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Discrete computational models and stress-strain state of a high-class mechanism taking into account friction forces in kinematic pairs under dynamic loading

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ABSTRACT: Quasi-static problems with respect to the stress-strain state and a class of problems with respect to forced vibrations acting in plain and spatial nodes of a high-class mechanism with elastic links are considered, taking into account the model of friction forces in kinematic pairs under the action of vertical and horizontal concentrated forces in kinematic pairs. The developed algorithms of solving quasi-static problems by the finite element method for the entire system of the elastic mechanism with the friction force in kinematic pairs are presented; the basic resolving equilibrium equations for the entire system are drawn up without and taking into account friction forces. The basic relations of the finite element method for plain and spatial nodes of high-class elastic mechanisms with the friction force in kinematic pairs under dynamic loading are presented.

1 INTRODUCTION

The finite element method is better than other methods provided with numerical procedures for studying the mathematical model of an object. Its most important advantage is the presence of implicit unconditionally stable methods of the numerical integration of systems of differential equations of motion that describe the motion of mechanisms and are compiled taking into account the links elasticity.

2 RESEARCH METHOD

For the finite element method in the variant of the displacement method when solving problems of structural mechanics and the theory of elasticity, internal small displacements, velocities, and accelerations are used as unknown values. From the point of view of convergence, when solving this class of problems, there are imposed certain requirements (Zenkevich O. 1975) on the functions of the finite element shape, the most important of which in the case under consideration is the following: it has realized and has not accumulated the energy of elastic deformations.

Modeling the kinematic pairs of a plain and spatial elastic high-class mechanism (HCM) with allowance for Coulomb friction is shown in Figure 1 (Volmir A.S. et al. 1989).

$$\bar{F}_{mp} = -\bar{R}_{jX}, F_{mp} = fN_{Д}\ signR_{jX}, N_{Д} = R_{jY} \tag{1}$$

If a translational pair in a flat mechanism (Figure 1) moves along a fixed guide, then the sliding friction $\bar{F}_{mp}$ force will be directed oppositely to the horizontal component $\bar{R}_{jx}$ of the complete reaction.

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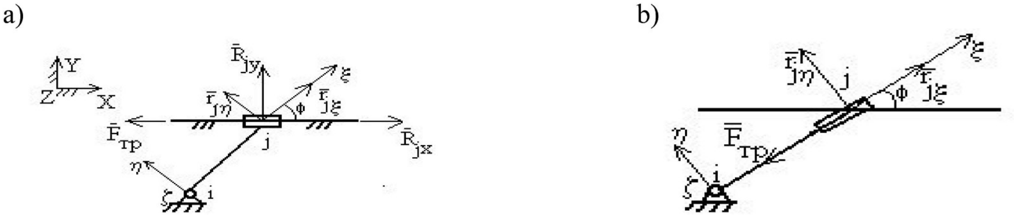

Figure 1. The friction forces in the translational kinematic pair of the plain mechanism.

The R_{jx}, R_{jy} nodes of the complete reaction are determined through the reactions in the $r_{j\xi}, r_{j\eta}$ local coordinate system (LCS):

$$\begin{bmatrix} R_{jX} \\ R_{jY} \\ M_{jZ} \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_{j\xi} \\ r_{j\eta} \\ m_{j\zeta} \end{bmatrix} \tag{2}$$

In turn, reactions in the LCS are expressed through the unknown internal forces N, Q_η, M_ζ at the nodes in such a way:

$$\begin{bmatrix} r_{i\xi} \\ r_{i\eta} \\ r_{i\zeta} \\ r_{j\xi} \\ r_{j\eta} \\ r_{j\zeta} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & l & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} N \\ Q_\eta \\ M_\zeta \end{bmatrix} \tag{3}$$

