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APPLICATION OF THE NEW B3Y-FETAL POTENTIAL IN THE SEMI-MICROSCOPIC ANALYSIS OF THE SCATTERING OF ACCELERATED ^{6}LI - LITHIUM AND ^{16}O - OXYGEN NUCLEI FROM THE ^{12}C - CARBON NUCLEUS

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The phenomenological and semi-microscopic values of the potentials found in the angular distribution of scattering of ions ^{6}Li - lithium, ^{16}O - oxygen from the target nucleus ^{12}C- carbon accelerated at low energies are determined. The experimental data of elastic scattering were analyzed based on the optical model and the double folding model. Based on the folding model, the density-dependent new Fetal potential of Yukawa 3 terms of effective nucleon-nucleon interactions was first used. The density-dependent Fetal, Paris, Reid microfolding potentials were constructed in the double folding model as a real part of the optical potential. The efficacy of the new Fetal micropotential at laboratory energies of 28 MeV and 30 Mev for the ^{16}O+^{12}C nuclear system was studied in comparison with the Reid, Paris variants. The efficacy of the new Fetal potential at laboratory energies of 12.3 MeV and 20 MeV for the ^{6}Li+^{12}C system has been studied in comparison with Reid and Paris potentials. The relative errors of phenomenological theoretical analysis and experimental cross-sections were determined in the range of 1.1 -3.0. As a result of semimicroscopic analysis, the coefficients of renormalization of folding potentials in the range Nr=0.8-1.0 were determined. The data obtained will be used in various fundamental research, in particular in future thermonuclear installations and nuclear astrophysics.

Keywords: nuclear matter, elastic scattering, semi-microscopic analysis, Botswana three Yukawa-Fetal potential

1. Introduction

The study of the interaction of accelerated ions with heavy nuclei will become an urgent topic as fundamental research in nuclear astrophysics and thermonuclear energy. Analysis of experimental data based on various models, the construction of the equation of state of nuclei is a theoretically important issue. The article presents a phenomenological and semi-microscopic analysis of the angular distribution of elastic scattering of low-energy weakly coupled ^{6}Li+^{12}C, ^{16}O+^{12}C nuclear systems. The significance of the phenomenological and microfolding potentials was determined within the framework of the Optical Model (OM) and double folding model (DFM) core. For the microscopic study of nuclear interactions , the equation of state can be constructed depending on the saturation property of the nucleus. K - nuclear incompressibility is the only value characterizing the saturation of nuclear matter [1, 2].

The Yukawa's Michigan three Yukawa (M3Y)-Reid, M3Y-Paris potentials were calculated based on effective nucleon-nucleon (NN) interactions as a real part of the optical potential. Effective NN interactions are generated in the G - matrix and consist of central, spin-orbital, and tensor members. In the calculation of their matrix interaction, all spin, isospin interaction components are formed. When calculating the folding potential, it is important to be an isospin independent center. Taking into account the spin-orbital interaction of two nuclei with NN interactions at low energy gives a successful characteristic for asymmetric systems. To understand the reaction, direct and exchange potentials are created based on the transformation of the isovector from a microscopic point of view. D.T. Khoa, G.R. Satchler scientists introduced density-dependent parameters into the NN interaction [3-5].

The purpose of introducing density-dependent parameters was to clarify the saturation property of nuclear matter. The new potential of B3Y-Fetal was obtained by applying a lowest-order constrained variational (LOCV) to the elements of the nuclear matrix of two bodies [6]. Based on these studies, Ochala

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[7, 8] first used the B3Y-Fetal potential obtained in the LOCV approach. The novelty of this article is that we introduce density-dependent parameters for B3Y-Fetal potency.

