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The proceedings are the papers of students, undergraduates, doctoral students and young researchers on topical issues of natural and technical sciences and humanities.

В сборник вошли доклады студентов, магистрантов, докторантов имолодых ученых по актуальным вопросам естественно-технических и гуманитарных наук.

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$$0 = A^2\dot{B}\left(AB - \dot{C}B + B(C - A)\right) + C^2(B - C)^{\cdot} \times \left(B - \dot{C}\right) \times A^2\left(A - \dot{B}\right) + A^2\dot{B} - -A\dot{B}\left(B - \dot{C}\right) + A\dot{B} \tag{12}$$

Қорытынды

Кармаркар шарты кеңістік уақытының бірінші класта болуы үшін қажетті шарт болып табылады.[10] Кармаркар күйінің туындысы геометриялық сипатта, Риман тензор компоненттері арасындағы қатынастарды қамтамасыз етеді. Бұл өз кезегінде метрикалық потенциалдарды бір-бірімен байланыстырады және осылайша модельдердің дамуына ықпал етті. Кармаркар күйі қысым изотропиясымен бірге ішкі материяның жалғыз шектелген конфигурациясы ретінде беріледі.

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METHODS FOR THE INVESTIGATION OF COSMOLOGICAL SOLUTIONS IN THEORIES WITH MODIFED GAUSS-BONNET GRAVITY

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1. Introduction

According to Einstein's general relativity, the gravitational influence exerted by all the radiation, matter, and dark matter in the universe should decelerate the expansion of the universe. However, observational evidence contradicts this, as galaxies are receding from us at an increasing rate. This suggests the presence of an energy density counteracting the natural gravitational pull. Furthermore, this energy dominates over other forms of energy in the current universe, a phenomenon termed dark energy (DE), constituting 68% of the total energy in today's observable universe [1-5]. One hypothesis explains this phenomenon by positing that the source of DE has always existed and came to dominate everything in the past 7 billion years. This is known as the cosmological constant Λ. Another perspective suggests a nature evolving with time, where it was initially subdominant in the early universe, but over time, it diminished in value and came to dominance. This concept is commonly referred to as quintessence. In contrast, modified gravity theories propose that DE presents the first evidence that Einstein's general theory of relativity requires modifications. General Relativity has proven highly effective in explaining phenomena within our solar system and even on a cosmological scale. Consequently, any adjustments proposed by modified theories of gravity must be exceptionally small. However, if these modifications prove accurate, it will eliminate the necessity to postulate exotic forms of energy density to account for DE. The change in the curvature of the universe would explain the observation that the universe seems to be undergoing accelerated expansion. The Gauss–Bonnet term is a higher-order curvature invariant that is used in modified gravity theories. This paper focuses on the application of new and old methods tailored to investigate cosmological solutions within the framework of modified Gauss-Bonnet gravity theory.

2. Basic equations in $f(G)$ gravity

In this section, we review the $f(G)$ gravity theory in details, starting with the Einstein-Hilbert action for the $f(G)$ gravity theory:

$$S = \int \sqrt{-g}\left[\frac{1}{2\kappa^2}R + f(G)\right] d^4x, \quad (1)$$

where $f(G)$ is an arbitrary function of the Gauss-Bonnet invariant (we take it as $f = \alpha G^n$, where α and n are arbitrary constants), g denotes the metric tensor, R is the Ricci scalar and κ is the Einstein gravitational constant. It is defined as $\kappa = 8\pi G_N c^4$ (choosing $8\pi G_N = c = 1$, where G_N is the Newtonian constant).

The Friedmann–Lemaître–Robertson–Walker metric (FLRW metric) is:

$$ds^2 = \mathrm{d}t^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right), \quad (2)$$

where $a(t)$ is the scale factor. In this study, the scale factor is defined in the following hybrid form: $a(t) = e^{\frac{t}{\beta}} t^{\gamma}$.

