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Substantiation of methods for calculation of traction forces redistribution indicators on modular front and rear wheels of the vehicle (4X4)

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ABSTRACT

An analysis of the existing study of the redistribution of traction forces on the wheels of four-wheel drive wheeled vehicles and the need to take them into account when assessing the stability and efficiency of vehicles have been vehicleried out. Proposed phenomenon of redistribution of traction forces of vehicle is considered on front and rear drive wheels separately as modular ones with reduction of all parameters to equation for each wheel separately. In such a statement, equations are justified for calculating indicators of redistribution of traction forces on modular front and rear wheels of a vehicle (4 × 4). Calculations of tangential traction forces on modular front and rear wheels of low-speed self-propelled wagon ВС5Э (4 × 4) have been vehicleried out according to the presented equations.

1. Introduction

In various industries, the most mobile and maneuverable vehicles are widely used for the transportation of goods and mechanization of various works, which, depending on their types and design, are constantly expanding. Therefore, thanks to the mass use of vehicles, it is relevant to continuously conduct scientific research aimed at improving their active safety and efficiency.

Road accidents entail numerous losses, including those related to the death and loss of ability to work. Therefore, all scientific research aimed at improving the safety of vehicles is relevant. It is known that traffic safety is determined by the active and passive safety of vehicles, road infrastructure and the driver. The design of the vehicle must first of all meet the requirements of active safety, which entails a decrease in the likelihood of accidents on the roads. Nevertheless, in ensuring road safety, the main criterion is the necessary qualification of the driver [1].

The performance properties of vehicles of different designs vary differently depending on the specific road conditions. Therefore, depending on the road conditions, in each specific case, the more significant operational properties of the vehicle are those that primarily determine its safe operation.

The automotive industry is developing in the direction of improving the transmissions of new and modernized vehicles [2]. The basics of which are, firstly, the need to save energy resources, and secondly, the need for higher quality vehicles. Therefore, the problem of improving the operational properties of vehicles belongs to one of the main ones in mechanical engineering and requires a solution, including by reducing losses in the transmission. In recent years, thanks to the use of automatic systems, vehicles have become more adapted to operating conditions that have increased their active safety, but reserves are still significant along the way. One of the main reserves is in the vehicle chassis, transmission and control mechanisms of the vehicle, since the parameters of these parts primarily determine its active safety in certain road conditions.

It is known that when moving four-wheel drive vehicles, even along rectilinear trajectories, traction forces are redistributed between the wheels of various vehicle axles, therefore, studies on the problems of the emergence of redistribution of traction forces during the movements of four-wheel drive vehicles are paramount.

When assessing the passability and stability of vehicles, the definition of their traction and traction properties is of particular importance. Therefore, the assessment of the influence of the redistribution of traction forces on the wheels of different axles of the vehicle on stability and passability, as well as on its effectiveness, is an urgent problem.

In this work, the authors have considered the possibility of taking into account the redistribution of traction forces separately on the front

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and rear driving wheels of the vehicle for a full assessment of its traction and traction properties in the road conditions.

2. Analysis of the phenomenon of redistribution of traction forces on the driving wheels of vehicles

Despite the achievements of scientists in the field of vehicle dynamics, such important issues as the establishment of patterns of internal dynamic processes in the nodes and units of the vehicle during its movement have not been sufficiently investigated. According to Yu.V. Pirkovsky, the current theory does not allow answering many questions that inevitably arise in the design of four-wheel drive vehicles.

Therefore, rational design and operation of wheeled vehicles is impossible without theoretical analysis of their operational properties [3,4]. In particular, when operating vehicles in various road conditions, the issues of their effective and safe operation, which directly depend on the processes of interaction of their wheels with the road, are urgent problems [5,6].

All operational properties of vehicles are interconnected, but depending on its purpose and operating condition, those operational properties that are the main ones come to the fore.

As for one or another operational property of vehicles, it is a complex indicator that depends on a number of factors. The parameters by which this or that operational property of vehicles is evaluated also have different significance.

