

The reaction of the Earth's surface to twisting and normal loads moving through the tunnel

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ABSTRACT: The problem of the action on the surface of a circular cylindrical cavity located in an elastic half-space (array) uniformly moving along its forming normal and twisting loads is solved. Dynamic equations of elasticity theory are used to describe the motion of the array. On the basis of the obtained solution, the stress-strain state of the Earth's surface is investigated under the action of axisymmetric loads of this type uniformly moving along an unsupported tunnel of shallow laying.

1 INTRODUCTION

The stress-strain state (SSS) of the earth's surface under the action of a twisting load uniformly moving along an unsupported tunnel is investigated in the article [1]. In this paper, a similar study was carried out in the case when, in addition to the twisting load, a normal load moving at the same speed acts on the tunnel. Such an impact occurs, for example, when cleaning devices are operating in a circular tunnel.

2 MATERIALS AND METHODS

The research uses the method of mathematical modeling involving models of the theory of elasticity.

Formulation and analytical solution of the problem.

For research, we will present the design scheme of the tunnel in the form of an extended circular cylindrical cavity with radius R , located in a linearly elastic, homogeneous and isotropic half-space (array), assigned to fixed Cartesian x, y, z and cylindrical r, θ, z coordinate systems, the z axis of which coincides with the axis of the cavity and is parallel to the load-free horizontal boundary of the half-space (Earth's surface), the x – axis – perpendicular to this boundary: $x \leq h$ (Figure 1), where h – is the distance from the axis of the cavity to the boundary of the half-space (earth's surface). The physical and mechanical properties of the array are characterized by the following constants: ν – Poisson's ratio, μ – shear modulus, ρ – density.

In the direction of the z axis, the normal $P_r(\theta, \eta)$ and twisting $P_\theta(\theta, \eta)$ loads move along the surface of the cavity with the same constant velocity c

$$\sigma_{rr}|_{r=R} = P_r(\theta, \eta), \quad \sigma_{r\theta}|_{r=R} = P_\theta(\theta, \eta), \quad \sigma_{r\eta}|_{r=R} = 0. \quad (1)$$

where $r, \theta, \eta = z - ct$ – movable cylindrical coordinate system, σ_{ij} – components of the stress tensor in the array, $j = r, \theta, \eta$.

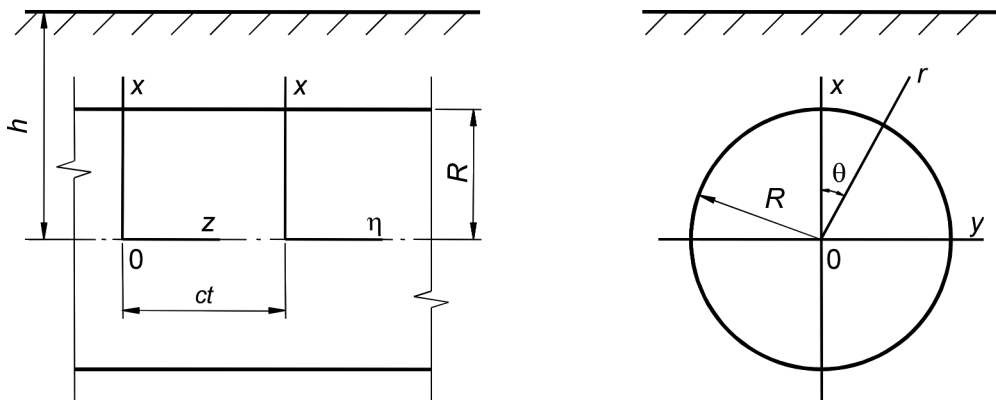


Figure 1. Design diagram of an unsupported tunnel.

Since the boundary of the half-space is free of loads, then, for $x = h$

$$\sigma_{xx} = \sigma_{xy} = \sigma_{x\eta} = 0 \quad (2)$$

To describe the motion of the half-space, we use the dynamic equations of the theory of elasticity in a moving coordinate system [2]

$$\left(M_p^{-2} - M_s^{-2}\right) \text{grad div} \mathbf{u} + M_s^{-2} \nabla^2 \mathbf{u} = \partial^2 \mathbf{u} / \partial \eta^2, \quad (3)$$

where \mathbf{u} – displacement vector of elastic medium; $M_p = c/c_p$, $M_s = c/c_s$ – Mach numbers, $c_p = \sqrt{(\lambda + 2\mu)/\rho}$, $c_s = \sqrt{\mu/\rho}$ – velocities of expansion-compression and shear waves in the medium, $\lambda = 2\mu\nu/(1 - 2\nu)$, ∇^2 – Laplace operator.

