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# Application of the form invariance transformations of the scalar cosmological model in inflation theory

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**Abstract.** In this work, it is shown that the equations of motion of the scalar field for spatially flat, homogeneous, and isotropic space-time Friedmann-Robertson-Walker have a form-invariance symmetry, which is arising from the form invariance transformation. Form invariance transformation is defined by linear function  $\bar{\rho} = n^2 \rho$  in general case. It is shown the method of getting potential and the scalar field for the power law scale factor. The initial model is always stable at exponent of the scale factor  $\alpha > 1$ , but stability of the transformation model depends on index  $n$ . Slow roll parameters and spectral induces is obtained and at large  $\alpha$  they agree with Planck observation data.

## 1. Introduction

The inflationary era came after the era of quantum gravity in the universe. Inflation is considered the most promising candidate for describing its post-Planck era. Inflation scenario is interesting so that most of inflation theories decide complex problems of the standard cosmology of the Big Bang, like horizon problem and flatness problem. However, to date, it has not been possible to direct verification the inflation of the universe. The simplest description of inflation realizations by introducing the one scalar field so-called inflaton, which slowly rolls from the peak of the self-interacting potential to the point of minimum in the context of the slow roll approximation [1]-[4]. When inflaton decays on the last stage, in the process of reheating [5]-[7], inflation comes to the end. The paper researches the scalar field inflationary model, using form invariance transformation (FIT) methods.

Currently, the universe is undergoing an acceleration phase of expansion. This is confirmed different cosmological observation data [8]-[11]. Due to the lack of a complete understanding of the nature of accelerated expansion, this is a source called dark energy. Previously, many papers have been proposed on dark energy [12]-[28].

In the present work, symmetry the Einstein for the Fridmann-Robertson-Walker space-time is researched with several sources like the perfect fluid and the homogeneous scalar field in the power law expansion of the universe. The basic conditions of linear FIT was introduced, which contains identical transformation and dual transformation. This allows to us make clear some consequences following from the Lie group structure of the above transformations. The slow roll parameters [29] were introduced for describing inflation.

The work is organized as follows: in section 2 outlines model is considered and general equations for form invariance of transformations are obtained; in section 3, linear form invariance



transformation is defined; in section 4, dynamical equations of the scalar field are built and transformation expression is introduced; in section 5, slow roll parameters are introduced, in section 6, finding the solution of dynamic equations is realized, and this solution was analysed by the slow roll parameters method and spectral indices method and was compared with cosmological observational data and section 7, results of the work are presented.

## 2. Model

Let us to introduce Einstein-Gilbert action in form

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left\{ \frac{1}{2} R + \mathcal{L}_m \right\}, \quad (1)$$

where  $R$  is the scalar curvature and  $\mathcal{L}_m$  is the matter Lagrangian.

We consider action (1) together with Fridmann-Robertson-Walker(FRW) metric

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2), \quad (2)$$

where  $a(t)$  is scale factor of the universe. The chief purpose of the investigation is research internal symmetry of the Einstein equations together with spatially flat, homogeneous and isotropic the FRW universe (2).

Invariant with relative to each other cosmological models are investigated, for which it is possible to introduce a relationship between the energy densities

$$\bar{\rho} = \bar{\rho}(\rho). \quad (3)$$

Fridmann equations contain three unknown parameters ( $H, p, \rho$ ), using the connection (3), we can find relation for that parameters [30]

$$\frac{\bar{\rho}}{\rho} = \frac{3\bar{H}^2}{3H^2}, \Rightarrow \bar{H} = H \left( \frac{\bar{\rho}}{\rho} \right)^{\frac{1}{2}}, \quad (4)$$

$$\bar{\rho} + \bar{p} = \frac{\dot{\bar{\rho}}}{\dot{\rho}} \left( \frac{\rho}{\bar{\rho}} \right)^{\frac{1}{2}} (\rho + p) = \frac{d\bar{\rho}}{d\rho} \left( \frac{\rho}{\bar{\rho}} \right)^{\frac{1}{2}} (\rho + p), \quad (5)$$

$$\bar{p} = -\bar{\rho} + \frac{d\bar{\rho}}{d\rho} \left( \frac{\rho}{\bar{\rho}} \right)^{\frac{1}{2}} (\rho + p). \quad (6)$$

