Smart Geotechnics for Smart Societies – Zhussupbekov, Sarsembayeva & Kaliakin (Eds)

Studying the state of stress parameters of stress intensity factors in reinforced concrete bending elements

Zh.S. Nuguzhinov
KazMIRR Institute of the NJC "KarTU named after Abylkas Saginov", Karaganda, Kazakhstan

M.Zh. Bakirov
NJC "KarTU named after Abylkas Saginov", Karaganda, Kazakhstan

D.T. Tokanov, O. Habidolda & I.A. Kurokhtina
KazMIRR Institute of the NJC "KarTU named after Abylkas Saginov", Karaganda, Kazakhstan

A.T. Mukhamejanova & D.K. Zhilkibayev
Eurasian National University, Nur-Sultan, Kazakhstan

ABSTRACT: The work determines the stress intensity factor (SIF) at the crack tip in bending reinforced concrete beams. The problem has been solved in a three-dimensional formulation using special elements that take into account peculiarities of the stress field at the crack tip.

Calculations have been carried out for beams of rectangular cross-section with pure bending to obtain analytical calculated dependences on variable factors. To obtain analytically calculated dependences on variable factors, a matrix of rational planning a multifactorial computer experiment has been compiled to determine the SIF and stress state parameters in bending rectangular reinforced concrete beams with a crack. Regression dependences for the SIF and the stress in the armature on the parameters of the beam and loading are obtained. The numerical calculation method is general and applies to beams of an arbitrary section under arbitrary loading. The obtained dependencies make it possible to evaluate the crack resistance of reinforced concrete beams according to the force criterion of fracture mechanics without the use of computer technology and expensive software complexes. In this paper, there is considered determining the SIF in bending reinforced concrete beams by the finite element method. The calculation results are presented in the form of the SIF regression dependences on the varied parameters.

1 INTRODUCTION

Cracks are a very common type of defect in reinforced concrete structures [1, 2]. They can appear both at the manufacturing stage and the stage of mounting and operation. Operational cracks appear in the tensile zone of concrete when the maximum stress exceeds the ultimate strength of concrete. Since concrete is a brittle material and has lower tensile strength than compressive strength, cracks appear even before the bearing capacity of the reinforced concrete element is exhausted.

It is necessary to determine the stress state in the section with a crack to assess the bearing capacity of a reinforced concrete element. To assess crack resistance, it is necessary to determine the parameters of fracture mechanics that correspond to the accepted fracture criterion. The brittle fracture force criterion is applied for concrete, expressed through the stress intensity factor (SIF). When bending, it is necessary to determine the SIF of the first type (a separation crack). It is necessary to know the nominal stresses at the crack tip to determine it.

In the general case, the calculation of the SIF is performed by the finite element method (FEM) in a linear formulation. Such a calculation for bending reinforced concrete elements is

DOI: 10.1201/9781003299127-360

rarely performed. The one is due to the complexity of the calculations associated with the above-mentioned character of crack development and the need for a volumetric formulation of the problem.

Since cracks exist in any reinforced concrete elements, determining the stress state in the section with a crack and calculating the SIF are very important for assessing the real state of the structure in operation.

In this paper, there is considered determining the SIF in bending reinforced concrete beams by the finite element method. The calculation results are presented in the form of the SIF regression dependences on the varied parameters.

2 METHODS

The finite element method is the main method of determining the parameters of fracture mechanics in metal structures. Because of the complexity of implementing the algorithm of determining the SIF in reinforced concrete structures, it is rarely used and has its characteristics. This calculation is carried out in a linear setting based on a three-dimensional model. The SIF is determined by the field of displacements or stresses at the crack tip. In the ANSYS software package, all the stages of calculations are automated, especially in the Workbench interactive mode.

A common disadvantage of numerical calculation methods is the difficulty of analyzing the final results to control the calculated value. It is not clear from the solution, which input parameters, in which direction, and how much is better to be changed to get the desired result. It is necessary to carry out multiple calculations with step-by-step changes in many variable parameters to catch the change trends. In addition, the results obtained cannot be used with a quantitative change in the properties of materials, structural parameters or loading.

