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SIMPLIFIED METHOD OF CALCULATING A BEAM ON A TWO-PARAMETER ELASTIC FOUNDATION

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ABSTRACT: When calculating beams lying on a solid elastic foundation, the simplest foundation model proposed by Winkler-Zimmerman is often used. This hypothesis has been repeatedly subjected to well-founded criticism, because the deformation of the base occurs only in the area of the load applied to it. In order to clarify the Winkler-Zimmerman hypothesis, many authors have proposed other models that allow mitigating the disadvantage of this model to varying degrees. The paper deals with the method of obtaining an analytical solution to the problem of beam bending on a two-parameter elastic foundation. A new version of the elastic foundation model is proposed. The account of the elastic foundation is produced by means of parameter of the flexural rigidity. The resolving differential equation of bending of beams resting on a two-parameter elastic foundation is obtained. An analytical solution for this equation is derived, and solutions for various boundary conditions of beams resting on a two-parameter elastic foundation are discussed. The accuracy of the proposed method is verified through examples, and they showed an excellent agreement with the Winkler and Vlasov models.

Keywords: Beam, A Two-Parameter Elastic Foundation, Present Model, Winkler Model, Vlasov Model

1. INTRODUCTION

When calculating beams lying on a solid elastic foundation, the simplest foundation model proposed by Winkler-Zimmerman [1-3] is usually used, according to which the reaction of the foundation at the considered point is proportional to the displacement of the foundation edge at the same point. In other words, the base is presented in the form of linearly elastic springs that are not connected to each other. Therefore, under the action of a concentrated force, only the spring over which this force is located turns out to be deformable. This hypothesis has been repeatedly subjected to well-founded criticism due to the uncertainty of the method of finding the coefficient of elastic resistance for soil foundations, and because it does not take into account the inclusion in the work of foundation the areas that are adjacent to the point of application of the concentrated force.

P.L. Pasternak developed an elastic foundation model that is characterized by two bedding values [4]. Such a model makes it possible to partially take into account deformations of the base not only under a concentrated force but also in neighboring sections. This is achieved due to the fact that the magnitude of the base reaction is considered to depend not only on the magnitude of the beam settlement at the point under consideration but also on the curvature of the beam deflection line at the same point.

In addition to these models, other options were proposed that were based on solutions to the problems of the theory of elasticity for an elastic half-plane [5] and an elastic half-space [6].

Vlasov and Leontiev [7], adopted the simplified–continuum approach based on the variational principle and derived a two-parameter foundation model. In the two-parameter Vlasov model, the foundation was considered as an elastic layer and, by introducing an arbitrary parameter γ, the vertical distribution of deformation in the soil was characterized. Jones and Xenophontos [8] established a relationship between the γ parameter and the displacement characteristics, but did not offer any method for calculating its actual value. Vallabhan and Das [9] determined the parameter γ as a function of the beam and foundation soil characteristics using an iterative procedure and named this model modified Vlasov foundation [10].

A number of scientists [11-13] studied beams on two-parameter foundation by the Galerkin method, the power series method and the method of differential operator series. Results of other approaches, such as iterative methods [14], discrete singular convolution method [15], and finite element method [16-18], can be found in the research.

In this paper, a new simplified method for calculating a beam on a two-parameter elastic foundation is proposed. This model is more accurate than the well-known model of Winkler, and at the same time it is simpler than the model of Vlasov. According to the proposed method, an elastic foundation is considered as a single-layer model, the properties of which are described by two elastic characteristics.

2. RESEARCH SIGNIFICANCE

At present, all presented models include an elastic foundation of the Winkler, Vlasov and Pasternak types. But with the development of the scale of construction, the interaction of soil and foundation beams, plates and other elements requires deeper theoretical studies. The development of more realistic foundation models and simplified methods is very important for the safe and economical design of such type of structure. Therefore, it is an important task to develop mathematical models and methods for assessing the stress-strain state of beams lying on an elastic foundation, taking into account their geometric and physical characteristics. An important difference of the method proposed by the authors from other models is that it allows taking into account the elastic foundation using the parameter of the flexural rigidity and the resolving differential equation of a beam on a two-parameter elastic foundation are obtained in a simplified form.

