Вращательные движения роторов вибровозбудителей 1,2, поворот корпуса относительно наголовника, а также перемещение свайного элемента вдоль оси z.

Таким образом за обобщенные координаты следует принять:

$$q_1 = \varphi_1 \tag{4}$$

За угловой поворот ротора 1 примем φ_1.

$$q_2 = \varphi_2 \tag{5}$$

За угловой поворот ротора 2 примем φ_2.

$$q_3 = \psi \tag{6}$$

За угловой поворот корпуса машины примем ψ

$$q_3 = z \tag{7}$$

Координата перемещение свайного элемента вдоль оси z.

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UDK 531

MOVEMENT OF CYLINDRICAL SOLID ON ROUGH VIBROPLANE

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The surface of the body has the shape of a radius of curvature in the vicinity. In this case, the body should be lower. A solid body rests on a rough horizontal plane at a point *D* (Fig. 1.).

Let the plane perform translational rectilinear harmonic oscillations according to the law $\xi(t) = A\sin(\omega t)$, directed at an angle β to the horizontal.

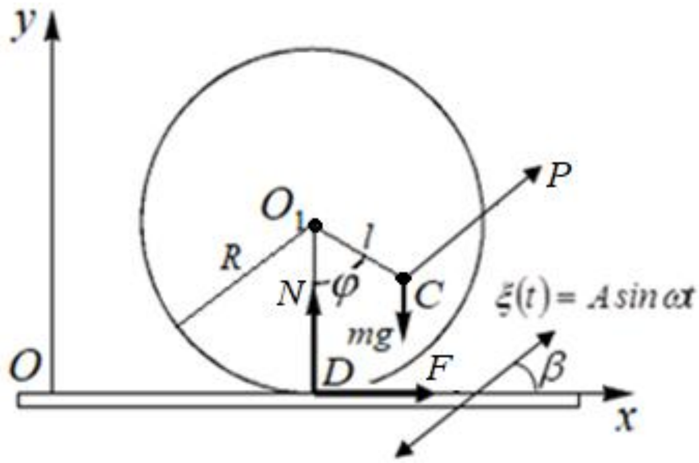


Fig. 1. A cylindrical solid on a rough surface

Here: A, ω – amplitude and frequency of oscillations, t – time. The inertial properties of the body are characterized by the mass m and the moment of inertia J_c relative to the center of mass *C*. We will set the position of the body by the coordinates of the center of mass χ_c, y_c in the *Oxy*, coordinate system associated with the rough plane and the rotation angle φ. The interaction of a solid with a plane occurs through the action of the normal reaction *N* and the sliding friction force F_{fr} (neglected rolling friction). We assume that friction obeys the law of Amonton-Coulomb:

$$F \le f \cdot N,$$

where f – coefficient of sliding friction. In this paper, we consider continuous motion, $N \ge 0$. The body is also under the influence of gravity *G*. In the relative motion to all forces it is necessary to add the portable inertia force:

$$P = m \cdot A \cdot \omega^2 \sin(\omega \cdot t) \tag{1}$$

The motion under investigation can consist of the following steps: rolling without sliding and rolling with sliding. All stages of relative motion are described by a system of differential equations arising from general theorems on the motion of the center of mass and on the change in the kinetic moment [1,2]:

$$\begin{cases} mx_C = F + P\cos\beta, \\ my_C = N - mg + P\sin\beta, \\ J_C\ddot{\varphi} = -Fy_C - Nl\sin\varphi. \end{cases} \tag{2}$$

For a more convenient recording, we consider the direction of the rotation angle clockwise to be positive. The coordinates of the center of mass *C* can be represented as:

$$x_c = x_{o_1} - l\sin\varphi,\ y_c = R - l\cos\varphi, \tag{3}$$

where $x_{o_1}, y_{o_1} = R$ – coordinates of the center of curvature $O_1; l = O_1C.$ When rolling without sliding, the instantaneous center of velocity is at the contact point D, i.e $v_D = 0,$ or

$$\dot{x}_{o_1} = \varphi R, \;\; \dot{y}_{o_1} = 0 \qquad (4)$$

Using (3), (4), we find:

$$\ddot{x}_C = \ddot{\varphi}(R - l\cos\varphi) + \dot{\varphi}^2 l \sin\varphi, \qquad (5)$$

and then from the first equation of system (2) we determine the friction force:

$$F = m[-A\cdot\omega^2 \cdot \sin(\omega t)\cdot\cos\beta + \ddot{\varphi}(R - l\cos\varphi) + \dot{\varphi}^2 l\sin\varphi]. \qquad (6)$$