Each of researching cosmological models is filled by perfect fluid with barotropic equation of state  $p = (\gamma - 1)\rho$  and  $\bar{p} = (\bar{\gamma} - 1)\bar{\rho}$  respectively. Relationship of barotropic indices  $\gamma$  and  $\bar{\gamma}$  equal to

$$\bar{\gamma} = \frac{\bar{\rho} + \bar{p}}{\bar{\rho}} = \frac{d\bar{\rho}}{d\rho} \left( \frac{\rho}{\bar{\rho}} \right)^{\frac{3}{2}} \gamma. \quad (7)$$

Form invariance symmetry is confirmed by form invariance transformation and shows the equivalence of the researched models.

## 3. Linear FIT

Lets define FIT as a linear function [30], [31]

$$\bar{\rho} = n^2 \rho, \quad (8)$$

where  $n$  is constant. In that case equations (4)-(6) take the form

$$\bar{H} = nH, \quad (9)$$

$$\bar{\rho} + \bar{p} = n(\rho + p), \quad (10)$$

$$\bar{p} = n[p + (1 - n)\rho]. \quad (11)$$

The given linear FIT induces linear expressions of variables  $(H, p, \rho)$ . By integrating (9) we get power law relation for the scale factor

$$\bar{a} = a^n \quad (12)$$

and form (7) transformation for barotropic index

$$\bar{\gamma} = \frac{\gamma}{n}. \quad (13)$$

#### 4. Scalar field

Let us explore the behaviour of the scalar field and show transformation in FIT accordance (8)-(11). Matter Lagrangian of the scalar field for FRW metric equal to

$$\mathcal{L}_b = \frac{1}{2}\dot{\varphi}^2 - V(\varphi), \quad (14)$$

here  $V(\varphi)$  is potential of the scalar field. In case of the scalar field, Einstein equation and Klein-Gordon equation take the form

$$3H^2 = \rho, \quad (15)$$

$$3H^2 + 2\dot{H} = -p, \quad (16)$$

$$\ddot{\varphi} + 3H\dot{\varphi} + V_\varphi = 0, \quad (17)$$

where energy density  $\rho$  and pressure  $p$  are defined by expressions

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi), \quad (18)$$

$$p = \frac{1}{2}\dot{\varphi}^2 - V(\varphi). \quad (19)$$

Using the equation (18) and (19) we can find dependence on time of potential  $V(t)$  and the scalar field for any scale factor  $a(t)$ . It means, that potential  $V(\varphi)$

$$\varphi = \int \sqrt{-2\dot{H}} dt, \quad (20)$$

$$V(\varphi) = \dot{H} + 3H^2. \quad (21)$$

The solution will be considered as neutral stable if the condition the speed of sound  $c_s^2 = 1 - \gamma > 1/5$  satisfies. To analyse the stability of the solutions, we obtain an expression relating the square of the derivative of the scalar field  $\dot{\varphi}^2$  and barotropic index  $\gamma$ , substituting into the equation of state  $p = (\gamma - 1)\rho$  the energy density  $\rho$  (18) and the pressure  $p$  (19). In this case it follows that

$$\gamma = \frac{\rho + p}{\rho} = \frac{\dot{\varphi}^2}{\rho}, \quad (22)$$

where  $0 < \gamma < 1$  and speed of sound equals to

$$c_s^2 = 1 - \frac{\dot{\varphi}^2}{\rho}. \quad (23)$$

The converted energy density and pressure of the scalar field are equal to

$$\bar{\rho} = \frac{1}{2}\dot{\bar{\varphi}}^2 + \bar{V}(\bar{\varphi}) = n^2 \left( \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \right), \quad (24)$$

$$\bar{p} = \frac{1}{2}\dot{\bar{\varphi}}^2 - \bar{V}(\bar{\varphi}) = \frac{n(2-n)}{2}\dot{\varphi}^2 - n^2V(\varphi), \quad (25)$$