3. THEORETICAL ANALYSIS OF A BEAM ON A TWO-PARAMETER ELASTIC FOUNDATION

In the Cartesian coordinate system $\left(-\frac{l}{2} \le x_1 \le \frac{l}{2}, -\frac{h_0}{2} \le x_3 \le \frac{h_0}{2}\right)$, a beam on an elastic foundation is considered, where l, h_0 are geometrical dimensions (length, height) of the beam, E is the elasticity modulus of the beam material, h is the elastic foundation thickness, $\overline{E}$ is the elasticity modulus of the foundation material.

To obtain the calculation theory are used the following hypothesis:

1) Linear deformation ε_3 in cross direction and shear deformation γ_{13} are absent

$$\varepsilon_3 = \frac{\partial U_3^0}{\partial x_3} = 0, \quad \gamma_{13} = \frac{\partial U_1^0}{\partial x_3} + \frac{\partial U_3^0}{\partial x_1} = 0. \tag{1}$$

2) Normal stress σ_3 in cross direction is absent

$$\sigma_3 = 0. \tag{2}$$

The components of the beam displacements are obtained by integrating Eq. (1)

$$U_3^0(x_1, x_3) = W_0(x_1),$$
$$U_1^0(x_1, x_3) = -h_0 \cdot \varphi_0(z) \frac{dW_0(x_1)}{dx_1}, \tag{3}$$
$$\varphi_0(z) = -C_0 + z, \quad z = \frac{x_3}{h_0},$$

where $\varphi_0(z)$ is the function of tangential displacements distribution (U_1^0), $W_0(x_1)$ is the function of the beam deflection, C_0 is an arbitrary constant, z is a dimensionless cross coordinate.

The components of the beam displacements taking into account the Hooke law $\sigma_1^0 = E \cdot \varepsilon_1 = E \frac{\partial U_1^0}{\partial x_1}$ and equilibrium equations $\frac{\partial \sigma_1^0}{\partial x_1} + \frac{\partial \tau_{13}^0}{\partial x_3} = 0, \ \frac{\partial \tau_{13}^0}{\partial x_1} + \frac{\partial \sigma_3^0}{\partial x_3} = 0$ have the form

$$\sigma_1^0 = -E \cdot h_0 \varphi_0(z) \frac{d^2 W_0(x_1)}{dx_1^2},$$
$$\tau_{13}^0 = E h_0^2 \psi_0(z) \frac{d^3 W_0(x_1)}{dx_1^3},$$
$$\sigma_3^0 = E h_0^3 \delta_0(z) \frac{d^4 W_0(x_1)}{dx_1^4}, \tag{4}$$
$$\psi_0(z) = A_0 - C_0 z + \frac{z^2}{2},$$
$$\delta_0(z) = B_0 - A_0 \cdot z + C_0 \frac{z^2}{2} - \frac{z^3}{6},$$

where σ_1^0, σ_3^0 is normal stress in the direction of the coordinate axes x_1, x_3, τ_{13}^0 is tangential stress, $\psi_0(z)$ is the function of cross tangential stresses distribution, $\delta_0(z)$ is the function of cross normal stresses distribution, A_0, B_0, C_0 are arbitrary constants.

The components of the elastic foundation displacements and stresses are determined as follows [19]

$$U_1(x_1, x_3) = -h \cdot \varphi(z_0) \frac{dW(x_1)}{dx_1},$$
$$U_3(x_1, x_3) = f(z_0) \cdot W(x_1),$$

$$\varphi(z_0)=\frac{(1+\nu)}{(1-\nu)}\delta'(z_0), \qquad (5)$$
$$f(z_0)=\delta''(z_0)-\frac{2}{(1-\nu)}k^2\delta(z_0),$$
$$\delta(z_0)=(C_1+C_2z_0)e^{-kz_0},\ z_0=\frac{x_3}{h},\ k^2=\bar{k}^2\cdot h^2,$$

where ν is the Poissons ratio of the elastic foundation material, z_0 is the dimensionless cross coordinate, $\varphi(z_0), f(z_0)$ are the functions of the displacements distribution (U_1,U_3), k is the deformed state parameter that depends on the boundary conditions.