The desired equation of motion for $f(G)$ gravity can be obtained by variating the action (1). The Gauss-Bonnet invariant G, the Ricci scalar R and the expression of metric tensor g in action can be written as functions that are solely dependent on the scale factor a:

$$G = 24H^2(\dot{H} + H^2), R = 6(\dot{H} + 2H^2), \sqrt{-g} = a^3, \quad (3)$$

here it is convenient to introduce the Hubble parameter, which is simply a function of scale factor $H = \frac{\dot{a}}{a}$, where dot is the time derivative. Variation of action (1) results in the point-like Lagrange function:

$$L = -\frac{3}{\kappa^2} a\dot{a}^2 + fa^3 - f_G G a^3 - 8 f_{GG} \dot{G} \dot{a}^3. \quad (4)$$

After substitution of the Lagrange function (4) into the Euler-Lagrange equations, we obtain the equations of motion for the $f(G)$ gravity:

$$\begin{cases} \frac{1}{\kappa^2}(3H^2 + 2\dot{H}) = -8H^2\dot{G}^2 f_{GGG} - 8H^2\ddot{G} f_{GG} - 16\dot{G}(H\dot{H} + H^3) f_{GG} + Gdf - f \\ \frac{3}{\kappa^2}H^2 = -24H^3\dot{G} f_{GG} + Gdf - f \end{cases} . \quad (5)$$

3. Pressure, energy density and equation of state parameter

In the standard formulation, the equations of motion (5) can be written as:

$$3H^2 + 2\dot{H} = -p \text{ and } 3H^2 = \rho, \quad (6)$$

where p and ρ are the pressure and the energy density of the perfect cosmological fluid, respectively. The first equation in (6) relates the rate of change of the Hubble parameter to the pressure of the contents of the universe. The second equation, known as the critical density equation, shows the relationship between the Hubble parameter and the energy density of the universe.

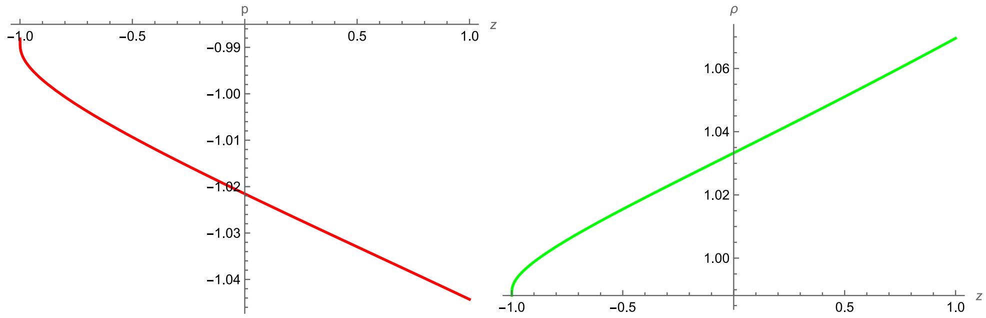

Figure 1. Evolution of the pressure p and energy density ρ as functions of redshift z with arbitrary constants $\alpha = -1,\ \beta = 5,\ \gamma = 0.951,\ n = 0.003$.

$$p = \omega\rho. \tag{7}$$

We should also consider the equation of state parameter (EoS) ω here. It is a dimensionless number, that can help us understand the expansion of the universe by describing the specifics of the cosmological perfect fluid.

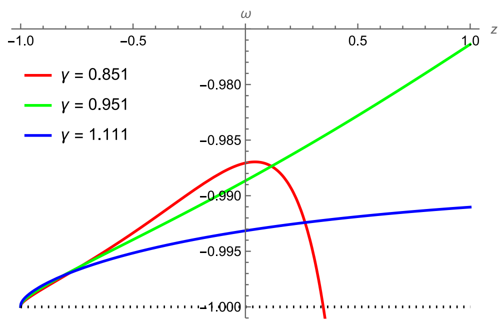

Figure 2. Evolution the EoS parameter as a function of redshift z with arbitrary constants $\alpha = -1$, $\beta = 5,\ n = 0.003$ and different value of the γ parameter. At $\gamma = 0.851$ the model seems unstable, at $\gamma = 0.951$ and $\gamma = 1.111$ the model corresponds to quintessence as $\frac{1}{3} < \omega < -1$, while $\omega = -1$ corresponds to ΛCDM.

4. Cosmographic analysis

Cosmographic parameters are a set of parameters that describes the expansion of the Universe [1]. They are derived from the Hubble parameter and its derivatives with respect to redshift. The most used cosmographic parameters are the deceleration parameter q, the jerk parameter j, the snap parameter s, the lerk parameter l. They are not reliant on a specific model, which allows them to impose constraints on the characteristics of DE.

