ҚАЗАҚСТАН РЕСПУБЛИКАСЫ ҒЫЛЫМ ЖӘНЕ ЖОҒАРЫ БІЛІМ МИНИСТРЛІГІ

«Л.Н. ГУМИЛЕВ АТЫНДАҒЫ ЕУРАЗИЯ ҰЛТТЫҚ УНИВЕРСИТЕТІ» КЕАҚ

Студенттер мен жас ғалымдардың
«ǴYLYM JÁNE BILIM - 2023»
XVIII Халықаралық ғылыми конференциясының
БАЯНДАМАЛАР ЖИНАҒЫ

СБОРНИК МАТЕРИАЛОВ
XVIII Международной научной конференции
студентов и молодых ученых
«ǴYLYM JÁNE BILIM - 2023»

PROCEEDINGS
of the XVIII International Scientific Conference
for students and young scholars
«ǴYLYM JÁNE BILIM - 2023»

2023
Астана

УДК 001+37
ББК 72+74
G99

«ǴYLYM JÁNE BILIM – 2023» студенттер мен жас ғалымдардың XVIII Халықаралық ғылыми конференциясы = XVIII Международная научная конференция студентов и молодых ученых «ǴYLYM JÁNE BILIM – 2023» = The XVIII International Scientific Conference for students and young scholars «ǴYLYM JÁNE BILIM – 2023». – Астана: – 6865 б. - қазақша, орысша, ағылшынша.

ISBN 978-601-337-871-8

Жинаққа студенттердің, магистранттардың, докторанттардың және жас ғалымдардың жаратылыстану-техникалық және гуманитарлық ғылымдардың өзекті мәселелері бойынша баяндамалары енгізілген.

The proceedings are the papers of students, undergraduates, doctoral students and young researchers on topical issues of natural and technical sciences and humanities.

В сборник вошли доклады студентов, магистрантов, докторантов и молодых ученых по актуальным вопросам естественно-технических и гуманитарных наук.

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So, the completeness of the eigenfunctions of the perturbation problem can be replaced with easier problem completeness of the eigenfunctions of Laplace operators. Since they similar operators their spectrum coincides. By changing the operator $Kf(x) = \varphi(x) \iint_\Omega f(\tau)\overline{g(\tau)}d\tau$ (choosing the functions φ and g) we will get completely new type of perturbation problem. In practice, if we confronted with such are perturbation problems which are defined on arbitrary domain $\Omega \subset R^n$, we can always (by calculating the functions φ and g) we bring the question of the completeness of the eigenfunctions with the completeness of the correct Laplace operators.

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UDC 517

CONVERGENCE OF SERIES WITH GENERALIZED MOTONE COEFFICIENTS

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Definition 1 Suppose, $\{a_{i,j}\}_{i=1,j=1}^{\infty}$ - consider a sequence of personal sums of a series along with a given sequence, and a numerical sequence

$$\{S_{k,l}\}_{k=1,j=1}^{\infty}.$$

Each of its elements forms the sum of some members of the original sequence

$$\{S_{k,l}\} = \sum_{i=1,j=1}^{i=k,j=l} a_{i,j}.$$

As a rule, the following symbol is used to designate series:

$$\sum_{i=1,j=1}^{\infty} a_{i,j}.$$

This shows the initial sequence of elements of the series, as well as the summation rule. According to this, it is said about the convergence of double numerical series:

• A double number series converges if its sequence of personal sums converges, i.e. the series $\{a_{i,j}\}$ converges and the sum is equal to s if

$$\forall \varepsilon > 0 : \exists m_o, n_o : m > m_o, n > n_o : \left||s_{mn} - s|\right| < \varepsilon.$$

It is also possible to write down the condition for convergence to the sum of s dual series as follows:

$$\lim_{n,m \to \infty} s_{mn} = s$$

- A double number series does not converge if its sequence of individual sums does not converge.
- A double number series is absolutely convergent if the series consisting of the modules of its members converges.