We transform equation (3) by expressing \mathbf{u} in terms of Lamé potentials [3]

$$\mathbf{u} = \text{grad } \varphi_1 + \text{rot}(\varphi_2 \mathbf{e}_\eta) + \text{rot rot}(\varphi_3 \mathbf{e}_\eta), \quad (4)$$

where \mathbf{e}_η – ort of the η axis.

It follows from (3) and (4) that the Lamé potentials φ_j satisfy the equations

$$\nabla^2 \varphi_j = M_j^2 \frac{\partial^2 \varphi_j}{\partial \eta^2}, \quad j = 1, 2, 3,$$

where $M_1 = M_p$, $M_2 = M_3 = M_s$.

Applying the Fourier transform with respect to η to the last equations, we obtain

$$\nabla_2^2 \varphi_j^* - m_j^2 \xi^2 \varphi_j^* = 0, \quad j = 1, 2, 3, \quad (5)$$

where ∇_2^2 – two-dimensional Laplace operator, $m_j^2 = 1 - M_j^2$, $m_1 \equiv m_p$, $m_2 = m_3 \equiv m_s$, $\varphi_j^*(r, \theta, \xi) = \int_{-\infty}^{\infty} \varphi_j(r, \theta, \eta) e^{-i\xi\eta} d\eta$.

By expressing the components of the stress-strain state of the medium through the Lamé potentials and applying the Fourier transform according to the η , it is possible to obtain expressions for stress σ_{lm}^* and displacement u_l^* transformants in cylindrical ($l, m = r, \theta, \eta$) and Cartesian ($l, m = x, y, \eta$) moving coordinate systems as a function of φ_j^* . To determine the stress-strain state of the array, you need to find φ_j^* .

If we limit ourselves to the case when the speed of movement of loads is less than the speed of propagation of shear waves in the array ($c < c_s$), then $M_s < 1$ ($m_s > 0$) and the solutions of equations (5) can be represented as follows [2]

$$\varphi_j^* = \Phi_j^{(1)} + \Phi_j^{(2)}, \quad (6)$$

where $\Phi_j^{(1)} = \sum_{n=-\infty}^{\infty} a_{nj} K_n(k_j r) e^{in\theta}$,

$$\Phi_j^{(2)} = \int_{-\infty}^{\infty} g_j(\xi, \zeta) \exp\left[iy\zeta + (x-h)\sqrt{\zeta^2 + k_j^2}\right] d\zeta.$$

Here $K_n(k_j r)$ – MacDonald functions, $k_j = m_j \xi$; $g_j(\xi, \zeta)$, a_{nj} – unknown, to be determined, functions and coefficients, $j = 1, 2, 3$.

It follows from [2] that when representing potentials in the form (6), expressions for transformants of potentials in the Cartesian coordinate system have the form

$$\varphi_j^* = \int_{-\infty}^{\infty} \left[\frac{e^{-xf_j}}{2f_j} \sum_{n=-\infty}^{\infty} a_{nj} \Phi_{nj} + g_j(\xi, \zeta) e^{(x-h)f_j} \right] e^{iy\zeta} d\zeta, \quad (7)$$

where $f_j = \sqrt{\zeta^2 + k_j^2}$, $\Phi_{nj} = \left(\frac{\zeta + f_j}{k_j}\right)^n$, $j = 1, 2, 3$.

The functions $g_j(\xi, \zeta)$ are expressed in terms of coefficients a_{nj} from the boundary conditions (2), rewritten for voltage transformers taking into account (7). To do this, it is necessary to allocate the coefficients at $e^{iy\zeta}$ and equate them, due to the arbitrariness of y to zero. Then:

$$g_j(\xi, \zeta) = \sum_{k=1}^3 \frac{\Delta_{jk}}{\Delta} e^{-hf_k} \sum_{n=-\infty}^{\infty} a_{nk} \Phi_{nk}, \quad j = 1, 2, 3. \quad (8)$$

Here $\Delta = (2\rho_0^2 - \beta^2)^2 - 4\rho_0^2 \sqrt{\rho_0^2 - \alpha^2} \sqrt{\rho_0^2 - \beta^2}$, $\alpha = M_p \xi$, $\beta = M_s \xi$, $\rho_0^2 = \xi^2 + \zeta^2$. The type of algebraic complements Δ_{jk} is defined in [2].

It should be noted that $\Delta(\rho_0)$ – is the Rayleigh determinant, which, as is well known, has two real roots: $\pm\rho_R = \pm\xi M_R$. Here $M_R = c/c_R$ – Mach number, c_R – Rayleigh surface wave velocity ($c_R < c_s$) in an elastic half-space. Since the corresponding $\pm\zeta_R = \pm\xi \sqrt{M_R^2 - 1}$, Z , then $\Delta(\rho_0)$ does not turn to zero on the real axis if $M_R < 1$, or $c < c_R$ (Dorelean speeds of movement of loads). In this case, all integral functions in (6) are continuous and tend to zero rather quickly at infinity. Therefore, integrals exist and satisfy the conditions of attenuation at infinity. To calculate integrals (7), you can use one of the numerical integration methods, having previously determined the coefficients a_{nj} .