$$\sigma_1=-\frac{\bar{E}h}{12}\cdot\psi'(z_0)\frac{d^2W(x_1)}{dx_1^2},$$
$$\tau_{13}=\frac{\bar{E}h^2}{12}\cdot\psi(z_0)\frac{d^3W(x_1)}{dx_1^3},$$
$$\sigma_3=\frac{\bar{E}h^3}{12}\cdot\alpha(z_0)\frac{d^4W(x_1)}{dx_1^4}, \qquad (6)$$
$$\psi(z_0)=12\left[\delta(z_0)+\frac{\nu}{k^2}\delta''(z_0)\right],$$
$$\alpha(z_0)=\frac{12}{k^4}\left[\delta'''(z_0)-(2+\nu)k^2\delta'(z_0)\right],$$

where $\psi'(z_0)$, $\psi(z_0)$, $\alpha(z_0)$ are the functions of stresses distribution $\sigma_1,\tau_{13},\sigma_3$.

To determine the arbitrary constants are used the following conditions:

a) contact conditions $\left(z=-\frac{1}{2}\right)$

$$1)\ W_0(x_1)=f(0)\cdot W(x_1),$$
$$2)\ -h_0\varphi_0\left(-\frac{1}{2}\right)\frac{dW_0(x_1)}{dx_1}=h\cdot\varphi(0)\cdot\frac{dW(x_1)}{dx_1}, \qquad (7)$$
$$3)\ Eh_0^2\psi_0\left(-\frac{1}{2}\right)\frac{d^3W_0(x_1)}{dx_1^3}=\frac{\bar{E}h^2\psi(0)}{12}\cdot\frac{d^3W(x_1)}{dx_1^3},$$
$$4)\ Eh_0^3\delta_0\left(-\frac{1}{2}\right)\frac{d^4W_0(x_1)}{dx_1^4}=-\frac{\bar{E}h^3\alpha(0)}{12}\cdot\frac{d^4W(x_1)}{dx_1^4}.$$

b) boundary conditions $\left(z=\frac{1}{2}\right)$

$$1)\ \tau_{13}^0=0:\ \psi_0\left(\frac{1}{2}\right)=0,\quad A_0-\frac{C_0}{2}+\frac{1}{8}=0,$$
$$2)\ \sigma_3^0=q:\ Eh_0^3\delta_0\left(\frac{1}{2}\right)\frac{d^4W_0(x_1)}{dx_1^4}=q(x_1), \qquad (8)$$
$$\delta_0\left(\frac{1}{2}\right)=B_0-\frac{A_0}{2}+\frac{C_0}{8}-\frac{1}{48},$$

where $q(x_1)$ is the intensity of the distributed external load.

From the equality of the beam deflection and the elastic foundation functions $W_0(x_1)=W(x_1)$ contact conditions (7) are written down in the form

$$1)\ 1=f(0),$$
$$2)\ C_0+\frac{1}{2}=\frac{h}{h_0}\varphi(0),$$
$$3)\ A_0+\frac{C_0}{2}+\frac{1}{8}=\frac{1}{12}\frac{\bar{E}h^2}{Eh_0^2}\beta_0,\ \beta_0=\psi(0), \qquad (9)$$
$$4)\ B_0+\frac{A_0}{2}+\frac{C_0}{8}+\frac{1}{48}=-\frac{\alpha_0}{12}\frac{\bar{E}h^3}{Eh_0^3},\ \alpha_0=\alpha(0).$$

From conditions (8) and (9) are determined the arbitrary constants

$$C_0=\frac{1}{12}\frac{\bar{E}h^2}{Eh_0^2}\beta_0,\ A_0=-\frac{1}{8}+\frac{1}{24}\frac{\bar{E}h^2}{Eh_0^2}\beta_0,$$
$$B_0=\frac{1}{24}-\frac{1}{32}\frac{\bar{E}h^2}{Eh_0^2}\beta_0-\frac{1}{12}\frac{\bar{E}h^3}{Eh_0^3}\alpha_0, \qquad (10)$$

where β_0,α_0 are the functions of distribution of the elastic foundation stresses (τ_{13},σ_3) at the contact point.