To assess the active safety of a vehicle in various road conditions, knowledge is needed in the field of its operational properties, in particular, to assess the stability and passability of a vehicle, and firstly knowledge in the field of its traction and traction properties is needed. In particular, for assessing the stability and patency of all-wheel drive vehicles, the comparison of wheel adhesion forces with the road with traction forces should be accompanied by taking into account their redistribution on the wheels. In general, the determination of the parameters of the front and rear driving wheels of the vehicle separately with its uneven movement to assess its traction dynamics was not considered in the literature. It is known that even under all other equal conditions there appears a redistribution of traction forces on the wheels of the vehicle due to the different nature of interaction with the support of its front and rear driving wheels [7–11].

From literary sources, the equation of the traction balance of the vehicle is known, which is obtained from the condition of interaction of the entire vehicle with the road. The inertia force included in the traction balance equation is calculated through the factor of accounting for the rotating masses of the entire vehicle. In addition, various schemes for the interaction of single wheels by road are given in the literature. At the same time, loading modes of longitudinal force and torque of front and rear wheels are not separately given. Therefore, the equations of unequal movement of the front and rear driving wheels of the vehicle separately are also not given.

It is known that in the operation of four-wheel drive vehicles, along with their advantages, their disadvantages are manifested, which are to reduce the traction qualities on the ascents of the front and on the descents of the rear drive wheels due to the redistribution of their weight. It is known that even when moving horizontally, there is also a redistribution of traction forces on the front and rear driving wheels of vehicles due to kinematic mismatch of their wheels. However, in the calculations, the influence on each other of the parameters of the front and rear driving wheels and their relationship have not been taken into account. Meanwhile, it is known from practice that when vehicles move even along a rectilinear trajectory, power circulations occur in their transmission, respectively, the traction forces on their wheels are redistributed, which will determine their stability and efficiency.

Therefore, it is necessary to further develop the theoretical substantiation of the criteria for evaluating the effectiveness of various types of transmissions when driving on roads [4,12].

It is known that four-wheel drive vehicles are operated in a wide range of road and climatic conditions. practically, there are no four-wheel drive vehicles that would be operated only off-road or only on hard roads. therefore, there is a need to control transmission parameters because many methods and means that improve off-road efficiency and passability degrade them when driving on hard roads.

Currently, in order to reduce internal power losses in the drive of three-axis four-wheel drive vehicles, various differential mechanisms are mainly used in the inter-wheel and inter-axis circuit, the choice of a certain gear ratio of the inter-wheel differential is used, which does not quite allow reducing the cost of driving on hard roads.

It is known that at least 50% of the total mileage of a triaxial four-wheel drive vehicle occurs on hard roads. The operation of such vehicles is considered irrational due to the additional power costs of driving transmission units, which are not necessary in these driving conditions, it is known that the type of power drive determines the amount of power loss in the transmission and in wheel propulsors. According to research by various authors in the balance of power of an four-wheel drive vehicle, the power consumption in the transmission and wheel propulsors can reach 30… 35%. Therefore, it is considered a promising path associated with the complete disconnection of individual driving vehicle axles of a triaxial four-wheel drive vehicle and the substantiation of the need to turn them off on hard roads [4]. Reducing transmission losses by disconnecting individual driving axles is an effective way to improve the efficiency of a three-axis four-wheel drive vehicle when driving on hard roads.

The analysis showed that in order to ensure high passability and economy of four-wheel drive vehicles, the power distribution between the wheels of the driving vehicle axles must meet the known requirements:

- operation of wheels of each axle shall ensure the identity of torque moments and rolling resistance moments, that is, operate in the mode of no redistribution of traction forces
- if it is necessary to create a longitudinal traction force to overcome the resulting resistances (acceleration, rotation and others) of power supplied to the wheels of the driving vehicle axles as a result of their intensive skidding, taking into account the redistribution of traction forces, should not exceed the load-bearing capacity limit for soil destruction, that is, skidding of the wheels of the driving vehicle axles should be optimal, corresponding to the values at which the maximum traction force is formed.