When studying the dynamics of hinge mechanisms taking into account the friction forces, the reaction $\bar{r}_{j\eta}$ is considered passing through the center of the pivot; in kinematic pairs, the frictional moment is also taken $m_{j\zeta}^{mp}$ into account. In the presence of a rotational $m_{j\zeta}$ pair in a plain mechanism (Figure 2), the frictional moment is directed opposite to the internal moment. It is $\bar{F}_{mp}$ assumed that the normal pressure forces are concentrated at point A and the friction force is applied at the same point. Then:

$$m_{j\zeta}^{mp} = -m_{j\zeta} = -F_{mp} \cdot r = -frN_{Д} = -fr\sqrt{r_{j\zeta}^2 + r_{j\eta}^2} \;\; N_{Д} = \sqrt{r_{j\zeta}^2 + r_{j\eta}^2} \tag{4}$$

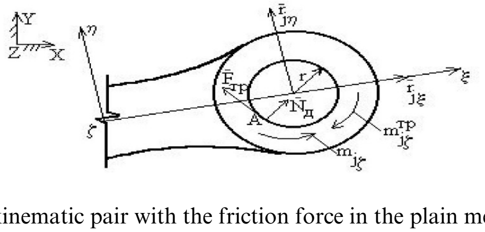

Figure 2. The rotational kinematic pair with the friction force in the plain mechanism.

Figure 3 conventionally shows the slider of the spatial translational pair of the fifth class, which can move along the axis. In such a kinematic pair, it is assumed that the contacts of the links occur in two planes.

Sliding friction forces arise in these planes. The friction force F_{mp1} is proportional to the second component of the reaction in the LCS, and the friction force F_{mp2} in the side face is proportional to the third component of the reaction in the LCS:

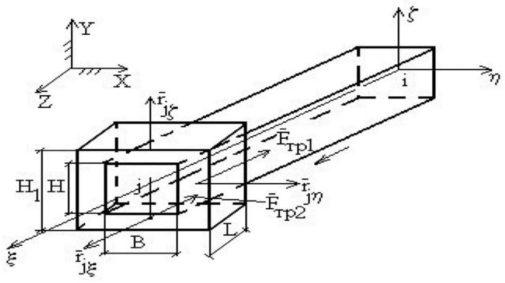

Figure 3. Translational kinematic pair with a frictional force in the spatial mechanism.
B – the slider contact width, L – its length, H1 – its height, H – is the slider contact height

$$F_{mp1} = fr_{j\eta} \cdot signr_{j\eta}, \overline{F}_{mp2} = -\bar{r}_{j\xi}, F_{mp2} = fr_{j\zeta} \cdot signr_{j\zeta}, F_{mp} = F_{mp1} + F_{mp2}. \quad (5)$$

In turn, reactions in the LCS are expressed through the unknown internal forces $N_j, Q_{j\eta}, Q_{j\zeta}, M_j, M_{j\eta}, M_{j\zeta}$ are the nodes and are presented in the form:

$$\{\bar{r}\} = [d_{r,s}] \cdot \{N\} \quad (6)$$

Where $\{\bar{r}\}^T = (r_{i\xi}, r_{i\eta}, r_{i\zeta}, \ldots, m_{j\zeta})$, $\{\bar{N}\}^T = \{N_j, Q_{yj}, Q_{zj}, \ldots, M_{z\zeta}\}$ $a_{kk} = -a_{k+6,k} = -1$; $a_{5,3} = a_{6,2} = l$; the rest elements are equal to zero.

In the spatial model of a rotational kinematic pair, it is assumed (Figure 4) that contacts occur in the end planes of the hinge at two different points A and B.

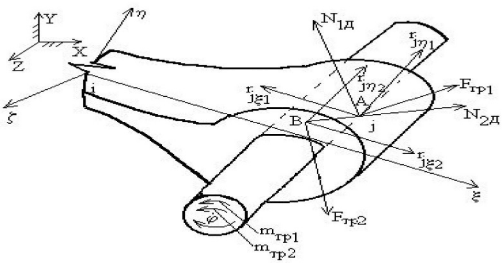

Figure 4. Rotational kinematic pair with a frictional force in a spatial mechanism.