Taking into account functions of displacements (5) from the first two conditions (9), are obtained the following equations

$$2k\alpha_0-(1-\nu)\beta_0=-\frac{2n}{k},$$
$$(1-\nu)k\alpha_0-\beta_0\cdot m=n\frac{h_0}{h}, \qquad (11)$$
$$n=\frac{6(1-\nu^2)}{k},\ m=2+\frac{(1-\nu^2)}{k}\frac{\bar{E}h}{Eh_0}.$$

The parameters of stress distribution functions are found from Eq. (11)

$$\alpha_0=-\frac{6(1-\nu^2)}{k^3}\cdot P_0,\ \beta_0=-\frac{12(1-\nu^2)}{k^2}\cdot P_1,$$
$$P_0=\frac{2\cdot m+\frac{h_0}{h}k(1-\nu)}{2m-(1-\nu)^2},\ P_1=\frac{k\frac{h_0}{h}+(1-\nu)}{2m-(1-\nu)^2}. \qquad (12)$$

The resolving equation for the beam on a two-parameter elastic foundation is obtained from the second condition (8) taking into account Eqs. (10) and (12)

$$\gamma \cdot \frac{d^4 W_0(x_1)}{dx_1^4} = \frac{q(x_1)}{EJ}, \quad J = \frac{h_0^3}{12},$$
$$\gamma = 1 + \frac{6(1-\nu^2)\cdot P_1}{k^2}\frac{\bar{E}h^2}{Eh_0^2} + \frac{6(1-\nu^2)\cdot P_0}{k^3}\frac{\bar{E}h^3}{Eh_0^3}, \quad (13)$$

where γ is the parameter accounting the elastic foundation, J is the axial moment of inertia of the beam cross section.

Taking into account the equations of transitional processes

$$\frac{d^2 W_0(x_1)}{dx_1^2} = -\bar{k}^2 W_0(x_1), \quad \bar{k}^2 = \frac{k^2}{h^2},$$

Eq. (13) are written in the stationary form

$$\frac{d^4 W_0(x_1)}{dx_1^4} - 2r^2 \frac{d^2 W_0(x_1)}{dx_1^2} + S^4 W_0(x_1) = \frac{q(x_1)}{EJ},$$
$$2r^2 = \frac{6(1-\nu^2)\cdot P_1 l^2}{h^2}\frac{\bar{E}h^2}{Eh_0^2}, \quad (14)$$
$$S^4 = \frac{6(1-\nu^2)\cdot kl^4 \cdot P_0}{h^4}\frac{\bar{E}h^3}{Eh_0^3}.$$

Based on Eq. (4), the beam external forces are presented as follows

$$M = -EJ\cdot g_1 \cdot \frac{d^2 W_0(x_1)}{dx_1^2}, \; Q = EJ \cdot g_2 \cdot \frac{d^3 W_0(x_1)}{dx_1^3},$$
$$g_1 = 12\int_{-1/2}^{1/2} \varphi_0(z)\cdot z dz, \quad g_2 = 12\int_{-1/2}^{1/2}\psi_0(z) dz, \quad (15)$$

where M is a bending moment, Q is a shear force, g_1, g_2 are parameters of the beam flexural rigidity.

The parameters of the beam flexural rigidity taking into account Eqs. (3), (4), (10) and (12), are written down in the form:

$$g_1 = 12\int_{-1/2}^{1/2}\left(-C_0 \cdot z + z^2\right)dz = 1,$$
$$g_2 = 12\int_{-1/2}^{1/2}\left(A_0 - C_0\cdot z + \frac{z^2}{2}\right)dz = -g_0, \quad (16)$$
$$g_0 = 1 + \frac{6(1-\nu^2)P_1}{k^2}\frac{\bar{E}h^2}{Eh_0^2},$$

where g_0 is the parameter of the beam shear force.