In the case of a locked transmission, the power distribution depends on the kinematic mismatch, the ratio of the reduced tangential elasticity of the wheels and the ground, and the overall resistance to motion. Comparative analysis of the optimal power distribution law with the actual transmission design showed that the transmission should be automatic, able to adapt to wheel conditions that vary depending on soil characteristics and road flatness.

In the works of Y.V. Pirkovskii [4], the need for an automatic power distribution system for wheels in sharply changing traffic conditions has been justified. The experimental data of the law of changing the optimal distribution of torques on the vehicle axles of a biaxial vehicle have been analyzed for the case of movement on flat loamy soil depending on the traction force on the hook. In near-ideal driving conditions, with an even distribution of the load along the axles, the ratio of the torque on the front axle to the total torque should be from 80% (traction force on the hook is 0) to 40% (traction force on the hook is 22, kN) [4]. For similar ground conditions, these values for triaxial vehicles lie within 67…25%, for four-axis - within 57…19%. The wide limits of torque change are explained by the fact that on compacted soils on which the experience was vehicleried out, the rolling resistance of the front wheels is greater than those following them. therefore, to provide rolling in a near-free mode, more torque is required to be applied to the wheels in front than to the subsequent wheels. If the soil is not compacted after the passage of the wheel, then the rolling conditions of all wheels with their uniform load are approximately the same, therefore, the optimal torques on the wheels should be equal. When pulling force is applied on the hook, it is necessary to redistribute the moments in such a way that the rear axis is more loaded. Analysis of movements on uneven surfaces with a large redistribution of vertical loads on wheels showed that the optimal distribution of moments is even more complicated. In this case, you must use an automatic system [4].

To a greater extent, the blocked wheel drive meets the conditions for ensuring high passability. It provides better adaptability to ground condition and surface profile-varying road conditions. With a locked wheel drive and in the absence of kinematic inconsistency due to different reduced tangential elasticity of tires and soil, due to the unequal supporting surface under the wheels of different axes, the required unevenness of the distribution of moments along the wheels and axes can be approximately ensured on most soils. In this case, it will be the larger the greater the difference in the state of the soil under the wheels determining the reduced tangential elasticity of the propulsor. The occurrence of a large long-acting kinematic mismatch can dramatically change the law of distribution of torques on wheels. Thus, with curvilinear movement on the same soil, the pattern of the distribution of moments on vehicle axles changes.

On deformable compacted soils during straight-line movement, a greater moment is supplied to the front axle than to the rest of the vehicle axles, which is required by the traffic conditions. When turning, on the contrary, a smaller moment is applied to the front controlled vehicle axles and the rear driving vehicle axles are overloaded. This is explained by the reasons: firstly, the occurrence of kinematic inconsistency between vehicle axles as a result of curvilinear movement; secondly, by increasing the overall resistance to movement due to the fact that each wheel must lay its own track, and thirdly, by equalizing the reduced tangential elasticity of the wheels, since the ground conditions under each wheel are practically the same. It has been experimentally established [4] that with certain turning radii, differential distribution of power over the wheels becomes more expedient than a blocked wheel drive. Thus, the resistance to the movement of the vehicle along a circular trajectory with a radius of 30 to 12 m turns out to be more than the resistance to rectilinear movement on the same ground with a locked drive 2.1…2.3 times, and with a differential drive only in 1, 5…1.6 times. Given the exceptional inhomogeneity of the mechanical properties and profile of many natural soil surfaces, for a mechanical transmission, a blocked wheel drive should be considered preferable due to the condition of high passability. This conclusion is supported by comparative tests of multi-axis vehicles on the ground. Tests of vehicles showed [4] that blocking differentials when moving on relatively flat surfaces (on snow and deformable soils) does not have a significant effect and does not give sharp advantages. The average speed of the vehicles was almost the same, the vehicles were stuck in equal conditions and for reasons independent of the type of wheel drive. The influence of the type of wheel drive on passability is sharply manifested when driving on roads and terrain with large irregularities (the height is greater than the total suspension travel), and when overcoming various obstacles. With large redistributions of vertical loads between axis and individual wheels, the traction capabilities of a vehicle with a differential drive are reduced. Tests conducted by various researchers have shown that blocking differentials on multi-axis vehicles to increase passability is necessary. It is caused not so much by the difference in the adhesion coefficients of individual axis and wheels, but by the large difference in vertical loads [6], which appear when moving along irregularities, and the heterogeneity of the soil under the axis. An analysis of foreign vehicles high passability showed that in most cases, even on commercial vehicles, forced blocking of differentials with cockpit control is used.