where we used the FIT (8) and expression for pressure (11). From (13) and (22)

$$\dot{\bar{\varphi}}^2 = n\dot{\varphi}^2. \quad (26)$$

After integrating the equation (26), we obtain a transformation expression for the scalar field  $\bar{\varphi} = \sqrt{n}\varphi$ . The transformation conditions for the potential of the scalar field and the speed of sound are

$$\bar{V} = \frac{n(n-1)}{2}\dot{\varphi}^2 + n^2V(\varphi), \quad (27)$$

$$\bar{c}_s^2 = 1 - \bar{\gamma} = 1 - \frac{\dot{\varphi}^2}{n\rho}. \quad (28)$$

The transformations of the time dependence of the potential  $\bar{V}(t)$  and the scalar field  $\bar{\varphi}(t)$  are equal

$$\bar{\varphi} = \int \sqrt{-2\dot{\bar{H}}dt} = \int \sqrt{-2n\dot{H}dt}, \quad (29)$$

$$\bar{V}(\varphi) = \dot{\bar{H}} + 3\bar{H}^2 = n\dot{H} + 3n^2H^2. \quad (30)$$

## 5. Slow roll parameters and spectral indices

For description of inflation, we introduce the slow roll parameters in terms of the Hubble parameter [29], [32]

$$\epsilon_H = -\frac{\dot{H}}{H^2}, \quad \eta_H = -\frac{\ddot{\varphi}}{H\dot{\varphi}} = -\frac{1}{2}\frac{\ddot{H}}{H\dot{H}}. \quad (31)$$

The slope of the potential  $\epsilon_V(\varphi)$  and the curvature  $\eta_V(\varphi)$ , which are called the parameters of the slow roll are defined through the potential and the scalar field function, as follows [33]

$$\epsilon_V(\varphi) = \frac{1}{2}\left(\frac{V_{,\varphi}}{V}\right)^2, \quad \eta_V(\varphi) = \frac{V_{,\varphi\varphi}}{V}. \quad (32)$$

If the potential of the scalar field and the scalar field function are expressed by the respect with terms of a time derivative, then expressions (32) can be rewritten as [29], [34]

$$\epsilon_V(t) = \frac{1}{2\dot{\varphi}^2}\left(\frac{\dot{V}}{V}\right)^2, \quad \eta_V(t) = \left(\frac{\ddot{V}}{\dot{\varphi}^2} - \frac{\dot{V}\ddot{\varphi}}{\dot{\varphi}^3}\right)\frac{1}{V}. \quad (33)$$

If they are small, they will equal to slow roll in Hubble parameter terms

$$\epsilon_V \approx \epsilon_H, \quad (34)$$

$$\eta_V \approx \eta_H + \epsilon_H. \quad (35)$$

For the emergence and continuation of the inflationary stage, it is necessary that these parameters are in the area

$$\epsilon_V(\phi) \ll 1, \quad |\eta_V(\phi)| \ll 1. \quad (36)$$

A useful quantity is the number of e-folds defined as  $N = \ln(a)$ , which measures the amount of space-time expansion. The slow roll approximation yields a  $N$  given by

$$N = \int_{t_1}^{t_2} H dt = \int_{\varphi_{end}}^{\varphi} \frac{H}{\dot{\varphi}} d\varphi, \quad (37)$$

where  $\varphi_{end}$  is the value at the end of inflation, when (36) executes, and the upper limit in the integral is the value relative to the value of  $\varphi$  at crossing the horizon. If the value of the potential function  $V(\varphi)$  is known one can make predictions about inflation, that can be experimentally verified by measuring the power spectrum [35]. It is counted backwards in time from the end of inflation. In other words,  $N = 60$  is before the end of inflation.

Slow roll parameters approximately describe of the inflation dynamic and observational features of different models. We use the following spectral indices, which we find through the parameters of the slow roll [35], to compare our model with observational data

$$\begin{aligned} n_S - 1 &= -4\epsilon_H + 2\eta_H \approx -6\epsilon_V + 2\eta_V, \\ n_T &= -2\epsilon_H \approx -2\epsilon_V, \\ r_* &= 16\epsilon_H \approx 16\epsilon_V, \end{aligned} \quad (38)$$

where  $n_S$  is the scalar spectral index,  $n_T$  is the tensor spectral index and  $r_*$  is the tensor-to-scalar ratio.