For $c < c_R$, taking into account (8), the relations (7) will take the form

$$\varphi_j^* = \int_{-\infty}^{\infty} \left[\frac{e^{-xf_j}}{2f_j} \sum_{n=-\infty}^{\infty} a_{nj} \Phi_{nj} + e^{(x-h)f_j} \sum_{k=1}^3 \frac{\Delta_{jk}}{\Delta} e^{-hf_k} \sum_{n=-\infty}^{\infty} a_{nk} \Phi_{nk} \right] e^{iy\zeta} d\zeta. \quad (9)$$

Using the known decomposition $e^{ikr \cos \theta} = \sum_{n=-\infty}^{\infty} i^n J_n(kr) e^{in\theta}$, can get $\exp\left[iy\zeta + (x-h)\sqrt{\zeta^2 + k^2}\right] = \sum_{n=-\infty}^{\infty} I_n(kr) e^{in\theta} \left(\frac{\zeta + \sqrt{\zeta^2 + k^2}}{k}\right)^n e^{-h\sqrt{\zeta^2 + k^2}}$ and, using (8), represent (6) in a cylindrical coordinate system:

$$\phi_j^* = \sum_{n=-\infty}^{\infty} [a_{nj} K_n(k_j r) + b_{nj} I_n(k_j r)] e^{in\theta},$$

where $b_{nj} = \sum_{k=1}^3 \sum_{m=-\infty}^{\infty} a_{mk} A_{nj}^{mk}$, $A_{nj}^{mk} = \int_{-\infty}^{\infty} \frac{\Delta_{jk}}{\Delta} \Phi_{mk} \Phi_{nj} e^{-h(f_k + f_j)} d\zeta$.

The coefficients $a_{nj} Z$ are found from the boundary conditions (1), rewritten as

$$\sigma_{rr}^*|_{r=R} = P_r^*(\theta, \zeta), \dots, \sigma_{r\theta}^*|_{r=R} = P_\theta^*(\theta, \zeta), \quad \sigma_{r\eta}^*|_{r=R} = 0,$$

where $P_j^*(\theta, \zeta) = p_j(\theta)p_j^*(\zeta)$, $p_j(\theta) = \sum_{n=-\infty}^{\infty} P_{nj}e^{in\theta}$, $p_j^*(\zeta) = \int_{-\infty}^{\infty} p_j(\eta)e^{-i\zeta\eta}d\eta$, $j = r, \theta$.

When the coefficients of the Fourier-Bessel series are equated at $e^{in\theta}$, an infinite system of linear algebraic equations with a determinant of normal type is obtained, for the solution of which the reduction method or the method of successive reflections (approximations) can be used [2]. Moreover, as calculations show, the determinant of this system can turn to zero only when $c \geq c_R$.

After determining the coefficients a_{nj} , applying the inverse Fourier transform to the displacement and stress transformants, it is possible to calculate the components of the stress-strain state of the array in cylindrical and Cartesian coordinate systems.