Reactions in the LCS are $r_{j\xi1}, r_{j\xi2}, r_{j\eta1}, r_{j\eta2}$ determined through the internal forces by formula (6). The friction forces moments are determined by the following formulas:

$$m_{mp1} = -F_{mp1} \cdot r = -fr|N_{1Д}| = -fr\sqrt{r^2_{j\xi_1} + r^2_{j\eta_1}}, \quad (7)$$

$$m_{mp2} = -F_{mp2} \cdot r = -fr|N_{2Д}| = -fr\sqrt{r^2_{j\xi2} + r^2_{j\eta2}}, \quad (8)$$

$N_{1Д}, N_{2Д}$ where are the normal pressure forces; r is the pivot radius. Reactions *are* determined $r_{j\xi1}, r_{j\xi2}, r_{j\eta1}, r_{j\eta2}$ by formula (6).

Based on the finite element method and analysis of the stress-strain state of plain and spatial structures based on the HCM with elastic links and friction forces in kinematic pairs, elastic displacements, internal forces in the elements of the rods are determined with and without regard to friction forces.

The relationship equations between the vectors of nodal forces $\{f_\Lambda^e\}$ and displacements are $\{\delta_\Lambda^e\}$ represented in the form:

$$\{f_\Lambda^e\} = [K_\Lambda^e] \cdot \{\delta_\Lambda^e\} \tag{9}$$

where $[K_\Lambda^e]$ is the symmetrical square matrix of rigidity of the 12x12 FRE (finite rod element) order. It is called the matrix of rigidity of a spatial rod element:

$$[K_\Lambda^e] = [K_{ij}],\ (i,j = 1, 2,\ \ldots, 12) \tag{10}$$

The elements of the matrix of rigidity of the FRE (finite rod element) connecting the force and kinematic parameters at the nodes of a given FRE are determined through its geometric parameters of the cross section, elastic characteristics and length (Postnov et al., 1979, Archer 1965).

If there is a hinge at nodes *i* and *j* of the design element "e", bending moments M_i or M_j in them are equal to zero. In this case, the equations for linking the forces at the nodes with the displacements of the nodes are transformed accordingly. If the node *i* has a hinge, then the corresponding row of the element matrix of rigidity is excluded in the usual way.

If the spatial structure consists of *m* rod elements, then the basic system of equilibrium equations is written as follows:

$$\{F_k\}^T = (F_1, \ldots, F_m), \tag{11}$$

where $\{F_k\}^T = (F_k, F_{yk}, F_{zk}, M_{xk}, M_{yk}, M_{zk}),\ k = 1, 2, \ldots, m$ is the vector of external forces applied to the k-th node in the global coordinate system (GCS) OXYZ, m is the total number of nodes.

To determine displacements, angles of rotation and forces in the system of *m* rod elements that make up the structure, it is necessary to satisfy the deformation compatibility conditions and equilibrium equations.

The kinematic characteristics at the nodes of each element automatically satisfy the first condition. Therefore, it is enough to satisfy the equilibrium conditions at the nodes of the structures. Essentially, satisfying these conditions at all nodal *m* points, we construct the main resolving system of linear algebraic equations for the components of the nodes displacements and angles of rotation in the form:

$$\{F\} = [K]\{U\}, \tag{12}$$

where $[K] = [K_{rs}]$, $(r, s = 1, 2, \ldots, 6m)$ is the square matrix of the 6m*6m order; it is called the system matrix of rigidity (SMR);

$$\{U\}^T = (u_1, \vartheta_1, w_1, \phi_{x1}, \phi_{y1}, \phi_{z1}, \ldots, u_m, \vartheta_m, \ldots, \phi_{zm}),\ \{F\}^T = (N, Q_y, Q_z, M_x, M_y, M_z)$$

Thus, the SMR (the system matrix rigidity) is formed from the rigidity matrices of all m rod elements of the mechanism defined in the GCS. Taking into account the boundary conditions, the components of displacements and deformations of the nodes of the mechanism are found by solving system of equations (17) by the iterative Gauss-Seidel method. This method is easy to program, each equation is iterated, and the determined value is refined. The process is repeated as many times as needed to obtain the acceptable solution accuracy.