Arbitrary constants (10) taking into account the parameters of stresses distribution functions (12) are taken the following values

$$C_0 = -\frac{(1-\nu^2)P_1}{k^2}\frac{\bar{E}h^2}{Eh_0^2},$$
$$A_0 = -\frac{1}{8} - \frac{(1-\nu^2)P_1}{2k^2}\frac{\bar{E}h^2}{Eh_0^2}, \quad (17)$$
$$B_0 = \frac{1}{24} + \frac{3(1-\nu^2)P_1}{8k^2}\frac{\bar{E}h^2}{Eh_0^2} + \frac{(1-\nu^2)P_0}{2k^3}\frac{\bar{E}h^3}{Eh_0^3}.$$

The components of the beam stresses (4) are determined taking into account the formula of internal forces (15):

$$\sigma_1^0 = h_0 \cdot \varphi_0(z)\frac{M}{J},$$
$$\tau_{13}^0 = -h_0^2 \cdot \psi_0(z)\frac{Q}{g_0 J}, \quad (18)$$
$$\sigma_3^0 = h_0^3 \cdot \delta_0(z)\frac{q(x_1)}{\gamma \cdot J}.$$

Thus, the effect of the elastic foundation on the beam is taken into account by two parameters P_0, P_1 and is determined by formula (12).

At the beam edges, one of the following boundary conditions must be satisfied.

1) If the beam edge has a full contact (touches fully) with the elastic foundation, then the boundary conditions as follows

$$W_* = \frac{Q\cdot L^3}{3\bar{E}J_0}, \quad \varphi_* = \frac{M \cdot L}{\bar{E}J_0}, \quad J_0 = \frac{h^3}{12}, \quad (19)$$

where W_*, φ_* are the beam edge displacements, L is the foundation length beyond the beam, $\bar{E}J_0$ is rigidity with the foundation deflection.

2) If the ends of the beam are hinge-supported, then the boundary conditions as follows

$$W_0 = 0, \quad M = -EJ\frac{d^2 W_0(x_1)}{dx_1^2} = 0. \quad (20)$$

3) If the ends of the beam are fixed, then the boundary conditions as follows

$$W_0 = 0, \quad \theta = \frac{dW_0(x_1)}{dx} = 0. \quad (21)$$

4) If the ends of the beam are free, then the boundary conditions as follows

$$M = -EJ\frac{d^2 W_0(x_1)}{dx_1^2} = 0, \; Q = -EJg_0\frac{d^3 W_0(x_1)}{dx_1^3} = 0. \quad (22)$$

Reactive pressures of the elastic foundation are determined according to (14) and have the form

$$r(x_1) = EJ\left(-2r^2\frac{d^2W_0}{dx_1^2} + S^4W_0\right) = \quad (23)$$
$$= EJ\left[-\frac{6P_1(1-\nu^2)}{h^2}\frac{\bar{E}h^2}{Eh_0^2}\frac{d^2W_0}{dx_1^2} + \frac{6k\cdot P_0(1-\nu^2)}{h^4}\frac{\bar{E}h^3}{Eh_0^3}W_0\right].$$

The calculation of a beam on a two-parameter elastic foundation is performed according to the following algorithm:
1) Determining the deflection function $W_0(x_1)$ by solving Eq. (13) and satisfying one of the options for boundary conditions (19) - (22).
2) Finding the internal forces of the beam (Q, M) by formula (15).
3) Determining the components of stresses (18) and components of the beam displacements (3).
4) Determining the components of displacements and stresses of the elastic foundation by formulas (5) and (6), respectively.
5) Determining reactive pressures of the elastic foundation by formula (23).

4. ANALYTICAL RESULTS AND VERIFICATION OF THE SIMPLIFIED METHOD

To verify the calculation accuracy of the simplified method, the following examples are considered. The examples show the analytical solution of beams on a two-parameter elastic foundation with different boundary conditions, various loadings, geometrical and physical characteristics of the beam and elastic foundation. The solutions of the problems are obtained by using the computer program MathCad.