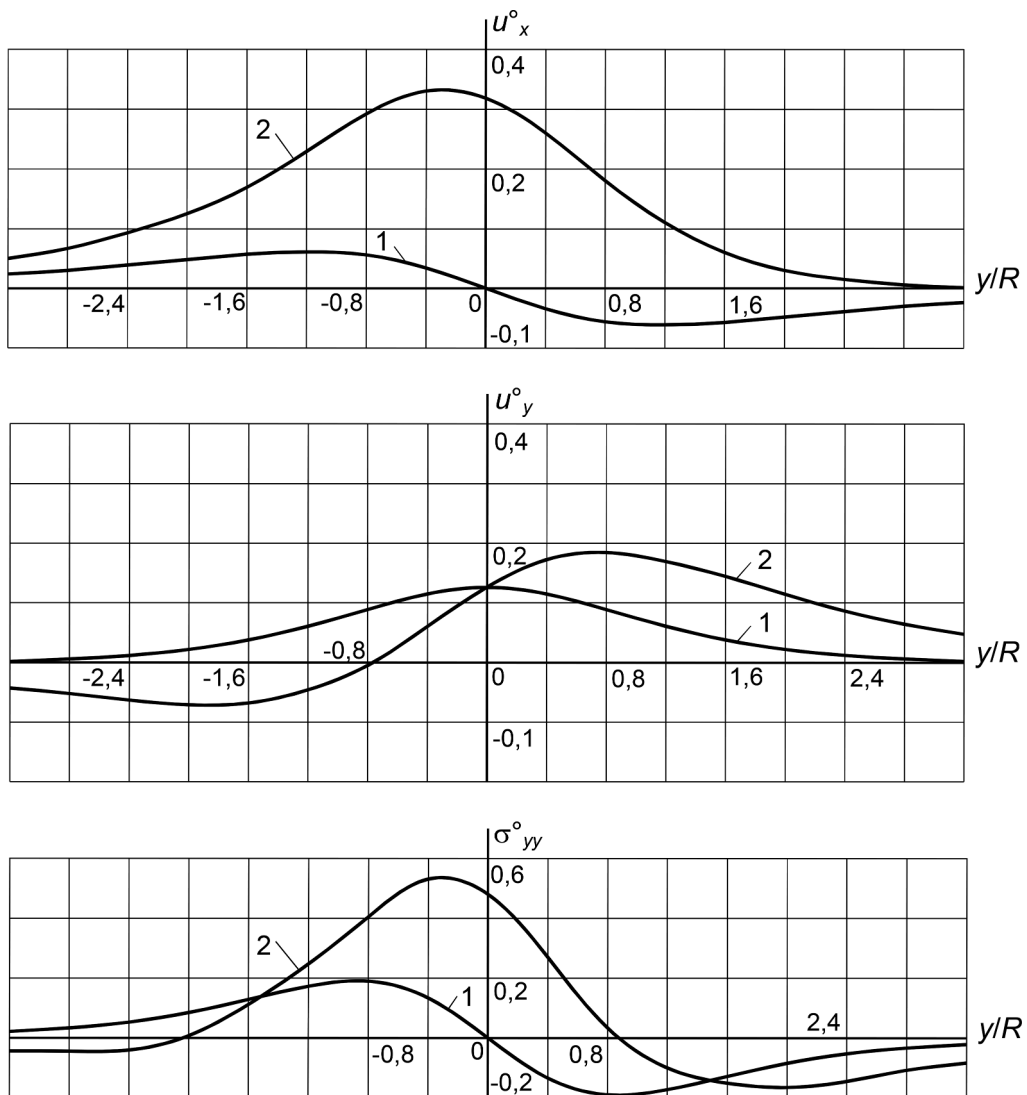


Figure 2. Change u_x^o (m), u_y^o (m) and σ_{yy}^o on the Earth's surface in the coordinate plane xy ($\eta=0$).

3 RESULTS AND DISCUSSION

To study the dynamic effect of the loads discussed above on the stress-strain state of the Earth's surface, consider a circular cylindrical tunnel with a radius of $R = 1$ m and a depth of laying $h = 2R$ in a siltstone massif ($\nu = 0,2$, $\mu = 2,532 \cdot 10^9$ Pa, $\rho = 2,5 \cdot 10^3$ kg/m³, $c_p = 1643,4$ m/c, $c_r = 1643,4$ m/c, $c_R = 917$ m/c).

Axisymmetric compressive normal P_r and twisting P_θ loads of intensity q (Pa), uniformly distributed over the tunnel surface in the interval $|\eta| \leq l_0 = 0,2R$ $Z=G$, move in the direction of the z Y axis with a constant Dornel velocity $c = 100$ m/c. The intensity q of each of the loads is selected in such a way that the total load along the entire length of the loading section $2l_0$ is equal to the equivalent concentrated ring load intensity $P^{\circ\circ}$ (N/m), that is, $q = P^{\circ\circ}/2l_0$.

Let 's introduce the notation: $u_x^{\circ} = u_x \mu / P^{\circ}$ (M), $u_y^{\circ} = u_y \mu / P^{\circ}$ (M), $\sigma_{yy}^{\circ} = \sigma_{yy} / P^{\circ}$, where $P^{\circ} = P^{\circ\circ}/m$ (Pa).

Figure 2 shows the curves of change u_x° (M), u_y° (M) and σ_{yy}° on the Earth's surface in the coordinate plane xy ($\eta=0$). Curves 1 correspond to the action of a twisting moving load only on the tunnel, curves 2 correspond to the joint action of moving twisting and compressing normal loads.

As follows from the analysis of the behavior of curves, with the combined action of twisting and compressing normal moving loads on the tunnel, the maximum displacements and stresses on the earth's surface are significantly greater than with the action of twisting loads alone. Therefore, in the case of calculating the stress-strain state of the Earth's surface on the load from the work of cleaning devices moving along the axis of the tunnel and rotating relative to it, it is possible to represent their impact on the tunnel as a moving twisting load only when the pressure exerted on its surface is much less than the intensity of the twisting load.

4 CONCLUSION

In a strict mathematical formulation, the problem of the action of normal and twisting loads uniformly moving along its generatrix on the surface of a circular cylindrical cavity located in an elastic half-space is solved. The solution obtained for the Dorelev speeds of the movement of loads and the software package developed on its basis allows using mathematical modeling methods to study the dynamics of the rock mass under the influence of these loads at different depths of the tunnel laying.

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