Accounting for the boundary conditions in the GCS in the presence of friction forces in kinematic pairs is carried out by excluding the corresponding degrees of freedom. This is done using the procedure of calculating the number of zeros in the given matrix ID, the degree of freedom of nodes IID, the boundary condition of internal force factors, and replacing them with the ordinal numbers of the corresponding degrees of freedom, and replacing units with zeros. When frictional forces act on a structure, corresponding bending moments are generated in rotational kinematic pairs. To study the impact of forces and moments of friction forces of translational and rotational pairs at a certain quasi-static position of the structure, the values of forces and moments of friction are calculated using the determined internal forces at the nodes according to the corresponding formulas [4].

Solutions by the finite element method for plain and spatial dynamic problems on the stress-strain state of structures based on the HCM with elastic links and friction forces in kinematic pairs at their different positions and subject to external dynamic loads.

Methods of accounting for friction forces in various kinematic pairs of plain and spatial elastic HCMs are considered when obtaining the basic systems of differential equations of motion relative to the kinematic parameters of all the structural units for the development of computational algorithms and quantitative analysis of their state at different fixed positions, taking into account the corresponding boundary, initial conditions and specified external variable forces.

The system of equations for the dynamic equilibrium of a set of finite rod elements, with the help of which the considered structure is discretized at the t moment with the mechanism position under study has the form:

$$[M]\{\ddot{U}\} + [C] \cdot \{\dot{U}\} + [K] \cdot \{U\} = \{F(t)\} \tag{13}$$

where $[M]$, $[C]$, $[K]$ are respectively the matrices of the system mass, damping, and rigidity that are obtained by summing up the corresponding matrices of all the computed rod elements; $\{\ddot{U}\}, \{\dot{U}\}, \{U\}$ are vectors of accelerations, velocities and displacements of the nodes with the use of which the computed rood structures are divided into finite two-node rod elements; $\{F(t)\}$ is the given vector of the external variable forces acting at the nodes.

In the calculations of the dynamic stress state of plain and spatial elastic HCMs taking into account the friction forces in kinematic pairs, the matrix of the system damping $[C]$ according to Rayleigh is taken in the form of the combination of the mass matrix $[M]$ and system rigidity $[K]$, i.e. $[C] = \alpha[M] + \beta[K]$; the α and β constants are determined from the values of the damping coefficients related to the two lowest frequencies.

When studying and analyzing the stress state of plain and spatial elastic HCMs taking into account the friction forces in kinematic pairs from the action of external variable forces, system of differential equations (18) was solved by the convenient absolutely stable stepwise method of direct integration according to the Newmark scheme with the step $\Delta t = 0,01$ (Hack & Becker 1999, Harlecki Andrzej. 1999).

3 CONCLUSION

In accordance with the theoretical solution of the quantitative and qualitative analysis of the dynamic stress state of plain elastic HCMs of II, IV classes and the spatial mechanism of VShD-8, taking into account the friction forces in kinematic pairs, algorithms have been developed and object-oriented software packages have been compiled in the Fortran high-level algorithmic language.

In the numerical calculations of the forced vibrations and the dynamic stress state, when the concentrated variable force $F = F_0 \sin \omega t$ acts on them, the dynamic analysis of the stress state of plane elastic HCMs of II, IV class was first carried out taking into account the friction forces in kinematic pairs at different static positions (Chernousko 2000, Massanov, et al. 1994). The elastic physical and geometric characteristics of the link materials are the same as when considering the static problems.

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