The deceleration parameter q characterizes a measure of how the expansion of the universe is changing over time. Mathematically, is it expressed as:

$$q = \frac{d^2a}{dt^2}\left(\frac{1}{a}\frac{da}{dt}\right)^{-2} = \frac{1+z}{H}\frac{dH}{dz} - 1. \tag{8}$$

It provides crucial information about the dynamics of the cosmic expansion. When the value of the q is positive, it implies that the expansion of the universe is slowing down. When it is negative, it implies that the universe is accelerating. In Fig. 3, the deceleration drops from the positive to negative value, which means that our model predicts that the universe is accelerating with time. At $\gamma = 0.851$ and $\gamma = 0.951$ projection regarding the evolution of q is consistent with the ΛCDM model.

The jerk parameter is third time derivative of the scale factor and the third-order term in the Taylor series expansion of the Hubble law. It provides insight into transitions between phases of accelerated expansion. For example, within the flat ΛCDM model, the jerk parameter is fixed at $j = 1$, attributed to the cosmological constant Λ. As depicted in Fig. 3, our model forecasts a transition of the jerk parameter at high redshifts while $\gamma = 0.851$ and $\gamma = 0.951$, suggesting a high rate of change in q. Moreover, it is defined as:

$$j = \frac{1}{a}\frac{d^3a}{dt^3}\left(\frac{1}{a}\frac{da}{dt}\right)^{-3} = \frac{(1+z)^2}{H}\frac{d^2H}{dz^2} - q^2. \tag{9}$$

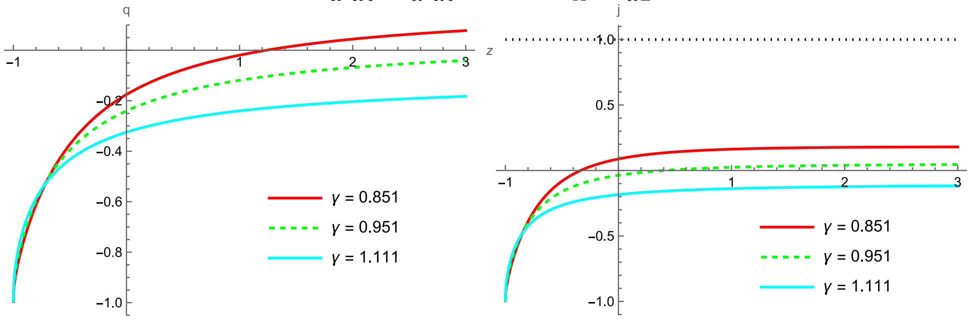

Figure 3. Evolution of the deceleration q and jerk j parameters as functions of redshift z.

Snap parameter s, also known as jounce, is the fourth time derivative of the scale factor. It determines the level of variation from the standard ΛCDM model, which assumes a cosmological constant Λ serving as the primary source of DE. In Fig. 4, at $\gamma = 0.851$ and $\gamma = 0.951$ we observe a transition from a negative to a positive value of snap parameter s. This behavior aligns with the preference for quintessence-like model, indicated by j. Snap is defined as:

$$s = \frac{1}{a}\frac{d^4a}{dt^4}\left(\frac{1}{a}\frac{da}{dt}\right)^{-4} = -\frac{(1+z)^3}{H}\frac{d^3H}{dz^3} - 3q^2 - 3q^3 + 4qj + 3j. \tag{10}$$

Lerk parameter l is the fifth derivative of the scale factor. It also provides information on how the higher-order derivatives of the Universe's acceleration offers insights into transitions between different epochs. In Fig. 4, at $\gamma = 0.851$ and $\gamma = 0.951$ lerk slowly decreases to a negative value of $l \cong -1$. Mathematically, it is written as:

$$l = \frac{1}{a}\frac{d^5a}{dt^5}\left(\frac{1}{a}\frac{da}{dt}\right)^{-5} = \frac{(1+z)^4}{H}\frac{d^4H}{dz^4} + 12q^2 + 24q^3 + 15q^4 - 32qj - 25q^2j - 7qs - 12j + 4j^2 - 8s. \tag{11}$$

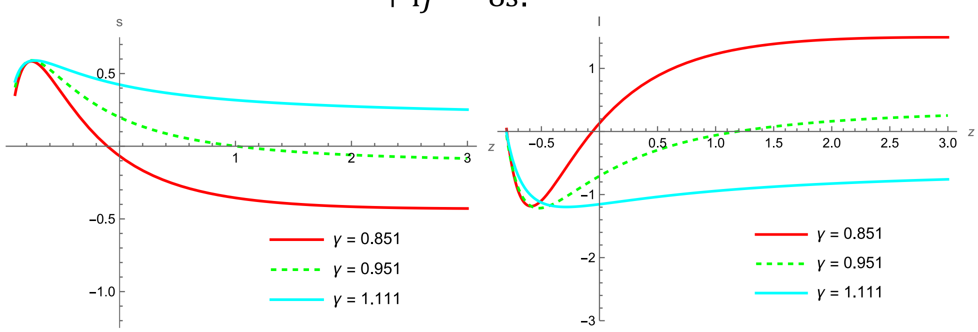

Figure 4. Evolution of the snap s and lerk l parameters as functions of redshift z.