The analysis showed [4] that at curvilinear motion 50% of the hitch weight and can no longer be used to generate traction force, the remaining driving wheels are overloaded with input torque. As a result, the passability of a vehicle in difficult road conditions and the economy of its work on roads of all types are reduced.

We shall give some examples of the redistribution of traction forces on the driving wheels of four-wheel drive vehicles. The redistribution of traction forces on the front and rear driving wheels of vehicles is recorded by experiments [4,12–15]. This fact is still not sufficiently substantiated, accordingly, it is not described theoretically. For example, when moving four-wheel drive vehicles with a locked transmission, even along a rectilinear trajectory at low speed, power circulation in the transmission is observed due to the redistribution of traction forces on wheels. The reason for such phenomena in relation to vehicles with 4 × 4 wheel arrangement is that their front wheels are in relation to the rear laggards (if the weight states of the vehicle axles and the air pressures in the tires are equal) when starting during acceleration [13,16]. Torques on the wheels of the vehicle during its movement (at constant air pressures in the tires of all wheels) will follow the condition:

$$M_{k2} > M_{k1}$$

where $M_{\kappa 1}$, $M_{\kappa 2}$ - torques on the front and rear wheels, respectively, which is confirmed by experiments vehicleried out on the vehicle 4BC−10 **(**4x4**)** (4 × 4) [15]. At that overpressure in front wheels tires over air pressure in rear wheels tires ensures exceeding of front wheels rolling radius over rear wheels rolling radius [13,16], respectively, the condition $M_{\kappa 1} > M_{\kappa 2}$ is fulfilled, which is also confirmed in operation [16].

In vehicles of Ural 377 (6 × 4) and Ural 375 (6 × 6) brands, the reasons for the redistribution of traction forces are also established regularities in the differences in the rolling radii of the wheels of various vehicle axles [13,16]. All this is also consistent with experiments [14].

The given analysis showed that as a design scheme, a 4 × 4 vehicle is suitable for more accurate consideration of the redistribution of its traction forces on the wheels of various axles, assuming that the supporting surface of the wheels is solid.

Therefore, consideration separately of the processes of interaction of single wheels with the road is necessary to establish relationships both between them and with the vehicle as a whole. In particular, this concerns the issues of assessing the stability and passability of the vehicle, taking into account the redistribution of traction forces on its wheels. Therefore, there is a need to assess the dynamicity of the vehicle, taking into account the redistribution of traction forces that arise due to the kinematic mismatch of its front and rear wheels. Moreover, the movement of the vehicle is considered stable when, upon exposure to certain specified traction forces, its deviations from the specified certain law of movement do not exceed the limit value on a certain given section of the track.

As a result of the analysis of literature and practical data on the operation of vehicles in various industries, it was found that the assessment of their different operational qualities is vehicleried out by various methods. At the same time, in calculations of traction-traction properties of the vehicle, respectively, its stability and passability, the issues of assessing the redistribution of traction forces on wheels are not considered. In this regard, in order to take into account the redistribution of traction forces on the wheels of different axles of the vehicle, it is proposed to consider these wheels separately using the Dalembert principle [13,16].

3. Substantiation of equation for calculation of traction forces redistribution indicators on front and rear wheels of the vehicle (4 × 4)

As noted above, in order to assess the stability and passability of a vehicle (4 × 4) in various modes of its acceleration, it is necessary to determine the indicators of redistribution of its traction forces on the front and rear driving wheels in the form of separate modules using the D'Alembert's principle.