For reasonable selecting of the parameters of reinforced concrete elements and controlling the loading parameters, it is necessary to have analytical dependences of the SIF on the varied parameters. A multifactorial computer experiment must be planned and performed to obtain these dependencies. By processing the results of such an experiment, the desired regression dependencies can be obtained.

3 RESULTS

The SIF in a flexible reinforced concrete beam was calculated by the finite element method in a linear formulation based on a three-dimensional model. The SIF was determined from the displacement or stress field at the crack tip. ANSYS (Ansys, Inc., Canonsburg, PA, USA) was used in Interactive Workbench mode. The main stages of solving the problem were as follows.

According to the above algorithm, there has been calculated a reinforced concrete beam of rectangular cross-section with the size of (15x30) cm of concrete grade B25 (Rbt=1.6 MPa, Rb=14.5 MPa, Eb=24000 MPa) with reinforcement made of AIII steel (Rs=370 MPa, E=200000 MPa). Figure 1 shows the scheme of fastening and loading the beam. In the middle section of the beam, a pure bending occurs with the moment M=Fa.

The beam has an initial crack l0=6 cm in the pure bending and reinforcement zone with the area of As=6.75 cm2 in the tension zone at a distance of 3 cm from the edge of this zone. The calculation of the beam has been carried out under the action of the torque of 8 kN·m. As a result of the calculation, there has been obtained KI=0.41 MN/m3/2.

4 PLANNING A COMPUTER EXPERIMENT

The purpose of the experiment is to obtain an analytical dependence of the SIF on the varied parameters. To assess the bearing capacity of a beam, one can simultaneously determine such dependences for the stress in the reinforcement and for the maximum compression stress in concrete. The analysis of numerous calculations to determine the stress state in reinforced

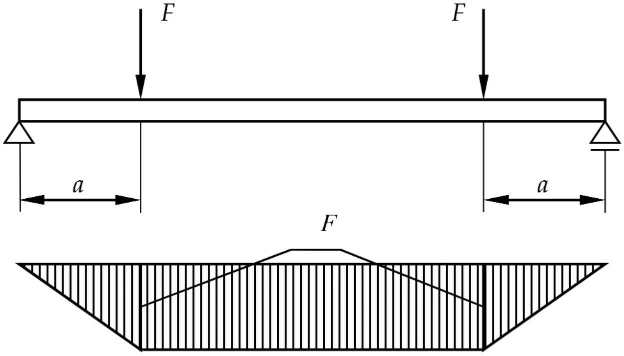

Figure 1. The calculation scheme with a moment diagram.

concrete beams with a bending crack shows that the stress in the reinforcement reaches standard resistance earlier than the stress in concrete. Therefore, alongside the SIF, regression dependences for the stress in the reinforcement σ_s are determined.

To obtain these dependencies, one first needs to select the parameters that significantly impact the output function. Then it is necessary to determine the real limits of changes in these parameters. Each argument's levels should be set so that they meet the requirements of the arithmetic progression (have the same intervals). These actions make it possible to plan the experiment properly and can reduce the required volume significantly.

When planning experiments, selecting the factor levels is important. It is made taking into account two conflicting requirements: the levels of factors should cover the entire possible range of its change, the total number of levels for all the factors should not lead to an excessive volume of tests.

For testing, we take a rectangular beam with reinforcement in the tension zone at a distance of 3 cm from the edge of this zone.

The main factors influencing the output values are the section dimensions *h, b,* the crack length *l*, the percentage of reinforcement μ, the elastic moduli of concrete E_b and reinforcement E_s, and the bending moment *M*. Since the calculation is performed in a linear formulation, the output values will vary linearly to the moment. Therefore, it can be excluded from the number of variable factors and taken fixed in the experiment. The performed test calculations and the analysis of analytical solutions [3, 4] show that the impact of the elastic moduli of materials can be taken into account by one parameter equal to their ratio $\alpha=E_s/E_b$. This leaves five variable factors: h, b, $z=l/h$, μ, α.