4.1 Example 1

A fixed-end beam on a two-parameter elastic foundation is considered. The uniformly distributed load was chosen as $q = 200\frac{kN}{m}$. A beam of length $\ell = 8\ m$, width $b_0 = 1\ m$ and height $h_0 = 5\ m$. The beam's material has a modulus of elasticity $E = 40\cdot 10^4\ Pa$. The physical and geometry parameters of the elastic foundations were $\bar{E} = 40\ Pa$, $\nu = 0.25$ and $h = 4\ m$.

The comparison results of the beam displacement, bending moment and shear force are shown in Tables 1-3 and Figures 1-3, which indicate that the values obtained by the present method is good agreement with the Winkler and Vlasov models.

Table 1 Vertical displacement values

Case	The length of the beam				
	0	2	4	6	8
Ww	0	0.2879	0.5119	0.2879	0
Wv	0	0.2879	0.5119	0.2879	0
Wp	0	0.2879	0.5119	0.2879	0

Table 2 Bending moment values

Case	The length of the beam				
	0	2	4	6	8
Mw,$\cdot 10^5$	-10.66	1.333	5.333	1.333	-10.66
Mv, 10^5	-10.66	1.333	5.333	1.333	-10.66
Mp, 10^5	-10.66	1.333	5.332	1.333	-10.66

Table 3 Shear force values

Case	The length of the beam				
	0	2	4	6	8
Qw,$\cdot 10^5$	7.9995	3.9996	0	-3.9996	-7.9995
Qv, 10^5	7.9998	3.9998	0	-3.9998	-7.9998
Qp, 10^5	7.9990	3.9995	0	-3.9995	-7.9990

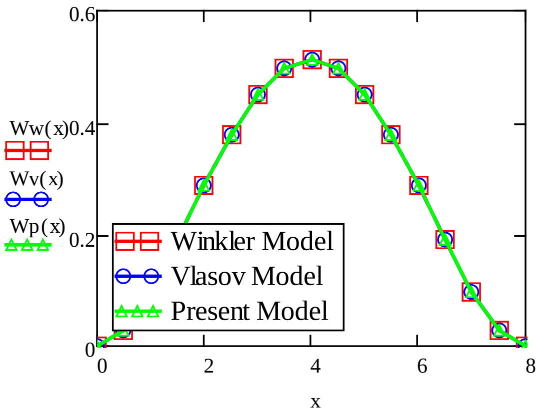

Fig.1 Vertical displacement of the beam on a two-parameter elastic foundation

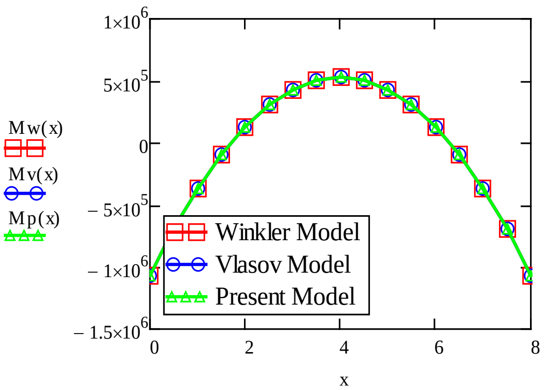

Fig.2 Bending moment of the beam on a two-parameter elastic foundation

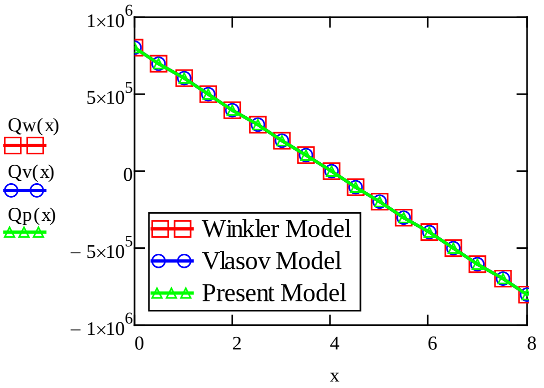

Fig.3 Shear force of the beam on a two-parameter elastic foundation

4.2 Example 2

A beam fixed at one end and supported at the other on a two-parameter elastic foundation is considered. The vertically uniform load was chosen as $q = 250\frac{kN}{m}$. The physical and geometry parameters of the elastic foundations were deformation modulus $\overline{E} = 50\,Pa$ and Poissons ratio $\nu = 0.25$, depth $h = 5m$. A beam of length $\ell = 20m$, width $b_0 = 1\,m$ and height $h_0 = 4m$, modulus of elasticity $E = 27 \cdot 10^5 Pa$.