Using the design scheme of interaction of the front driving wheel of the vehicle with the road (Fig. 1), the equations are obtained without taking into account the air resistance force [13,16]:

$$X_1 = X_{m1} + P_{i1} = X_{m1} + jm_1, \tag{1}$$

$$z_1 a + X_1 r + M_{i1} = M_{t1}, \tag{2}$$

where X_1 – tangential reaction of the road to the front wheels; X_{m1} – the reaction of the rear of the car to the front wheels; z_1 – the reaction of the road on the front wheels; M_{i1}– moment of inertia of wheel; M_{t1}– front wheel traction moment; P_{i1} – interial force of the front wheels; J_r – inertia moment of the rotor of the electric motor; J_1 – moment of inertia of front wheel; m_1 – mass, applied to the front wheels; j – acceleration; f – rolling resistance factor; r– wheel radius.

Solving Eqs. (1) and (2), equations are obtained [13,16–18].

$$X_{m1} = P_{t1} - z_1 f - jm_1\left(1 + \frac{J_r i_{tr}^2 \eta_{tr} + 2J_1}{m_1 r^2}\right), \tag{3}$$

$$X_1 = P_{t1} - P_{f1} - \frac{j}{r^2}\left(J_r - i_{tr}^2 \eta_{tr} + 2J_1\right), \tag{4}$$

where $\delta_{rm1} = 1 + \frac{J_1}{m_1 r^2}$ – coefficient of accounting of front drive wheel rotating masses [16];

$\delta_{grm1} = 1 + \frac{(J_r i_{tr}^2 \eta_{tr} + 2J_1)}{m_1 r^2}$ – coefficient of accounting of rotating masses, applied to front wheels;

P_{t1} – traction force of front wheels; P_{f1}– force of road resistance of front wheels.

According to the design scheme of interaction of the rear driving wheel of the vehicle with the road (Fig. 2), equations were also obtained without taking into account the air resistance force [13,16]:

$$X_2 = X_{m2} + P_{i2} = X_{m2} + jm_2, \tag{5}$$

$$z_2 a + X_2 r + M_{i2} = M_{t2}, \tag{6}$$

X_2 – the tangential reaction of the road to the rear wheels; X_{m2}– the reaction of the front of the vehicle to the rear wheels; z_2 – the normal reaction of the road to the rear wheel; M_{i2}– the inertia moment of the rear wheels; M_{t2}– the traction moment on the rear wheels; P_{i2} – interial force of the rear wheels; P_{t2} – the traction force of the rear wheels; J_r – the moment of inertia of the electric engine rotor; J_2 – the moment of inertia of the rear wheel; m_2 – the mass, applied on the rear wheels; j – acceleration; f – rolling resistance coefficient; r – wheel radius.

Solving Eqs. (5) and (6), equations [13,16–18] are obtained.

$$X_{m2} = P_{t2} - z_2 f - jm_2\left(1 + \frac{(J_r i_{tr}^2 \eta_{tr} + 2J_2)}{m_2 r^2}\right), \tag{7}$$

$$X_2 = P_{t2} - P_{f2} - \frac{j}{r^2}\left(J_r i_{tr}^2 \eta_{tr} + 2J_2\right), \tag{8}$$

$\delta_{rm2} = 1 + \frac{J_2}{m_2 r^2}$- coefficient of accounting for the rotating masses of the rear driving wheel [16];

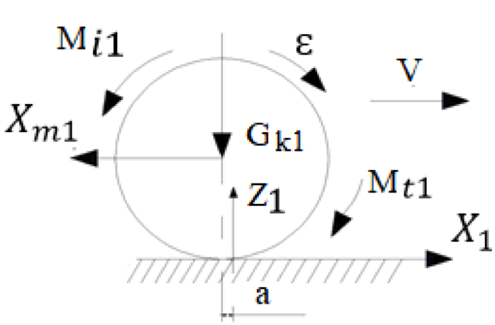

Fig. 1. Interaction of front drive wheel with road.