We will take the limits of changing these factors to most fully cover the entire range of their changes in real conditions:

$$h = (10\ldots50)\text{cm}, b = (5\ldots40)\text{cm}, z = 0.1\ldots0.7, \mu = (0,5\ldots3)\%, \alpha = 6\ldots11.$$

There are no interrelationships between the factors of the model. Therefore, we will use rational planning of the experiment. Since there are five variable factors, we will change them at five equal levels. Table 1 shows the values of the varied parameters at different levels.

Table 1. Varied parameters values.

Levels	Variable values				
	h, cm	b, cm	z	μ, %	A
1	10	5	0.1	0.5	6
2	20	14	0.25	1.1	7.2
3	30	23	0.4	1.7	8.4
4	40	32	0.55	2.3	9.6
5	50	41	0.7	2.9	10.8

With few numbers of factors ($k > 4$) for obtaining an experiment plan, it is more rational to use numerical matrices. The number of rows in the matrix is equal to the number of experiments, and the number of columns is equal to the number of influencing factors. The combination of numbers in each line is a combination of the factor levels in the corresponding experiment. Such plans are based on Latin squares. Only orthogonal Latin squares are suitable to obtain an optimal plan [5]. The technique of constructing mutually orthogonal squares is described in detail in [6].

According to the planning matrix, the full experiment will consist of 25 experiments divided into five orthogonal squares. The first three squares are shown below:

1	1 2 3 4 5	6	1 3 5 2 4	11	1 4 2 5 3
2	2 3 4 5 1	7	2 4 1 3 5	12	2 5 3 1 4
3	3 4 5 1 2	8	3 5 2 4 1	13	3 1 4 2 5
4	4 5 1 2 3	9	4 1 3 5 2	14	4 2 5 3 1
5	5 1 2 3 4	10	5 2 4 1 3	15	5 3 1 4 2

Each row of the matrix represents one experiment, the number of which is indicated by the first digit, and the subsequent digits indicate the levels of the corresponding factor. These levels correspond to the values of the varied parameters shown in Table 1. The first row of the planning matrix shows all the levels in ascending order. The rest of the rows are obtained by incrementing the level number by one in the matrix column with the order from the last to the first number. The first line of the next square consists of the diagonal of the previous square.

5 NUMERICAL CALCULATION

To carry out a computer experiment, we use the structural design shown in Figure 1. The bending moment at pure bending is set equal to 10 kN·m.

The calculation of the SIF in the beam is performed using the ANSYS Software with the rational planning matrix. In each experiment, as a result of the calculation, the SIF (K_I) and the stress in the reinforcement σ_s have been displayed for viewing. A fragment of the experimental results is shown in Table 2.

Table 2. A fragment of the experiment results.

No.	h, cm	b, cm	z	μ	α	K_I, MN/m$^{3/2}$	σ_s, MPa
1	10	14	0.4	2.3	10.8	0.433	229
2	20	23	0.55	2.9	6	0.266	27
3	30	32	0.7	0.5	7.2	0.195	98
4	40	41	0.1	1.1	8.4	0.13	15.3
5	50	5	0.25	1.7	9.6	0.6	57

Based on the results of the computer experiment, regression dependences are constructed for the output parameters. To do this, we will use an unconventional regression analysis method, which is implemented in the ANETR program created at Karaganda Technical University.

The program first arranges the factors in order of decreasing the impact on the output parameter value. For each factor, it selects the corresponding equation from 15 types of equations included in the program. At the same time, to identify the impact of some factors, it may be necessary to neutralize the impact of potent factors. The program ends with selecting a certain combination of partial equations to form the general model. The dependences obtained in this way are valid in the range of variation of the variable parameters.

The adequacy of the general model is assessed by the standard deviation (SD) of the calculated and experimental values of the output quantity and the multiple correlation coefficient of the model. Note that the program developers evaluate the resulting regression model as "excellent" if the standard deviation is lower than 25% and "good" if the standard deviation is lower than 40%.

The initial data of each experiment are entered into the ANETR program in the form of a 25x5 matrix and the value of the analyzed output parameter (response function) in the form of a 25x1 column vector to obtain regression dependences.