Table 4 Vertical displacement values

Case	The length of the beam				
	0	8	12	16	20
Ww	0	12.192	14.962	10.344	0
Wv	0	12.215	14.992	10.365	0
Wp	0	12.214	14.990	10.364	0

Table 5 Bending moment values

Case	The length of the beam				
	0	8	12	16	20
Mw,·10^6	-12.47	4.489	6.982	5.487	0
Mv, 10^6	-12.49	4.498	6.996	5.497	0
Mp, 10^6	-12.49	4.497	6.995	5.496	0

Table 6 Shear force values

Case	The length of the beam				
	0	8	12	16	20
Qw,·10^6	3.119	1.122	0.125	-0.872	-1.871
Qv, 10^6	3.124	1.124	0.125	-0.874	-1.874
Qp, 10^6	3.123	1.124	0.125	-0.875	-1.874

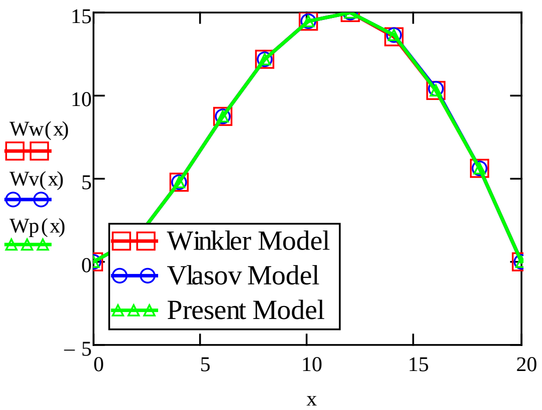

Fig.4 Vertical displacement of the beam on a two-parameter elastic foundation

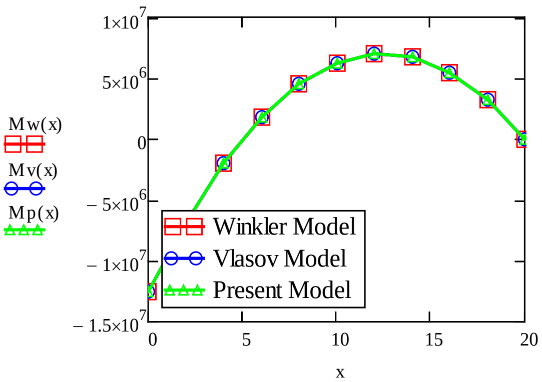

Fig.5 Bending moment of the beam on a two-parameter elastic foundation

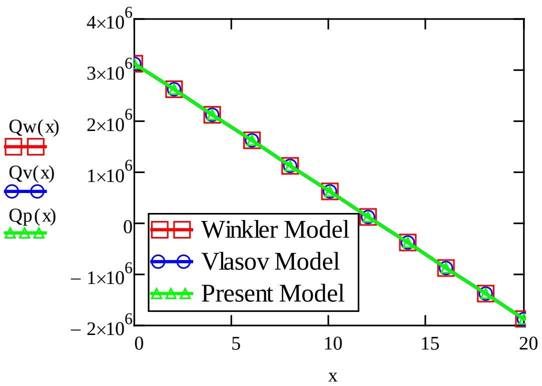

Fig.6 Shear force of the beam on a two-parameter elastic foundation

4.3 Example 3

A cantilever beam on a two-parameter elastic foundation is considered. The beam on a two-parameter elastic foundation was assumed to be subjected only to uniform vertical loads

$q = 100\frac{kN}{m}$. The physical and geometry parameters of the elastic foundations were deformation modulus $\overline{E} = 20\,Pa$ and Poissons ratio $\nu = 0.25$, and depth $h = 4\,m$. A beam of length $\ell = 10\,m$, width $b_0 = 1\,m$, height $h_0 = 2\,m$, and modulus of elasticity $E = 20 \cdot 10^5\,Pa$.