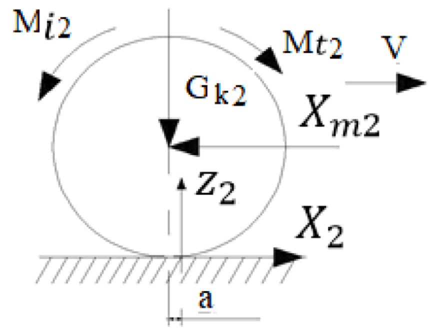

Fig. 2. Interaction of rear drive wheel with road.

$\delta_{grm2} = 1 + \frac{J_r i_{tr}^2 \eta_{tr} + 2J_2}{m_2 r^2}$– the coefficient of accounting for rotating masses, applied to rear wheels; P_{t2} – rear wheel traction force; P_{f2}– force of road resistance of rear wheels.

4. Results and discussion

According to the obtained equations, in relation to the self-propelled wagon ВС−5Э (4 × 4) (Table 1), calculations have been made for the redistribution of its traction forces on the front and rear wheels. In particular, on the basis of the traction calculation data, calculated data on the determination of tangential reactions (tangential traction forces) on the front and rear wheels of a self-propelled vehicle are obtained, which are shown graphically in Figs. 3, 4.

Calculation of the tangential reaction of the road to the front wheels X_1 and rear wheels X_2 according to the formula (4) and (8). According to the obtained design data, the dependence of the tangential reaction of the X_1 road to the front wheels and rear wheels X_2 on speed and acceleration has been built (Figs. 3 and 4).

Analysis of the obtained graphs shows that with an increase in the speed of the vehicle, the tangential forces of the traction of its wheels decrease, and with an increase in acceleration, on the contrary, increase. The analysis shows that with an increase in the speed of the vehicle, the tangential traction forces of the rear wheels X_2 decrease more intensively than X_1. This is confirmed by the fact that at speeds of 1 and 5 km/h, the tangential traction force of the rear wheels (X_2) exceeds X_1 by 7313.5 N and 4428 N, respectively. The increase in acceleration is accompanied, on the contrary, by a more intense increase in the tangential traction force of the rear wheels (X_2) than X_1.

This is also confirmed by the fact that at accelerations of 1.7 and 2.3 m/s^2, the tangential traction force of the rear wheels (X_2) exceeds X_1, respectively, by 7700, N and 6040, N.

Comparative analysis shows that the values of the tangential traction forces on the rear wheels exceed their values on the front wheels of the vehicle. This is due not only to the slight excess of the rear wheel hitch

Table 1
Parameters of self-propelled wagon.

Name of parameters of a self-propelled car ВС5Э	Value
load capacity, N	58800
weight of a self-propelled wagon without cargo with cargo, N	133300
weight of a wagon with cargo on the front axle, N	57700
weight of a wagon with cargo on the rear axle, N	75600
type of electric motor	АИУЕ 225 М−4/8
the moment of inertia of the wheel, kg m^2	19,6
moment of inertia of the motor rotor, kg m^2	0,918
wheel radius, M	0,475
Rolling resistance coefficient	0,06
coefficient of accounting for rotating masses, reduced to the front wheel	1,597
coefficient for accounting for rotating masses, reduced to the rear wheel	1,596