If a number series is summed, then the limit of S is called the sum of its sequence of separate sums:

$$S = \sum_{i=1,j=1}^{\infty} a_{i,j}$$

Properties:

Suppose summing in $\{a_{m,n}\}_{m,n=1}^{\infty}$ all the rows a double series with the sum S, and also to sum up the series consisting of their sums, i.e.,

$$s_{i^*} = \lim_{n\to\infty} \sum_{j=1}^{n} a_{i,j}$$

and

$$s' = \lim_{m\to\infty} \sum_{i=1}^{m} s_{i^*}$$

is the limit of the equation. Then $s = s'$. Likewise, if

$$s_{*_j} = \lim_{m\to\infty} \sum_{i=1}^{m} a_{i,j}$$

and

$$s'' = \lim_{n\to\infty} \sum_{i=1}^{n} s_{*_j}$$

if there are limits, then $s = s''$

- Markov's theorem: Suppose that the double series $\{a_{i,j}\}$ is summed in all the series
- $s_{m^*} = \sum_{n=1}^{\infty} a_{mn}$ and in all columns $s_{*_n} = \sum_{m=1}^{\infty} a_{mn}$.

Denote the sum of the series $s' = \sum_{m=1}^{\infty} a_{m^*}$.

where:

- the k-th $r_m^{(k)} = \sum_{n=k+1}^{\infty} a_{mn}$ residual series R_k forms series that converges with the $\sum_{m=1}^{\infty} r_m^{(k)}$ is sum.
- It is necessary and sufficient to have the limit $\lim R_k = R$, so that its series consisting of the sum of the columns converges $s'' = \sum_{n=1}^{\infty} s_{*_n}$.

to fulfill the equality $s = s'$ it is necessary and sufficient to have $R = 0$.

Definition 2. A sequence $\curlywedge = \{\curlywedge_{n_1,n_2}\}_{n_1,n_2}$ is said to be general monotone, written $\curlywedge \in GMS^2$ if the relations

$$\sum_{k_1=n_1}^{2n_1} \left|\curlywedge_{k_1,n_2} - \curlywedge_{k_1+1,n_2}\right| \le C\left|\curlywedge_{n_1,n_2}\right|,$$

$$\sum_{k_2=n_2}^{2n_2} \left|\curlywedge_{n_1,k_2} - \curlywedge_{n_1,k_2+1}\right| \le C\left|\curlywedge_{n_1,n_2}\right|,$$

$$\sum_{k_1=n_1}^{2n_1}\sum_{k_2=n_2}^{2n_2}\left|\curlywedge_{k_1,k_2}- \curlywedge_{k_1+1,k_2}-\curlywedge_{k_1,k_2+1}+\curlywedge_{k_1+1,k_2+1}\right| \le C|\curlywedge_{n_1,n_2}|,$$

hold for all integers n_1 and n_2, where the constant C is independent of n_1 and n_2.

If $\{\curlywedge_{n_1,n_2}\} \in GMS^2$, then

$$\left|\curlywedge_{k_1,k_2}\right| \le C|\curlywedge_{n_1,n_2}| \quad \text{for} \quad n_1 \le k_1 \le 2n_1, n_2 \le k_2 \le 2n_2.$$

This implies that the condition

$$\sum_{k_1=n_1}^{2n_1}\sum_{k_2=n_2}^{2n_2}\left|\curlywedge_{k_1,k_2}- \curlywedge_{k_1+1,k_2}-\curlywedge_{k_1,k_2+1}+\curlywedge_{k_1+1,k_2+1}\right| \le C(|\curlywedge_{n_1,n_2}|+|\curlywedge_{2n_1,2n_2}|)$$

is equivalent to the condition

$$\sum_{k_1=n_1}^{2n_1}\sum_{k_2=n_2}^{2n_2}\left|\curlywedge_{k_1,k_2}- \curlywedge_{k_1+1,k_2}-\curlywedge_{k_1,k_2+1}+\curlywedge_{k_1+1,k_2+1}\right| \le C|\curlywedge_{n_1,n_2}|.$$

Lemma 1 The sequence $\{\curlywedge_{n_1,n_2}\} \in GMS^2$ satisfies the following properties: there exists C > 0 such that

$n_1 \le k_1 \le 2n_1, n_2 \le k_2 \le 2n_2 \colon \left|\curlywedge_{k_1,k_2}\right| \le C|\curlywedge_{n_1,n_2}|,$

$n_1 < N_1 : \sum_{k_1=n_1}^{N_1}\left|\curlywedge_{k_1,n_2}-\curlywedge_{k_1+1,n_2}\right| \le C\left(\left|\curlywedge_{n_1,n_2}\right| + \sum_{k_1=n_1+1}^{N_1}\frac{|\curlywedge_{k_1,n_2}|}{k_1}\right),$