The DFM can study the real potential depending on the mass and energy of the nucleus. The Woods-Saxon potential form correctly describes the diffuse surface of the nucleus from the point of view of the nucleon density distribution. Accounting for effective NN forces and correlations is a way of constructing potentials in a microscopic approach. The article uses for the first time the potential of B3Y-Fetal for the $^{6}Li+^{12}C$ nuclear system. And for the $^{16}O+^{12}C$ system, the analysis is carried out for the first time at energies E_{lab}=28 MeV and 30 MeV. Based on the B3Y-Fetal interaction, the analysis of symmetric systems $^{12}C+^{12}C$, $^{16}O+^{16}O$ at energies E_{lab} =145 - 450 MeV was carried out [8, 9]. For an asymmetric $^{16}O+^{12}C$ system in the low-energy range E_{lab}=20 - 140 MeV, the analysis was carried out on the basis of a folding model [10, 11].

2. NN - interaction potentials

Effective NN interaction takes into account even and odd components of the central forces. The real potential is the sum of direct and exchange potentials [12].

$$\vec{V} = V^{D} + V^{EX} \tag{1}$$

The direct potential is completely elastic and is written as follows [13, 14]:

$$V^{D}(\vec{R}) = \int\int \rho^{(1)}(r_1)V_D(s)\,\rho^{(2)}(r_2)dr_1dr_2 \tag{2}$$

where, $V_D(s)$ - is the direct component of the effective NN interaction, $\rho^{(1)},\rho^{(2)}$ - densities of colliding nuclei, $s = r_2 - r_1 + R$.

When calculating the exchange potential, absorption processes are taken into account [15, 16]:

$$V^{EX}(\vec{R}) = \int\int \rho^{(1)}(r_1, r_1{}' + s)\upsilon_{EX}(s)\,\rho^{(2)}(r_2, r_2{}' - s)\,exp[\,i\vec{k}(R)s/\eta])dr_1dr_2 \tag{3}$$

where $\upsilon_{\mathrm{EX}}(s)$ - is the exchange component of the effective NN - interaction, $\rho^{(i)}(r, r')$ - is the density matrix of colliding nuclei.

Direct and exchange components of the M3Y-Reid potential based on the elements of the G-matrix [17]:

$$\upsilon_D(s) = 7999{,}0 \cdot \frac{e^{-4s}}{4s} - 2134{,}25 \cdot \frac{e^{-2{,}5s}}{2{,}5s} \tag{4}$$

$$\upsilon_{EX}(s) = 4631{,}4 \cdot \frac{e^{-4s}}{4s} - 1787{,}1 \cdot \frac{e^{-2{,}5s}}{2{,}5s} - 7{,}8474 \cdot \frac{e^{-0{,}7072s}}{0{,}7072s} \tag{5}$$

Components of the direct and exchange potential of M3Y-Paris, [3, 4]:

$$\upsilon_D(s) = 11061{,}6 \cdot \frac{e^{-4s}}{4s} - 2537{,}5 \cdot \frac{e^{-2{,}5s}}{2{,}5s} \tag{6}$$

$$\upsilon_{EX}(s) = -1524{,}0 \cdot \frac{e^{-4s}}{4s} - 518{,}8 \cdot \frac{e^{-2{,}5s}}{2{,}5s} - 7{,}8474 \cdot \frac{e^{-0{,}7072s}}{0{,}7072s} \tag{7}$$

Components of the direct and exchange potential of B3Y-Fetal [6, 8]:

$$\upsilon_D(s) = 10472{,}13 \cdot \frac{e^{-4S}}{4s} - 2203{,}11 \cdot \frac{e^{-2{,}5S}}{2{,}5s} \tag{8}$$

$$\upsilon_{\mathrm{EX}}(s) = 499{,}63 \cdot \frac{e^{-4S}}{4s} - 1347{,}77 \cdot \frac{e^{-2{,}5S}}{2{,}5s} - 7{,}8474 \cdot \frac{e^{-0{,}7072S}}{0{,}7072s} \tag{9}$$

3. Introduction of density-dependent parameters

The equation of state of the optical potential is constructed depending on the density and energy [18].

