the triangle PST is equilateral, we get the triangle PAS has angles equal to $30°, 60°, 90°$, respectively. Therefore, $\angle APS = 30°$.

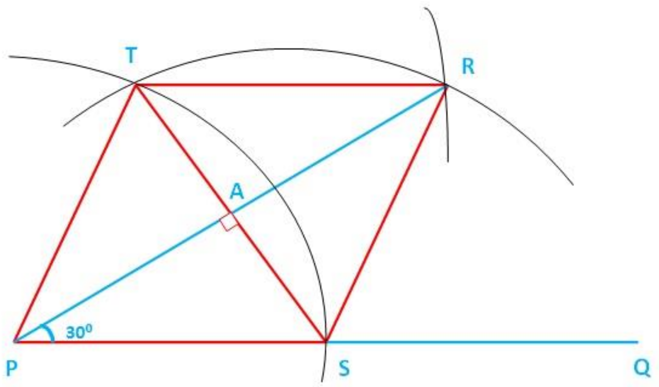

Figure 1

2. Now we will show how to make an angle equal to 45°.

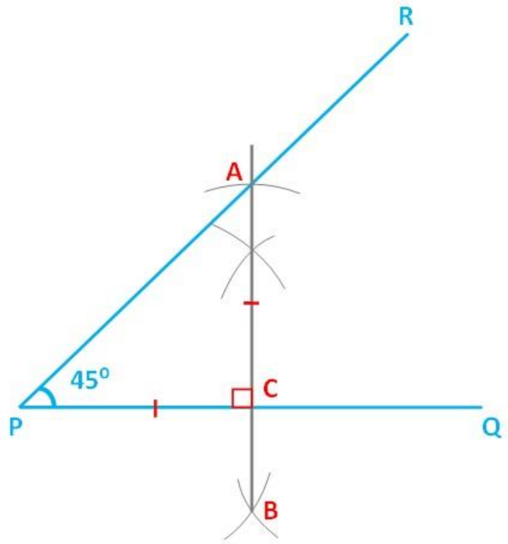

Figure 2

First we take the straight PQ and draw a straight line perpendicular to it. Let it be direct AB. Let C denote the intersection of these perpendicular lines. Next, we construct a circle with a radius equal to the length of the segment PC and center C. Connect the intersection of this circle and line AB with point P. Since the triangle APC is isosceles angles are equal at the base, i.e. ∠APC = 45°.

3. Build angle 60°

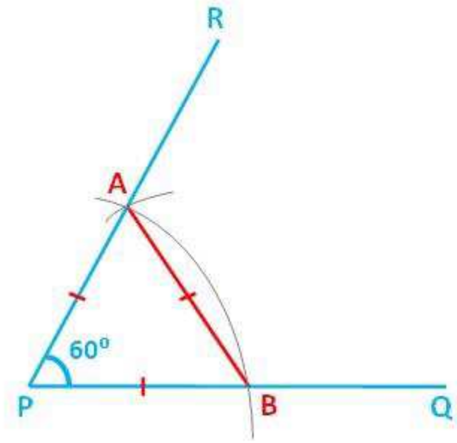

Figure 3

First we take the straight line PQ and draw a certain circle with the centrifugal P and the radius of an arbitrarily chosen length. Let B denote the point of intersection of the circle and the line PQ. Now draw a circle with center B and radius equal to the length of the radius of the previous circle. The intersection of these two circles is denoted by A. And we connect the points P and A. Then we obtain an equilateral triangle APB. From there, the angle APB is 60°.

References

1) Аргунов Б. И., Балк М.Б. Геометрические построения на плоскости: Пособие для студентов педагогических институтов. – М.: ФОРУМ: Москва, 1957. – 258 с.
2) A. Meskens, P. Tytgat, Exploring Classical Greek Construction Problems with Interactive Geometry Software, Compact Textbooks in Mathematics, DOI 10.1007/978-3-319-42863-5_7