## 6. Search for solution

Let's consider the case, when expansion of the universe is described by power law solution

$$a = a_0 t^\alpha, \quad (39)$$

where  $a_0$  and  $\alpha$  arbitrary and positive constants, and for the accelerated expansion of the universe it is necessary  $\alpha > 1$ . In that case equations (20) and (21) have the next solutions

$$\varphi(t) = \sqrt{2\alpha} \ln t + \varphi_0, \quad V(t) = \frac{V_0}{t^2}, \quad (40)$$

where  $\varphi_0$  integration constant and  $V_0 = \alpha(3\alpha - 1)$ . From (40) we can find  $V(\varphi)$  replacing  $t$

$$V = V_0 e^{\frac{\sqrt{2\alpha}}{\alpha}(\varphi_0 - \varphi)}. \quad (41)$$

Energy density  $\rho$  and pressure  $p$  are determined, respectively, from the expressions (18) and (19)

$$\rho = \frac{3\alpha^2}{t^2}, \quad p = \frac{\alpha(2 - 3\alpha)}{t^2}. \quad (42)$$

Slow roll parameters in Hubble terms, using (31) and into account scale factor (39) take the form

$$\epsilon_H = \frac{1}{\alpha}, \quad \eta_H = \frac{1}{\alpha}. \quad (43)$$

Slow roll parameters in potential terms, using (33) and into account scale factor (39) take the form

$$\epsilon_V = \frac{1}{\alpha}, \quad \eta_V = \frac{2}{\alpha}. \quad (44)$$

As follows from (43) and (44), conditions (34) and (35) are satisfied for the case under study.

The transformed scale factor is  $\bar{a} = \bar{a}_0 t^{\bar{\alpha}}$  with  $\bar{a}_0 = a_0^n$  and  $\bar{\alpha} = n\alpha$ . In this case, the equations (29) and (30) have the following solutions

$$\bar{\varphi}(t) = \sqrt{2\alpha n} \ln t + \bar{\varphi}_0, \quad \bar{V}(t) = \frac{\alpha n(3\alpha n - 1)}{t^2}, \quad (45)$$

$\bar{\varphi}_0 = \sqrt{n}\varphi_0$  is integration constant. From (45) we find  $\bar{V}(\bar{\varphi})$  replacing  $t$

$$\bar{V} = \bar{V}_0 e^{\frac{\sqrt{2\alpha n}}{\alpha n}(\bar{\varphi}_0 - \bar{\varphi})}, \quad (46)$$

where  $\bar{V}_0 = \alpha n(3\alpha n - 1)$ . The transformed energy density is  $\bar{\rho}$  and pressure  $\bar{p}$  are determined, respectively, from the expressions (24) and (25)

$$\bar{\rho} = \frac{3n^2\alpha^2}{t^2}, \quad \bar{p} = \frac{(2 - 3n\alpha)}{t^2}. \quad (47)$$

The transformed slow roll parameters in Hubble terms are

$$\bar{\epsilon}_H = \frac{1}{n\alpha}, \quad \bar{\eta}_H = \frac{1}{n\alpha}. \quad (48)$$

The transformed slow roll parameters in potential terms are

$$\bar{\epsilon}_V = \frac{1}{n\alpha}, \quad \bar{\eta}_V = \frac{2}{n\alpha}. \quad (49)$$

As follows from 48 and (49) for the transformed model, the conditions (34) and (35) are also satisfied.