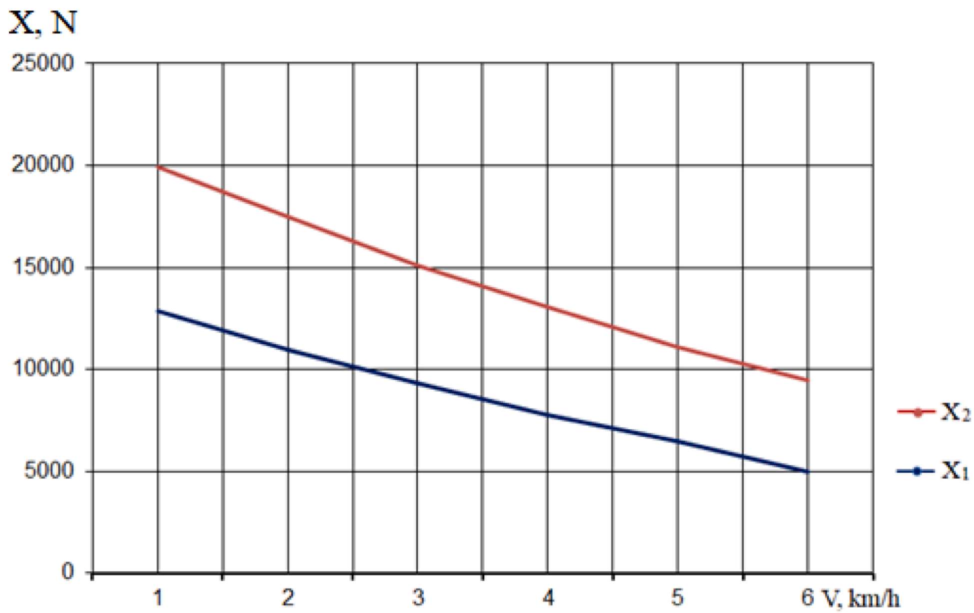

Fig. 3. Dependence of X_1 and X_2 on speed.

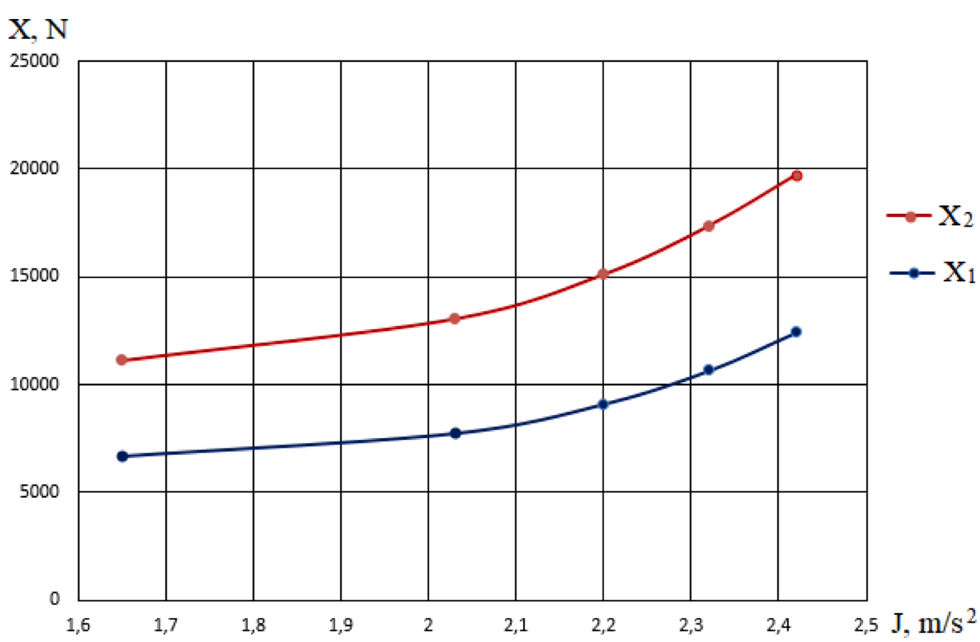

Fig. 4. Dependence of X_1 and X_2 on acceleration.

weight, but also to the excess of the front wheel rolling radius over the rear wheel rolling radius of the vehicle during its driving and acceleration. This is due not only to the slight excess of the rear wheel hitch weight, but also to the excess of the front wheel rolling radius over the rear wheel rolling radius of the vehicle during its driving and acceleration. The difference between the tangential traction forces on the rear and front wheels decreases as the speed of the vehicle increases. The difference between the tangential traction forces on the rear and front wheels increases as the acceleration of the vehicle increases.