Similarly, substituting the expression in the second equation

$$\ddot{y}_C = \ddot{\varphi} l\sin\varphi(R - l\cos\varphi) + \dot{\varphi}^2 l\sin\varphi, \qquad (7)$$

determine the normal reaction:

$$N = m[\ddot{\varphi} l\sin\varphi + \dot{\varphi}^2 l\cos\varphi - A\cdot\omega^2 \cdot\sin(\omega t)\cdot\sin\beta + g]. \qquad (8)$$

Finally, using (6), (8), from the third equation of system (2) we obtain the differential equation for the angle of rotation:

$$\ddot{\varphi} = \frac{K}{\frac{J_C}{m} + (R - l\cos\beta)^2 + l^2\sin^2\varphi}, \qquad (9)$$

where

$$K = A\cdot\omega^2\cdot\sin(\omega t)[R\cos\beta - l\cos(\varphi+\beta)] - gl\sin\varphi - \dot{\varphi}^2 l\sin\varphi.$$

We consider the equation of motion (9) of the rotation angle together with the initial conditions

$$t = 0: \varphi = \varphi_0, \; \dot{\varphi} = \dot{\varphi}_0. \qquad (10)$$

To solve problem (9)(10), using the method of partial discretization of nonlinear differential equations [3], we obtain

$$\ddot{\varphi}=\frac{1}{2}\sum_{i=1}^{n}(t_i+t_{i+1})\left\{\frac{K(t_i)}{\frac{J_C}{m}+(R-l\cos\beta)^2+l^2\sin^2\varphi(t_i)}\delta(t-t_i)-\right.$$
$$\left.-\frac{K(t_{i+1})}{\frac{J_C}{m}+(R-l\cos\beta)^2+l^2\sin^2\varphi(t_{i+1})}\delta(t-t_{i+1})\right\} \tag{11}$$

where $\delta(t)$ – Dirac delta function.

By integrating the right-hand side of differential equation (11) twice, we obtain the general solution in the form

$$\varphi(t)=\frac{1}{2}\sum_{i=1}^{n}(t_i+t_{i+1})\left\{\frac{K(t_i)}{\frac{J_C}{m}+(R-l\cos\beta)^2+l^2\sin^2\varphi(t_i)}(t-t_i)H(t-t_i)-\right.$$
$$=$$
$$\left.-\frac{K(t_{i+1})}{\frac{J_C}{m}+(R-l\cos\beta)^2+l^2\sin^2\varphi(t_{i+1})}(t-t_{i+1})H(t-t_{i+1})\right\}+C_1t+C_2, \tag{12}$$

where $H(t)$ – Heaviside function, C_1 and C_2 – are arbitrary integration constants.

Using the initial conditions (10), we have

$$\varphi(t)=\frac{1}{2}\sum_{i=1}^{n}(t_i+t_{i+1})\left\{\frac{K(t_i)}{\frac{J_C}{m}+(R-l\cos\beta)^2+l^2\sin^2\varphi(t_i)}(t-t_i)H(t-t_i)-\right.$$
$$\left.-\frac{K(t_{i+1})}{\frac{J_C}{m}+(R-l\cos\beta)^2+l^2\sin^2\varphi(t_{i+1})}(t-t_{i+1})H(t-t_{i+1})\right\}+\dot{\varphi}_0t+\varphi_0. \tag{13}$$

According to equation (13), the expression of the function $\varphi(t)$ at points t_i $(i=\overline{1,3})$ will be

$$\varphi(t_1)=\dot{\varphi}_0t_1+\varphi_0,$$

$$\varphi(t_2)=\frac{1}{2}(t_1+t_2)\frac{K(t_1)}{\frac{J_C}{m}+(R-l\cos\beta)^2+l^2\sin^2\varphi(t_1)}(t_2-t_1)+\dot{\varphi}_0 t_2+\varphi_0,$$

$$\varphi(t_3)=\frac{1}{2}(t_1+t_2)\frac{K(t_1)}{\frac{J_C}{m}+(R-l\cos\beta)^2+l^2\sin^2\varphi(t_1)}(t_3-t_1)+$$

$$+\frac{1}{2}(t_3-t_1)\frac{K(t_2)}{\frac{J_C}{m}+(R-l\cos\beta)^2+l^2\sin^2\varphi(t_2)}(t_3-t_2)+\dot{\varphi}_0 t_3+\varphi_0.$$