$$\upsilon_{D(EX)}(\mathrm{E}, \rho, s) = g(E)f(\rho)\upsilon'_{D(EX)}(s) \tag{10}$$

where, $g(E)$ - energy-dependent type of potential, $f(\rho)$ - two forms of density-dependent factor [2]:

1) $f(\rho) = C(1 + \alpha e^{-\beta\rho})$, the density-dependent M3Y (DDM3Y) – type
2) $f(\rho) = C(1 - \alpha\rho^{\beta})$, the β-parameter dependent M3Y (BDM3Y) - type (11)

Energy dependence [18, 2]:

$$g(E) = (1 - 0.003 \cdot E/A) \tag{12}$$

Density dependence function when introducing the γ – parameter [2]:

$$f(\rho) = C(1 + \alpha\, exp(-\beta\rho) - \gamma\rho) \tag{13}$$

The harmonic oscillator (HO) model [19] was used to distribute the matter density of nuclei ^{6}Li, ^{16}O and ^{12}C:

$$\rho(r) = \rho_0(1 + \alpha(r/a)^2)\, exp(-(r/a)^2) \tag{14}$$

4. Analysis of the ^{6}Li+^{12}C nuclear system

Experimental data for the ^{6}Li+^{12}C system were analyzed in the framework of OM, DFM at energies E_{lab}=12.3 MeV [20] and, E_{lab}=20 MeV [21]. Microfolding potentials were created in the C++ program. The density-dependent parameters included in the folding potentials are shown in the following table 1.

Table 1. Density-dependent parameters included in the γ-parameter dependent M3Y (CDM3Y2) and CDB3Y2 potential types created in the Yukawa 3 term (M3Y). K - incompressibility value [4, 9]

Density-dependent types	*C*	*a*	β (fm^3)	γ (fm^3)	*K* (MeV)
CDM3Y2, CDB3Y2	0.3346	3.0357	3.0685	1.0	204

As a real part of the optical potential, CDM3Y2, CDM3Y2, CDB3Y2 - folding potentials are used. The optical potential of the nuclear-nuclear interaction is written as follows:

$$U(r) = N_r V_F(r) - iW_0 f(r, r_W, a_W) + V_C(r) \tag{15}$$

where, N_r - renormalized factor, V_F - folding potential, W_0 - imaginary potential, a_W - diffusion, r_w - radius, and $V_C(r)$ - Coulomb potential.

The values of the σ_R - section of each analysis are presented in the table 2.

The phenomenological and semi-microscopic cross sections constructed on the basis of the parameters found in the energies E_{lab}=12.3 and E_{lab}=20 MeV are shown in the following figures 1-2.

Table 2. ^{6}Li+^{12}C - nuclear system, parameters detected as a result of the analysis of OM and DFM at energies of E_{lab} =12.3 MeV and E_{lab} =20 MeV. Coulomb radius fixed: R_C=1.3fm.

E_{Lab}, Mev	Potential	V_0, MeV	r_r, fm	a_r, fm	W_0, MeV	r_w, fm	a_w, fm	σ_R, *mb*	χ^2/N	N_r
12.3	OM	140.7	1.0	0.58	14.4	1.15	0.12	1005	1.1	-
	CDM3Y2-Reid				12.0	1.12	0.2		-	0.84
	CDM3Y2-Paris				12.0	1.12	0.2		-	0.84
	CDB3Y2- Fetal				12.0	1.12	0.2		-	0.84
20	OM	160.8	0.92	0.59	5.9	1.24	0.85	1261	1.8	-
	CDM3Y2-Reid				5.4	1.24	0.85	1357	-	0.85
	CDM3Y2-Paris				5.9	1.24	0.8	1369	-	0.80
	CDB3Y2- Fetal				5.4	1.24	0.8		-	0.82