E-folds accordingly (37) for the scale factor (39) take the form

$$N = \ln t^\alpha. \quad (50)$$

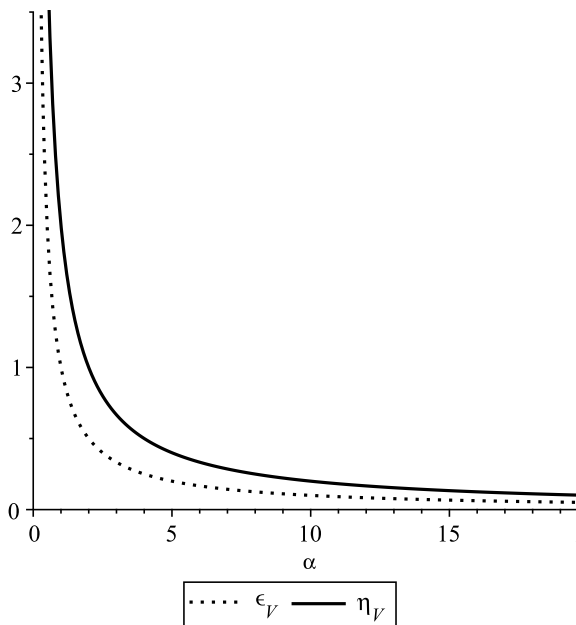
Time dependence of the function (50) is presented by figure 2. The spectral indices (38) in terms of the  $\alpha$  parameter and the e-folding (50) are equal

$$\begin{aligned} n_S &= 1 - \frac{2}{\alpha} = 1 + \ln t^{-\frac{2}{N}}, \\ n_T &= -\frac{2}{\alpha} = \ln t^{-\frac{2}{N}}, \\ r_* &= \frac{16}{\alpha} = \ln t^{\frac{16}{N}} = 8(1 - n_S). \end{aligned} \quad (51)$$

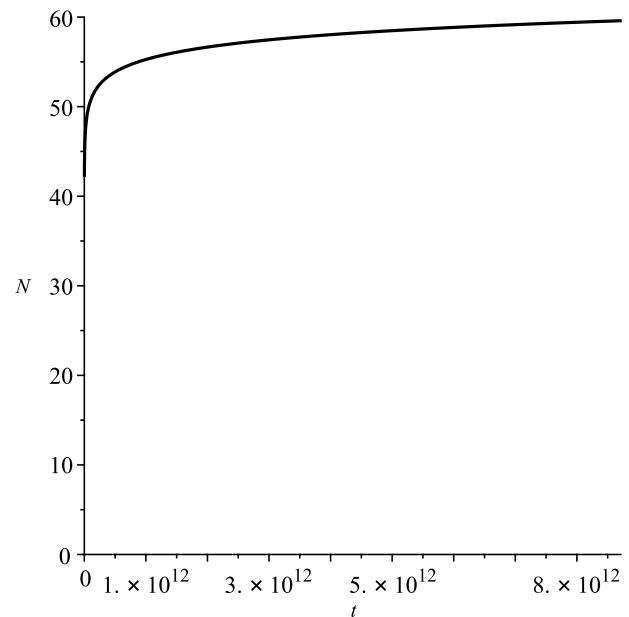
Dependence of the spectral indices via parameter  $\alpha$  and time  $t$ , at  $N = 60$  are shown in figure 3 and 4 respectively ( $n_S$  dotted line,  $n_T$  solid line and  $r_*$  dashed line). The values of the scalar spectral index  $n_S$  and the boundaries of the value of the tensor-scalar ratio  $r_*$  are given by the expression [36], Planck data accordingly

$$n_S = 0.9649 \pm 0.0042 \quad (68\% CL), \quad (52)$$

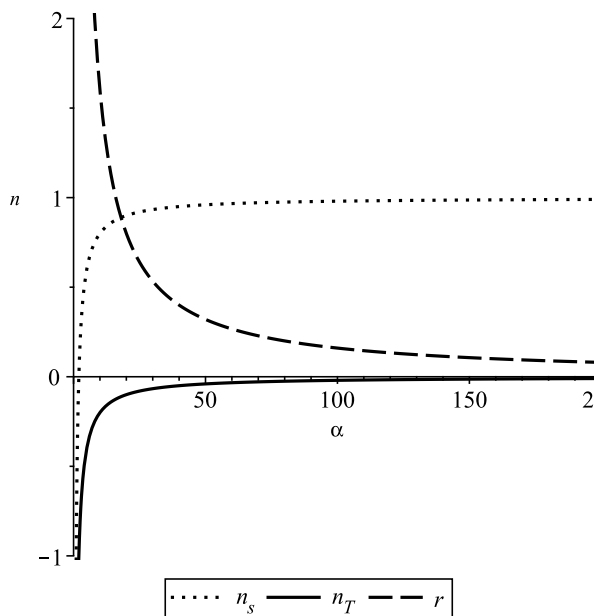
$$r_{0.002} < 0.056 \quad (95\% CL). \quad (53)$$