$n_2 < N_2 : \sum_{k_2=n_2}^{N_2}\left|\curlywedge_{n_1,k_2}-\curlywedge_{n_1,k_2+1}\right| \le C\left(\left|\curlywedge_{n_1,n_2}\right| + \sum_{k_2=n_2+1}^{N_2}\frac{|\curlywedge_{n_1,k_2}|}{k_2}\right),$

$n_1 < N_1, n_2 < N_2 : \sum_{k_1=n_1}^{N_1}\sum_{k_2=n_2}^{N_2}\left|\curlywedge_{k_1,k_2}-\curlywedge_{k_1+1,k_2}-\curlywedge_{k_1,k_2+1}+\curlywedge_{k_1+1,k_2+1}\right| \le C\left(\left|\curlywedge_{n_1,n_2}\right| + \sum_{k_1=n_1+1}^{N_1}\frac{|\curlywedge_{k_1,n_2}|}{k_1} + \sum_{k_2=n_2+1}^{N_2}\frac{|\curlywedge_{n_1,k_2}|}{k_2} + \sum_{k_1=n_1+1}^{N_1}\sum_{k_2=n_2+1}^{N_2}\frac{|\curlywedge_{k_1,k_2}|}{k_1k_2}\right).$

We consider the following cosine series

$$a_{oo} + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_{nm}\cos(nx)\cos(mx). \tag{1}$$

Theorem 1.

a) Let $\{a_{nm}\} \in GM^2$ and $\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{a^2{}_{nm}}{nm} < \infty$ then series (1) converges.

b) For any decreasing sequence $\{\gamma_{nm}\}$ satisfying the condition $\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\gamma^2{}_{nm}}{nm} = \infty$ there exists $\{a_{nm}\} \in GM$ such that $(a_{nm}) \le C\gamma_{nm}$ and series (1) diverge almost everywhere.

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UDC 517.946

A PRIORI ESTIMATES FOR THE SOLUTION OF THE DEGENERATE THIRD ORDER DIFFERENTIAL EQUATION

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Sufficient conditions for existence of the solution and coercive estimation for the solution of a third-order linear differential equation with a variable high coefficient in the following form

$$l_\lambda y = -\sqrt{5+x^2}\left(\sqrt{5+x^2}\,y'\right)'' + [q(x)+ir(x)+\lambda]y = f(x), \tag{1}$$

are obtained in this work, here $f \in L_p \equiv L_p(R),\ 1 \le p < +\infty,\ \ \lambda \ge 0$.

In the works of R.D.Akhmetkaliyeva, L.-E.Persson, K.N.Ospanov, P.Wall [1], which was published in 2015 various cases of the third-order linear differential equation with variable high coefficient are studied in details, and the results are presented. The fifth-order linear differential equation with a variable high coefficient was considered in the researching work of A.E.Muslim [2].

The general form

$$(L+\lambda E)y = -m_1(x)\left(m_2(x)\left(m_3 y'\right)'\right)' + [q(x)+ir(x)+\lambda]y = f(x), \quad f \in L_p,\ \lambda \ge 1$$

of the third order differential equations was considered in the dissertation work of R.D.Akhmetkaliyeva «Coercive estimate of the solution of the singular differential equation and its applications» [3].

Definition. A function $y(x) \in L_p(R)$ is called a solution of the differential equation in the following form

$$L_\lambda y := -m(x)\left(m(x)y'\right)'' + [q(x)+ir(x)+\lambda]y = f(x),$$

if there exists a sequence $\{y_n\}_{n=1}^{\infty}$ of three times continuously differentiable functions with compact support , and $\|y_n - y\|_p \to 0$, $\|L_\lambda y_n - f\|_p \to 0$, ($n \to \infty$) are fulfilled.

A symbol $C^{(k)}(R)$ is signified the set of all k times continuously differentiable functions $\varphi(x)$. $\sum_{j=0}^{k}\sup_{x\in R}\left|\varphi^{(j)}(x)\right| \prec \infty$ holds for a functions $\varphi(x)$. Let $W_\lambda(x) := \dfrac{|q(x)+\lambda+ir(x)|}{5+x^2}$.

Our main results in this work read:

Theorem. Suppose that $q(x)$ and $r(x)$ are continuous functions on R and satisfies the following conditions:

$$\frac{q(x)}{25+10x^2+x^4} \ge 1,\ r(x) \ge 1,$$