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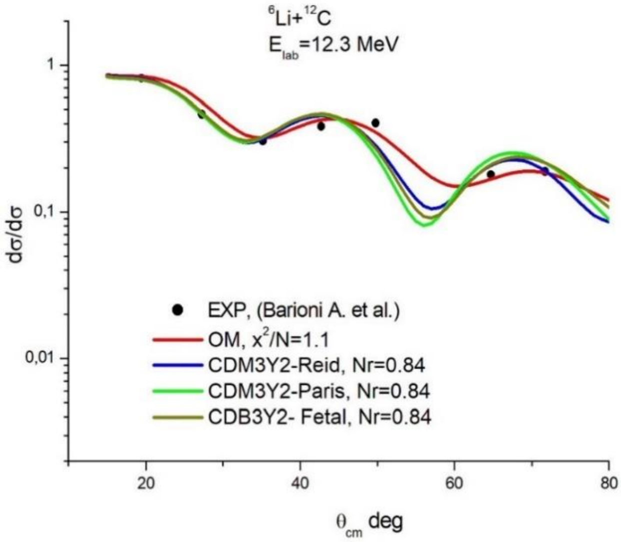

Fig. 1. $^{6}Li+^{12}C$ - nuclear system, cross sections in energies E_{lab}=12.3 MeV

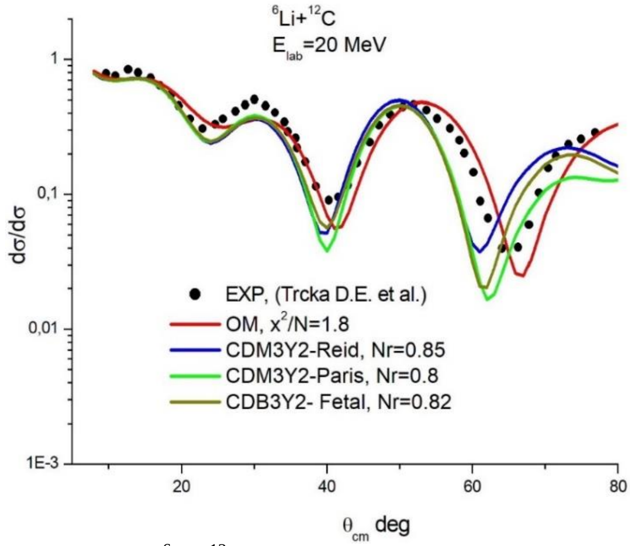

Fig. 2. $^{6}Li+^{12}C$ - nuclear system, cross sections in energies E_{lab}=20 MeV

5. Analysis of the 16O+12C nuclear system

Experimental data for the $^{16}O+^{12}C$ system were analyzed in the framework of OM, DFM at energies E_{lab}=28 MeV [22] and, E_{lab}=30 MeV [23]. The density-dependent parameters included in the folding potentials are shown in the following table 3-4.

Table 3. Density-dependent parameters included in the CDM3Y1, BDM3Y1 and DDB3Y1 potential types created in the Yukawa 3 term (M3Y). K - incompressibility value [4, 9]

Density dependent version	C	α	β	γ	K (MeV)
CDM3Y1-Paris	0.3429	3.0232	3.5512	0.5	188
BDM3Y1-Reid	1.2521	0.0 3.1757	1.7452	0.0	270
DDB3Y1-Fetal	0.2986		2.9605	0.0	176

Table 4. $^{16}O+^{12}C$ - nuclear system, Parameters detected as a result of the analysis of OM and DFM at energies of E_{lab} =28 MeV and E_{lab} =30 MeV. Coulomb radius fixed: R_C=1.3fm.