5. Energy Conditions

In the universe, the stress energy-momentum tensor stems from various matter fields, making it incredibly complex to precisely describe. Understanding matter's behavior under extreme conditions of density and pressure remains limited. Consequently, predicting singularities in the universe using Einstein field equation (12) seems challenging due to unknown factors. Nonetheless, certain reasonable inequalities regarding the stress energy-momentum tensor will be explored in this section. These inequalities are called the energy conditions. [2]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}. \tag{12}$$

Perfect fluids possess the stress energy momentum of the following form:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, \tag{13}$$

where u_μ is the fluid four-velocity, with total density ρ and pressure p, respectively.

The NEC or the Null Energy Condition posits that any null vector $l_{\mu\nu}$, the stress energy-momentum tensor $T_{\mu\nu}$ satisfies $T_{\mu\nu}n^\mu n^\nu \geq 0$, or equivalently $\rho + p \geq 0$. In simple terms, it implies that the local energy density is non-negative. It's the weakest of all energy conditions and is a component of all other energy conditions. It is also the easiest to work with and the one that all reasonable form of matter should satisfy. In Fig. 5, we can see that the NEC satisfied for our model.

The WEC or the Weak Energy Condition states that for any timelike vector t^{μ}, the stress energy-momentum tensor satisfies $T_{\mu\nu}t^{\mu}t^{\nu} \geq 0$, or equivalently $\rho \geq 0,\ \rho + p \geq 0$. This means that the energy density must be non-negative. It implies the NEC and is used to avoid solutions in general relativity that have negative energy densities. In Fig. 5, we can see that it is satisfied for our model.

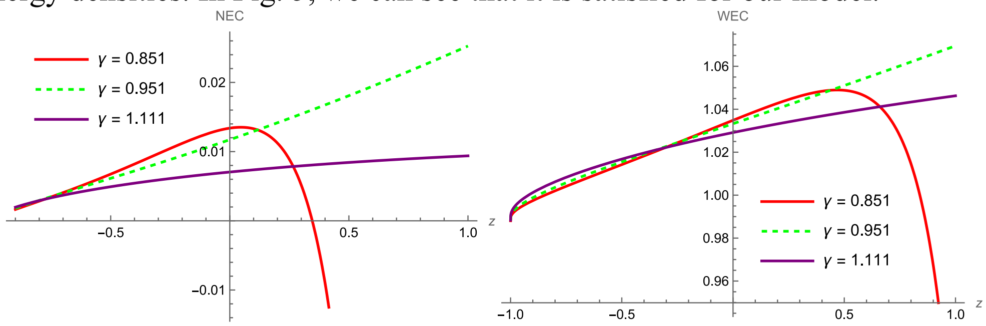

Figure 5. Evolution of NEC and WEC as functions of redshift z.

The SEC or the Strong Energy Condition asserts that for any timelike vector t^{μ}, $T_{\mu\nu}t^{\mu}t^{\nu} \geq \frac{1}{2}T_{\lambda}^{\lambda}t^{\sigma}t_{\sigma} \geq 0$, where T_{λ}^{λ} is the trace of the stress energy-momentum tensor, or equivalently $\rho + 3p \geq 0$. It is violated by dynamic DE, the cosmological constant and even simplest scalar field models, which is why it is the subject of much discussion nowadays. It also implies the NEC. In Fig. 6, we can see that our model violates SEC, much like most of the modern theories.

The DEC or the Dominant Energy Condition is the WEC with additional requirement that $T_{\mu\nu}t^{\mu}$ must be a non-spacelike vector, or equivalently $\rho \geq |p|$. This implies that the energy density as measured is non-negative, and the pressure is not too large compared to energy density. All reasonable forms of matter should satisfy this condition, and as we can see in Fig. 6, this energy condition is satisfied.

