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MODULI OF SMOOTHNESS AND LIOUVILLE-WEYL DERIVATIVES

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Let $L_p = L_p[0,2\pi]$ $(1 \le p < \infty)$ be a space of 2π periodic measurable functions, for which $|f|^p$ is integrable, and $L_\infty = C[0,2\pi]$ is a space of 2π periodic continuous functions with $\|f\|_\infty = \max\{|f(x),0 \le x \le 2\pi|\}$.

Let the function $f \in L_1$ have the following Fourier series

$$f(x) \approx \sigma(f) := \frac{a_0(f)}{2} + \sum_{\nu=1}^{\infty}(a_\nu(f)\cos\nu x + b_\nu(f)\sin\nu x) \tag{1}$$

The transformed Fourier series of (1) is given by

$$\sigma(f,\lambda,\beta) := \sum_{\nu=1}^{\infty}\lambda_\nu\left(a_\nu(f)\cos\left(\nu x + \frac{\pi\beta}{2}\right) + b_\nu(f)\sin\left(\nu x + \frac{\pi\beta}{2}\right)\right),$$

where $\beta \in \Re$ and $\lambda = \{\lambda_n\}$ is a sequence of positive numbers.

The function whose series coincides with $\sigma(f,\lambda,\beta)$ we will call (λ,β) derivative of the function f and denote it by $f^{(\lambda,\beta)}$. If $\lambda_n = n^r, r > 0, \beta = r$, then $f^{(\lambda,\beta)} = f^{(r)}$ -fractional derivatives in the sense of Weyl and for $\lambda_n = n^r, r > 0, \beta = r + 1$, $f^{(\lambda,\beta)} \equiv \bar{f}^{(r)}$, where $\overline{f}$ is conjugate function f.

The main task is to find estimates for the modulus of smoothness of a function with a transformed Fourier series in terms of the moduli of smoothness of the original function for different parameters $1 < p < q \le \infty$, for the average value of a bounded variational sequence.

Definition 1. A sequence $\lambda := \{\lambda_n\}_{n=1}^{\infty}$ belongs to the MVBVS class (mean value of a bounded variational sequence) if there exists $\mu \ge 2$ and the following condition is satisfied

$$\sum_{k=n}^{2n}|\lambda_k - \lambda_{k+1}| \le C\left(\frac{1}{n}\sum_{k=\frac{n}{\mu}}^{\mu n}|\lambda_k|^p\right)^p,$$

for all integers *n*, where the constant *C* does not depend on *n.*

Let $\omega_k(f,\delta)_p$ be the moduli of smoothness of the natural order $k \in N$ of the function , i.e.,

$$\omega_k(f,\delta)_p = \sup_{|h|\le\delta}\left\|\Delta_h^k(f)\right\|_p,$$

where

$$\Delta_h^k(f) = \Delta_h^{k-1}(\Delta_h(f(x)) \text{ and } \Delta_h(f) = f(x+h) - f(x)$$

The history of the Ulyanov-type inequality begins with the results of Hardy and Littlewood. In 1928 Hardy and Littlewood got the following result

$$H_p^\alpha = \left\{f \in L_p[0,2\pi] : \|f(x+h) - f(x)\|_p = o(h^\alpha)\right\} = Lip(\alpha, p)$$
$$\Rightarrow Lip(\alpha, p) \subseteq Lip(\alpha-\theta, q)$$

$$H_p^\alpha \subseteq H_p^{\alpha-\theta},$$

where $1 \le p < q < \infty, \quad \theta = 1/p - 1/q, \quad \theta < \alpha \le 1.$
In 1968 Ulyanov proved that

$$\omega_\alpha(f,\delta)_p \le C\left(\int_0^\delta \left(t^{-\theta}\omega(f,t)_p\right)^{q_1}\frac{dt}{t}\right)^{\frac{1}{q_1}}$$

where

$$1 \le p < q \le \infty, \ \theta = \frac{1}{p} - \frac{1}{q}, \quad q_1 = \begin{cases} q, q < \infty, \\ 1, q = \infty. \end{cases}$$

Here

$$\omega_\alpha(f,\delta)_p = \omega_1(f,\delta)_p.$$

DeVore, Riemenschneider, Sharpley in 1979 proved the following inequality

$$\omega_k(f,\delta)_p \le C\left(\int_0^\delta \left(t^{-\theta}\omega_k(f,t)_p\right)\frac{dt}{t}\right)$$

In 2005, Tikhonov and Ditzian obtained the following inequality

$$\omega_k(f^{(r)},\delta)_p \le C\left(\int_0^\delta \left(t^{-r-\theta}\omega_{k+r}(f,t)_p\right)^{q_1}\frac{dt}{t}\right)^{\frac{1}{q_1}}$$

where $r \in 0 < p < q \le \infty, \quad k, r \in \mathrm{N}.$

Ulyanov-type inequality for moduli of smoothness of fractional order was considered in the works of S. Tikhonov, B. Simonov, A. Jumabayeva and other authors.

Let $\omega_\alpha(f,\delta)_p$ moduli of smoothness of fractional order $\alpha, \alpha > 0$, of the function f,

$$\omega_\alpha(f,\delta)_p = \sup_{|h|\le\delta}\left\|\Delta_h^\alpha(f)\right\|_p,$$

where

$$\Delta_h^{\alpha}(f)=\sum_{\nu=0}^{\infty}(-1)^{\nu}\binom{\alpha}{\nu}f(x+(\alpha-\nu))$$

difference of the fractional order $\alpha, \alpha>0$, of the function $f\in L_p$ at a point x with step h.

We have obtained an Ulyanov-type inequality for moduli of smoothness of fractional order with MVBVS sequences.

Theorem 1. *Let* $f\in L_p$, $1<p<q<\infty, \theta=1/p-1/q$, $\rho>0$ *and* $\lambda=\{\lambda_n\}_{n=1}^{\infty}\in MVBVS$. *Then for any* $\alpha>0$,

$$\omega_{\alpha}\left(\varphi,\frac{1}{2^n}\right)_q\le C\left(\sum_{k=\frac{2^n}{\mu}}^{\infty}|\lambda_k|^q k^{\theta q-1}\omega_{\alpha+\theta+\rho}\left(f,\frac{1}{k}\right)_p\right)^{1/q}.$$

$$+2^{n(\theta+\rho)}\omega_{\alpha+\theta+\rho}\left(f,\frac{1}{2^n}\right)_p\max_{1\le l\le n}\sum_{k=\frac{2^l}{\mu}}^{\mu 2^l}\frac{|\lambda_k|}{k^{\rho+1}}$$

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ON THE SOLUTION OF FRACTIONAL q-DIFFERENCE EQUATION

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First we recall some elements of q-calculus, for more information see e.g. the books [1] and [2]. Throughout this paper, we assume that $0<q<1$ and $0\le a<b<\infty$.

Let $\alpha\in\mathbb{R}$. Then a q-real number $[\alpha]_q$ is defined by

$$[\alpha]_q:=\frac{1-q^{\alpha}}{1-q},$$

where $\lim_{q\to 1}\frac{1-q^{\alpha}}{1-q}=\alpha$.