Next, using the method of mathematical induction, we construct an analytical expression of the desired function at an arbitrary point $t_i(i=\overline{1,n})$

$$\varphi(t_j)=\frac{1}{2}(t_1+t_2)\frac{K(t_1)}{\frac{J_C}{m}+(R-l\cos\beta)^2+l^2\sin^2\varphi(t_1)}(t_j-t_1)+$$
$$+\frac{1}{2}\sum_{i=2}^{j-1}(t_{i+1}-t_{i-1})\frac{K(t_i)}{\frac{J_C}{m}+(R-l\cos\beta)^2+l^2\sin^2\varphi(t_i)}(t_j-t_i)+\dot{\varphi}_0 t_j+\varphi_0. \tag{14}$$

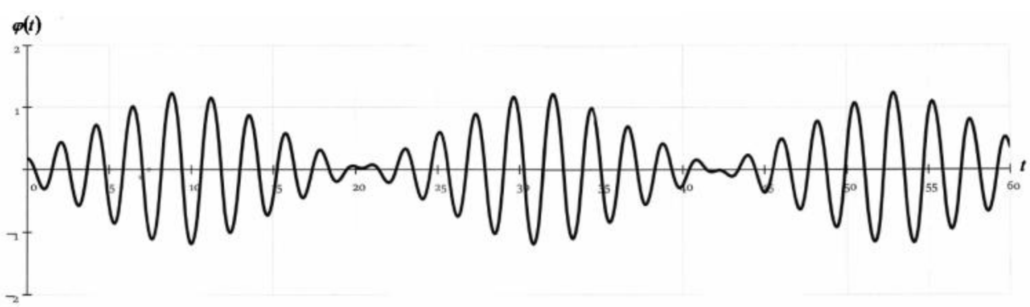
Figure 2 shows the curves of the angle of rotation $\varphi(t)$ of a cylindrical body located on a horizontal surface. The system parameters correspond to the values: $m=50kg,\ \beta=0{,}524rad,$ $R=0{,}5m,\ \varphi(0)=0{,}175rad,\ \dot{\varphi}(0)=0rad/s$.

In this case, the center of mass of the cylindrical body is offset from the geometric center by half the radius, i.e. $l=0{,}25m.$

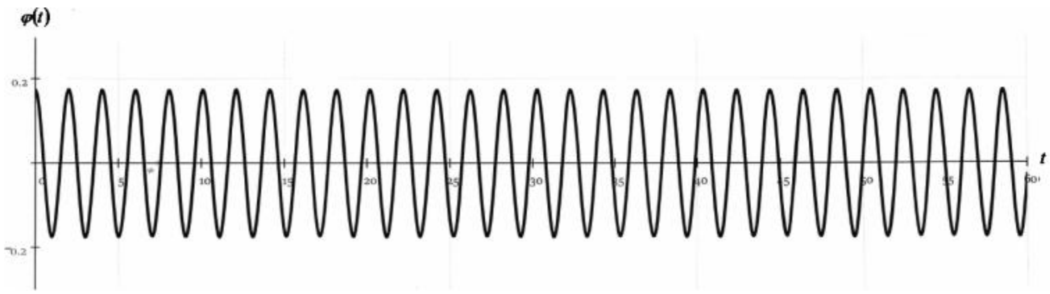
From graph 2*a* it follows that the nature of the beating occurs with the corresponding parameters $A=0{,}1m,\ \omega=3rad/s$

Figure 2*b* shows a graph of the change in $\varphi(t)$ at A=0,001m, $\omega=10rad/s.$

As follows from this graph, fluctuations in the angle of rotation obeys a harmonic law and has an established character.



a



b

Fig. 2. Curves of the angle of rotation $\varphi(t)$

It should be noted that for $l < 0{,}25m$, the period of oscillation of the rotation angle increases, and for $l > 0{,}25m$ the period decreases.

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LONGITUDINAL OSCILLATION OF THE RAIL DURING RAILWAY COMPOSITION MOVEMENT

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Annotation. The article investigates longitudinal wave processes in the rail under the influence of two mobile loads.

Key words: longitudinal oscillation, railway train, auto- coupling device.

1.Introduction

Most of the well-known works devoted to longitudinal vibrations occurring in a train solve problems mainly of unsteady longitudinal vibrations characteristic of transient conditions and special conditions of train movement and the influence of these vibrations on the realization of the traction force of the traction rolling stock. At the same time, in the process of train movement under the influence of constant or slowly changing forces, the regime of stationary longitudinal vibrations occurring in the train is established. In stationary fluctuations, the forces arising in the shock-traction devices (automatic coupler devices) are determined only by the applied external forces and are independent of the initial conditions. In this work, we study the longitudinal vibrations occurring in the composition and the influence of the longitudinal vibrations of the locomotive on these vibrations. In this case, the force arising between the electric locomotive and the train is external to the composition of the cars.

In this work, a train moving rectilinearly and uniformly with speed V (Figure 1) is presented as