The possibility of ensuring the stability of the vehicle when implementing longitudinal or lateral forces on its front and rear wheels mainly depends on the road condition and the load on the wheels of the vehicle. In this case, the conditions for the movement of the vehicle are

$$X_{1max} \leq P_{adh1}, \quad P_{adh1} = \varphi G_{k1},$$

$$X_{2max} \leq P_{adh2}, \quad P_{adh2} = \varphi G_{k2}$$

where P_{adh1}– traction force of the front wheels with the road, N; φ – traction coefficient; G_{k1}– weight falling on the front wheels, N; P_{adh2}– traction force of the rear wheels with the road, N; G_{k2}– weight falling on the rear wheels, N.

Analysis of the dependence of the tangential traction forces of the modular front and rear wheels of the vehicle on the speed of its movement shows, that with an increase in the speed of movement, they decrease, since with the increase in the speed of the vehicle, the traction forces on the wheels decrease, and in the case of their dependence on acceleration, the pattern is the opposite, i.e., with an increase in acceleration of the vehicle, the tangential traction forces on the wheels, on the contrary, increase. We will also explain this nature of dependencies, since it is known that with an increase in the speed of the vehicle, its acceleration decreases. As for compliance of tangential traction forces on the wheels of the vehicle with the condition of wheels adhesion to the road, when the vehicle moves along a dirt road with parameters C = 15 N/sm^2 (internal adhesion) and ϕ=15° (internal friction angle), the average adhesion coefficient will be equal to 0.55 and the adhesion forces of the front and rear wheels will be 31,735 N and 41,580 N, respectively.

Comparison of the adhesive forces with the graphical data shows that X_1 and X_2 are significantly less than the corresponding adhesive forces. Therefore, the results of the comparison show that the self-propelled wagon BC−5Э (4 × 4) is able to vehiclery out efficient and stable movement in these road conditions. The results of the calculations are sufficiently consistent with the results of the practice of operating 5-ton self-propelled wagon of the BC type.

Based on the obtained methods of calculating the indicators of redistribution of traction forces, it is possible to determine the coefficients of both the slippage of the wheels of the vehicle (4 × 4) and its insufficient rotation for automatic control systems both in operations [19–23,20,24]. At the same time, a promising direction in this area is the development of energy-efficient control algorithms as in the works [25–28].

5. Conclusions

1. A review and analysis of the redistribution of traction forces on the wheels of four-wheel drive wheeled vehicles in various industries and the need to take them into account when assessing the stability and efficiency of vehicles has been vehicleried out;
2. The initial prerequisites for assessing the redistribution of traction forces on the front and rear driving wheels of the vehicle separately have been substantiated;
3. It is substantiated that the front wheels of vehicles with a wheel arrangement (4 × 4), when starting during acceleration, are in relation to the rear laggards (if the weight states of the vehicle axles and the air pressures in the tires are equal);
4. Equations have been substantiated for calculation of traction redistribution indicators on modular front and rear wheels of a vehicle (4 × 4), which can be recommended for use both in operation and in design of vehicles.
5. The novelty is that the front and rear driving wheels are proposed to be considered separately independently of each other as modular, with all parameters reduced to the equation to each modular wheel separately;
6. Also, calculations of tangential traction forces on modular front and rear wheels of low-speed self-propelled wagon BC5Э (4 × 4) have been vehicleried out according to the presented equations. Calculations show that the difference between the tangential traction forces on the rear and front wheels decreases as the speed of the vehicle increases. The difference between the tangential traction forces on the rear and front wheels increases as the acceleration of the vehicle increases.
7. Analysis of the obtained design tangential traction forces on modular front and rear wheels has been vehicleried out and their comparison with the obtained hitch forces of the wheels of the self-propelled wagon BC5Э (4 × 4), which showed that this vehicle is capable of efficient and stable movement in these road conditions, which does not contradict the practical data of operation of 5-ton self-propelled vehicles of the BC type (4 × 4).
8. The methods proposed in the work for calculating the indicators of redistribution of traction forces on modular front and rear wheels of a vehicle (4 × 4) will allow for a more accurate assessment of its stability and passability, as well as efficiency.
9. The obtained methods of calculation of traction redistribution indicators can be used as the basis of algorithms for optimizing the coefficients of both the slippage of the vehicle wheels (4 × 4) and its insufficient rotation for automatic control systems.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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