Let us present the finally formed multidimensional models for the stress intensity factor:

$$K_I = (M/10)[1.415 \cdot 10^{-3}h^2 - 0.125h + 7.09/b - 1.086\lg z + \\ +0.537\mu^2 - 1.804\mu - 0.117\alpha + 4.209]\left(\mathrm{MN/m^{3/2}}\right). \tag{1}$$

In this expression, the variables must be substituted in the units in which they are indicated in Table 1. The results will also be in the units in which the output values are given in Table 2. The standard deviation of the model is 38.2 %, and the multiple correlation coefficient is $R = 0.82$.

Regression dependences for stress in reinforcement:

$$\sigma_s = (M/10)[25460/h^{1.718} + 1202.5/b + 1697z^2 - 1290z - 85.5\mu + \\ +310.9/\alpha + 185.7](\mathrm{MPa}). \tag{2}$$

The SD of the model makes 25.4%, and the coefficient of multiple correlation R=0.924.

To reduce the standard deviation, you can increase the number of levels of changing the factors, but this will increase the number of experiments. For example, for seven levels, there are needed 49 experiments

The SIF is a parameter of linear fracture mechanics. Therefore, its FEM calculation is carried out in a linear setting. With increasing the bending moment, there should be expected increasing the error in determining the SIF. This is due to the growing influence of physical nonlinearity at high stresses. Under these conditions, it is recommended to calculate the SIF only by the analytical method [4].

6 CONCLUSIONS

The technique and algorithm of determining the stress intensity factor in bending reinforced concrete beams with a crack by the finite element method are described. A matrix of rational planning of the multifactorial computer experiment to determine the SIF in bending rectangular reinforced concrete beams with a crack has been compiled. Based on this planning matrix, using the ANSYS software, machine experiments have been carried out for pure bending beams. By processing the experiment results using the program of unconventional regression analysis ANETR, dependencies have been obtained for determining the SIF and stress in reinforcement in reinforced concrete beams with cracks. The impact of the operational crack length on the SIF has been analyzed. It has been found that in a reinforced concrete beam, the SIF decreases with increasing the crack length. At the critical crack length, the SIF becomes zero and the effective moment is equal to the breaking moment. In reinforced concrete beams, unstable crack development does not lead to the rapid destruction of the beam. However, it reduces its bearing capacity due to increasing the crack length and increasing stresses in concrete and reinforcement. The obtained dependences make it possible to evaluate the crack resistance of reinforced concrete beams according to the force criterion of fracture mechanics without the use of computer technology and expensive software systems. They also make it possible to assess the real state of the beam depending on the reinforcement stress.

REFERENCES

[1] Kumar, S.; Barai, S. v *Concrete Fracture Models and Applications*; Springer Berlin Heidelberg: Berlin, Heidelberg, 2011; ISBN 978-3-642-16763-8. Concrete Fracture Models and Applications | SpringerLink.

[2] Guo, Z. *Principles of Reinforced Concrete*; Elsevier, 2014; ISBN 9780128008591. Principles of Reinforced Concrete (elsevier.com).

[3] Nuguzhinov, Z.; Vatin, N.; Bakirov, Z.; Khabidolda, O.; Zholmagambetov, S.; Kurokhtina, I. Stress-Strain State of Bending Reinforced Beams with Cracks. *Magazine of Civil Engineering* 2020, *97*, 9701–9701, doi:10.18720/MCE.97.1. Scopus - сведения о документе - Stress-strain state of bending reinforced beams with cracks.

[4] Nuguzhinov, Z.S.; Bakirov, Z.B.; Yu Kurokhtin, A.; Khabidolda, O.; Nuguzhinova, A. Assessment of Bending Reinforced Concrete Beams Crack Resistance. In Proceedings of the IOP Conference Series: Materials Science and Engineering; 2019; Vol. 690. Scopus - сведения о документе - Assessment of Bending Reinforced Concrete Beams Crack Resistance.

[5] Keedwell, A.D.; Dénes, J. *Latin Squares and Their Applications*; Elsevier, 2015; ISBN 9780444635556. Latin Squares and their Applications - 2nd Edition (elsevier.com).

[6] Draper, N.R.; Smith, H. *Applied Regression Analysis*; 2014; ISBN 9780471170822. Applied Regression Analysis, 3rd Edition | Wiley.