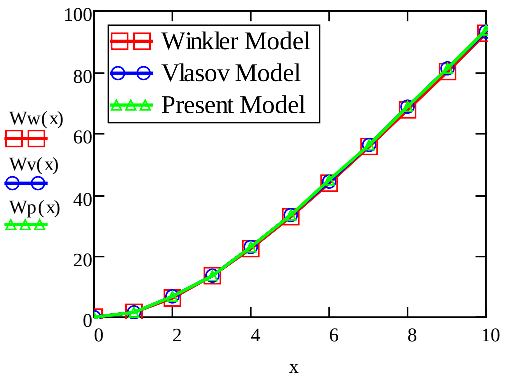

Fig.7 Vertical displacement of the beam on a two-parameter elastic foundation

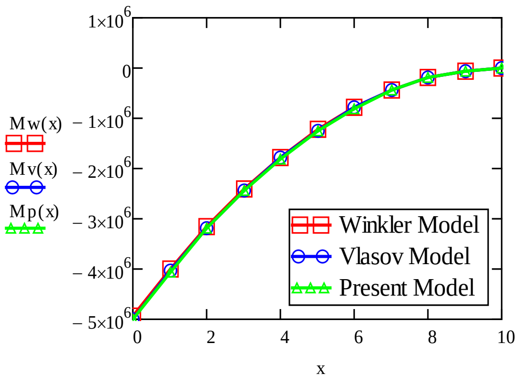

Fig.8 Bending moment of the beam on a two-parameter elastic foundation

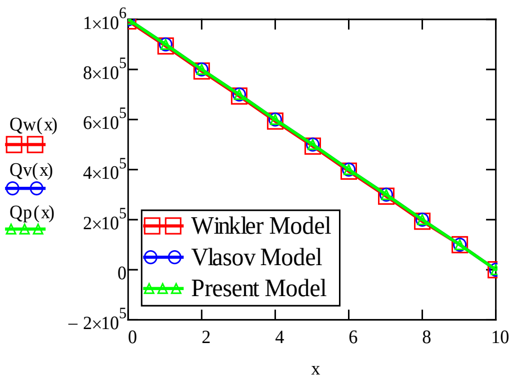

Fig.9 Shear force of the beam on a two-parameter elastic foundation

In tables and figures Ww, Mw, Qw means vertical displacement, bending moment, and shear force by Winkler, respectively Wv, Mv, Qv vertical displacement, bending moment, and shear force by Vlasov. Vertical displacement (Wp), bending moment (Mp), and shear force (Qp) had determined by the present analytical model.

The presented examples show the advantages of the suggested approach for an analytical solution of a beam on a two-parameter elastic foundation. The tables 1-6 and figures 1-9 show excellent agreement of the proposed method with the results obtained by Winkler and Vlasov models. These results are in good agreement with the results of the author's research, which were obtained using a different approach [20, 21].

This theory makes it possible to reduce solving various kinds of practically important problems to solving ordinary differential equations, which are well studied and are easily integrated. The simplicity of mathematical techniques and the clarity of the scheme make the theory under consideration very flexible and allow solving not only the main problems of calculating beams on an elastic foundation but a number of more complex issues. This includes, for example, the issues of calculating plates and shells on an elastic foundation.

5. CONCLUSION

An analytical solution has been developed for bending analysis of beams resting on a homogenous foundation. The presented mathematical model of a beam on an elastic foundation in a simple form takes into account the influence of an elastic foundation and makes it possible to carry out calculations based on ready-made results also for beams without an elastic foundation. The accuracy of the proposed method has been verified through three analytical examples. A comparative evaluation of the above results with the previously known theories of Winkler, Vlasov have shown their good agreement, which shows high reliability of the theoretical provisions and applied results of the approaches proposed by the authors. The proposed simplified theory of a beam on a two-parameter elastic foundation is of interest for structural engineers who deal with structural design of foundations in construction sites and the authors hope that it will be useful for further studies.

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