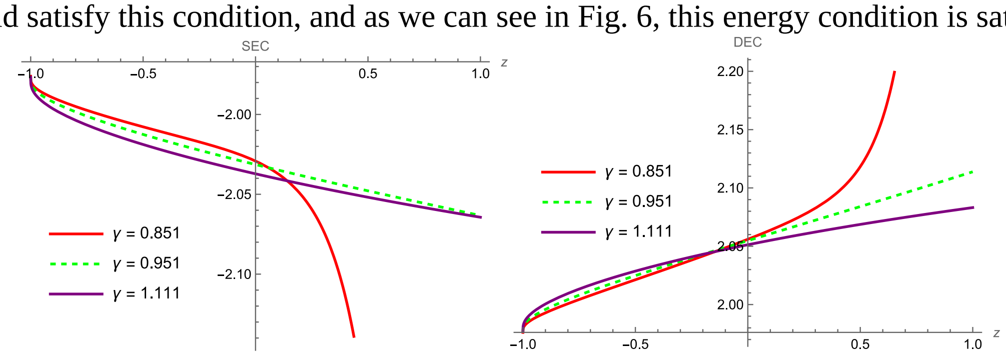

Figure 6. Evolution of DEC and WEC as functions of redshift z.

6. $\boldsymbol{Om(z)}$ diagnostic

In cosmology, Ω_{0m} represents the ratio of the actual density of matter in the universe to the critical density. $Om(z)$ [3] is a recently introduced combination of the Hubble parameter and cosmological redshift determined directly from observational data. It provides a null test of ΛCDM hypothesis and is defined as:

$$Om(z) = \frac{\left(\frac{\dot{H}(z)}{H_0}\right)^2 - 1}{(1+z)^3}. \tag{14}$$

These parameters are related as $Om(z) = \Omega_{0m}$ in ΛCDM case, whereas $Om(z) > \Omega_{0m}$ in quintessence while $Om(z) < \Omega_{0m}$ in phantom (ghost) energy. In Fig. 7, we can see that $Om(z)$ changes from positive to negative value. Here we assume three different values of matter density $\Omega_{0m} = 0.22$, $0.27, 0.32$ for the matter density. For ΛCDM $Om(z)$ should have a zero-curvature, however in our case it has a non-zero curvature, which is better at describing quintessence or phantom like DE. [3].

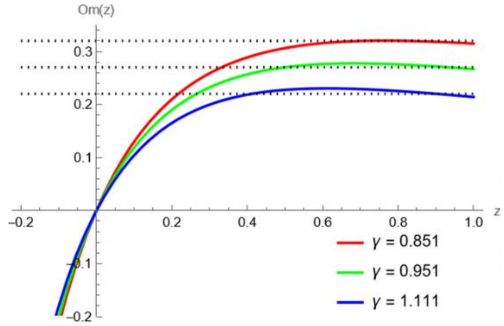

Figure 7. $Om(z)$ graph with $\alpha = -1,\ \beta = 5,\ n = 0.003$ and different value of the γ. At $\gamma = 0.851$ the model corresponds to phantom, at $\gamma = 0.951$ the model corresponds to ΛCDM and at $\gamma = 1.111$ the model corresponds to quintessence.

7. Concluding Remarks

In this study, we used cosmography, energy conditions and $Om(z)$ diagnostic to investigate the behavior of the $f(G)$ model. From our analysis we can state that the model aligns with the ΛCDM model in certain places, for example in evolution of deceleration q, and it differs from it, exhibiting a quintessence like behavior in its evolution of jerk parameter j and $Om(z)$ diagnostic parameter. Our model satisfies all the necessary energy conditions. It requires additional investigations and analyses to confirm its alignment with additional observational data and theoretical predictions. Nevertheless, the findings indicate that the $f(G)$ model has the potential to be provide useful solutions the field of cosmology.

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ЖАРЫҚ СӘУЛЕСІНІҢ ГРАВИТАЦИЯЛЫҚ АУЫТҚУЫ

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Салыстырмалылық теориясы — физикалық процестердің кеңістік-уақыттық қасиеттерін зерттейтін теория. 20-ғасырдың басында Альберт Эйнштейннің жасаған жалпы салыстырмалылық теориясы (ЖСТ) деп те аталатын Гравитация теориясы (GT) біздің гравитация, кеңістік пен уақытты түсінуімізге зор ықпал етті. Гравитацияны зерттеу тарихы ежелгі гректерден бастап көптеген ғасырларға созылады, бірақ ең маңызды жетістікті.

17-ғасырда Исаак Ньютон жасады. Ньютон бүкіл әлемдік тартылыс заңын: Әлемдегі әрбір дене