E_{Lab} MeV	Potential	V_0 MeV	r_V *(fm)*	a_V fm	W_0 MeV	r_W *(fm)*	a_W fm	χ^2/N	N_r
28	OM	96.0	1.18	0.507	6.05	1.15	0.854	3.0	-
	BDM3Y1-Reid				6.05	1.15	0.854	-	1.0
	CDM3Y1-Paris				6.05	1.15	0.854	-	0.8
	DDB3Y1-Fetal				6.05	1.15	0.854	-	0.9
30	OM	95	0.948	0.640	6.8	0.951	0.2	2.39	-
	BDM3Y1-Reid				6.8	0.951	0.2	-	1.0
	CDM3Y1-Paris				6.8	0.951	0.2	-	0.8
	DDB3Y1-Fetal				6.8	0.951	0.2	-	0.9

The phenomenological and semi-microscopic cross sections constructed on the basis of the parameters found in the energies E_{lab}=28 and E_{lab}=30 MeV are shown in the following figures 3-4.

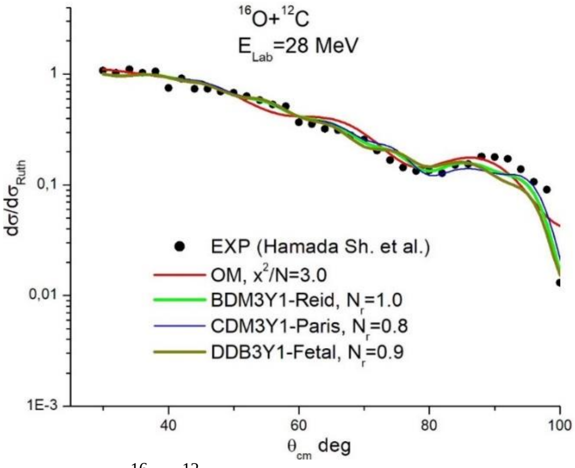

Fig. 3. $^{16}O+^{12}C$ - nuclear system, cross sections in energies E_{lab}=28 MeV

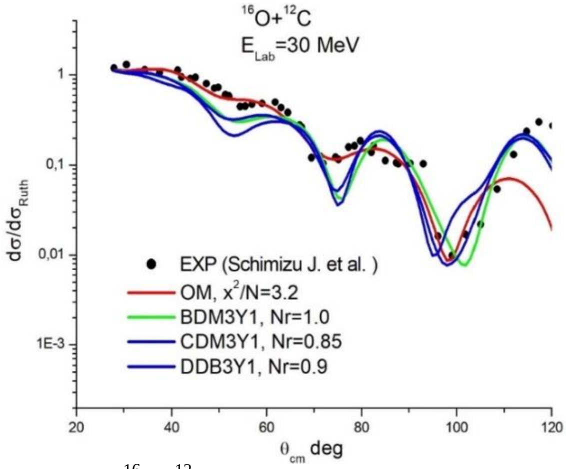

Fig. 4. $^{16}O+^{12}C$ - nuclear system, cross sections in energies E_{lab}=30 MeV

6. Conclusion

A phenomenological and semi-microscopic analysis of weakly coupled nuclear systems $^{6}Li+^{12}C$, $^{16}O+^{12}C$ was carried out. Microfolding potentials - BDM3Y1-Reid, CDM3Y1-Paris, DDB3Y1-Fetal, CDM3Y2-Reid, CDM3Y2-Paris, CDB3Y2-Fetal have been created in the DFM.

As a result of the phenomenological analysis, the relative errors of experimental and theoretical cross sections in the range χ^2/N=1.1 - 3.0 were revealed. As a result of semimicroscopic analysis, the coefficients of Nr - renormalization of microfolding potentials in the range Nr=0.8 - 1.0 were determined.

The efficacy of B3Y-Fetal potentials has been studied in comparison with M3Y - Reid, M3Y - Paris potentials. The values of the σ_R - cross-section of each analysis were determined.

The introduction of the density dependence in NN interactions in the study of collisions of heavy ions with light nuclei makes it possible to clarify the saturation property of nuclear matter, that is, to fully take into account nuclear nuclei.

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