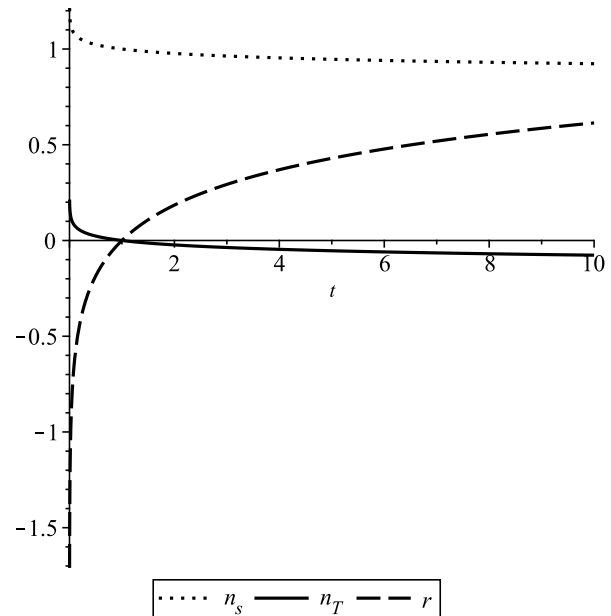
**Figure 1.** Dependence of the slow roll parameters (44) via  $\alpha$ .



**Figure 2.** Dependence of the e-folding via cosmic time  $t$  at  $\alpha = 2$



**Figure 3.** Dependence of the spectral indices (51) via  $\alpha$ .



**Figure 4.** Dependence of the spectral indices (51) via time  $t$ , at  $N = 60$ .

**7. Conclusion**

The researching results indicate that form invariance transformations can be used to obtain new solutions to the Einstein equations. Moreover, FIT allows to move from an unstable cosmology to a stable one and vice versa. The static universe containing a perfect fluid is always stable at



the speed of sound  $c_s^2 > 1/5$ . From the expression (22) for the model under study with (39) it follows that  $\gamma = \frac{\dot{\varphi}^2}{\rho} = -\frac{2\dot{H}}{3H^2} = \frac{2}{3\alpha}$ . Therefore, the speed of sound is  $c_s^2 = 1 - \gamma = 1 - \frac{2}{3\alpha} > 1/3$ , for  $\alpha > 1$ , which is necessary for the accelerated expansion of the universe and our original model always stable. After applying FIT, the speed of sound is  $c_s^2 = 1 - \frac{2}{3n\alpha}$  and since there are no restrictions on  $n$  the transformed model can be both stable and unstable.

Method for finding the time dependence of the potential  $V(t)$  and the scalar field  $\varphi(t)$  for any scale factor  $a(t)$  was illustrated. The scalar field potential can be used to control the expansion of the universe that has been verified.

The slow roll parameters were found and their graphs were plotted. From the graph 1 it can be seen that for  $\alpha \rightarrow \infty$  the slow roll parameters are  $\epsilon_V(\phi) \ll 1$ ,  $|\eta_V(\phi)| \ll 1$  and the model under study describes the inflationary stage. After applying FIT, the model also describes inflation, but the beginning or end of the inflationary stage will depend not only on the value  $\alpha$  and also on the value  $n$ . The graphs 3 and 4 show the dependence of the spectral indices (51) on the parameter  $\alpha$  and  $t$ . The results of the investigated model have been shown in the graph 3 at large values of  $\alpha$  are in good agreement with the observational data of Planck (52), (53).

### 7.1. Acknowledgments

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