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Cosmological model of $F(T)$ gravity with fermion fields via Noether symmetry

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Abstract. In this article, we investigate the modified $F(T)$ gravity, which is non-minimally coupled with the Dirac (fermion) field in Friedmann-Robertson-Walker space-time. Point-like Lagrangian is derived and modified Friedmann equations and Dirac equations for the fermion field are obtained by using the Lagrange multiplier. The Noether symmetry method related to differential equations is a useful tool for studying conserved quantities. In addition, this method is very useful for determining the unknown functions that exist in the point-like Lagrangian. Using this method, the form of the coupling between gravity and matter, the self-consistent potential, the symmetry generators, the form of $F(T)$ gravity and the first integral (Noether charge) or a conserved quantity for this model are determined. Cosmological solutions that have a power-law form and describe the late time accelerated expansion of the Universe are obtained.

1. Introduction

Latest observation indicated that expansion of universe is accelerated [1]. For explanation this late time accelerating, from scientists has been proposed two remarkable approaches. One is to assume contents of matter as scalar field in the right-hand side of the Einstein equation in the framework of general relativity, i.e. as scalar field may be consider phantom, quintessence, fermion, tachyon and etc. Another is to make modify the left-hand side of the Einstein equation, i.e. gravitational part.

At the present time, the modified theory of gravity is suitable approach for explanation late time acceleration. Recently, in analogy as well-known $F(R)$ gravity has been proposed a new modified gravity namely the so-called $F(T)$ gravity [2, 3]. Usually, in General Relativity used curvature scalar R defined by the Levi-Civita connection, but in teleparallel gravity used torsion scalar defined by Weitzenbock connection. In modified $F(T)$ gravity, torsion scalar T has been extended to a function of T. The idea of $F(T)$ gravity is equivalent to the concept of $F(R)$ gravity. In several literature, $F(T)$ gravity is assume alternative gravitational theory to general relativity and have been diversely explored its various properties.

In cosmology fermion fields have been studied as possible sources of early and late time expansion without the need of a cosmological constant term or a scalar field. In most of the papers are considered fermions fields are minimally coupled to gravity. But, in recently appears works effects of fermionic fields non - minimally coupling to gravity [4]. The fermion fields has been investigated via several approaches, with results including exact solutions, numerical solutions, cyclic cosmologies and anisotropy-to-isotropy scenario, perturbations, dark spinors.

The relation between general relativity and the equation for fermion fields is done via the tetrad formalism. The components of the tetrad play the role of the gravitational degrees of freedom.

The purpose of the present paper is to describe a spatially flat homogeneous and isotropic Friedmann - Robertson - Walker universe whose constituents are a fermionic field. We investigate models a fermion field non-minimally coupled with the $F(T)$ theory of gravitation. For our model we study Noether symmetry method. The Noether symmetry approach is very important tool because it allows to choose the potentials and the coupling compatible with the symmetry.

The Noether symmetry approach extensively studied in scalar field [5], fermion field [6], tachyon field [7], vector field [8], non-minimally coupled cosmology [9], $F(T)$ gravity [10] and etc. Also, the existence of Noether symmetry for teleparallel gravity which non-minimally coupled fermionic field is considered in literature [11].

The structure of this paper organizing as following. In Sect. II we briefly review $F(T)$ gravity. In Sect. III we consider model with a fermionic field that is non-minimally coupled to $F(T)$ gravity. In Sect. IV we applied Noether symmetry approach for our model. The cosmological solutions is considered in In Sect. IV.

The signature of metric used is $(+, -, -, -)$ and units have been chosen so that $8\pi G = c = \hbar = k = 1$.

2. $F(T)$ gravity
In this section, we briefly review of teleparallel gravity and $F(T)$ gravity.
Action for teleparallel gravity is given as

$$S = \int d^4x \, |e| \, T + S_m, \tag{1}$$

where T is the torsion scalar. $|e| = det(e^i_\mu) = \sqrt{-g}$ is the determinant of metric tensor.

Here the torsion scalar T is defined as

$$T = S^{\mu\nu}_\rho T^\rho_{\mu\nu}, \tag{2}$$

where

$$T^\rho{}_{\mu\nu} = -e^\rho_i \left(\partial_\mu e^i_\nu - \partial_\nu e^i_\mu \right) \tag{3}$$

$$S_\rho{}^{\mu\nu} = \frac{1}{2} \left(K^{\mu\nu}{}_\rho + \delta^\mu_\rho T^{\theta\nu}{}_\theta - \delta^\nu_\rho T^{\theta\mu}{}_\theta \right). \tag{4}$$

$$K^{\mu\nu}{}_\rho = -\frac{1}{2} \left(T^{\mu\nu}{}_\rho - T^{\nu\mu}{}_\rho - T_\rho{}^{\mu\nu} \right) \tag{5}$$

Now, in the context of a flat Friedmann-Robertson-Walker (FRW) model the metric is

$$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2], \tag{6}$$

The metric tensor $g_{\mu\nu}$ of spacetime is given by the following formulas:

$$g_{\mu\nu}(x) = \eta_{ij} e^i_\mu(x) e^j_\nu(x), \tag{7}$$

where

$$\mathbf{e}_i \cdot \mathbf{e}_j = \eta_{ij}, \quad \eta_{ij} = diag(1, -1, -1, -1). \tag{8}$$

In the teleparallel gravity, the torsion scalar corresponding to the FRW metric (6)

$$T \equiv -6\frac{\dot{a}^2}{a^2} \equiv -6H^2 \tag{9}$$

where H is the Hubble parameter. $a(t)$ is scale factor and dot means a derivative with respect to time. If we replace T in action for teleparallel gravity by any function of torsion scalar

$$\mathcal{S} = \int d^4x \, |e| \, F(T) + \mathcal{S}_m \,, \tag{10}$$

The field equations for the action (10) reads

$$[e^{-1}\partial_\mu(eS_i^{\mu\nu} - e_i^\lambda T_{\mu\lambda}^\rho S_\rho^{\nu\mu}]F_T + S_i^{\mu\nu}\partial_\mu T F_{TT} + \frac{1}{4}e_i^\nu F = \frac{1}{2}k^2 e_i^\rho T_\mu^\nu, \tag{11}$$

In the FRW background, from (6) we obtain modified Friedmann equations as

$$-2TF_T + F = 0, \tag{12}$$

$$-8\dot{H}TF_{TT} + (2T - 4\dot{H})F_T - F = 0. \tag{13}$$

As we know that $F(T)$ gravity has first-order derivatives in gravitational field equation. In general relativity, modified gravity as $F(R)$ where the gravitational field equation is fourth-order in derivatives.

Now we examine model with a fermionic field that is non-minimally coupled to gravity in the framework of $F(T)$ gravity.

3. Action and field equations

Action for model with a fermionic field that is non-minimally coupled to gravity in the framework of $F(T)$ gravity written as

$$\mathcal{S} = \int d^4x \sqrt{-g} \Big\{ h(u)F(T) + \frac{i}{2}\Big[\overline{\psi}\Gamma^\mu D_\mu \psi - (\overline{D}_\mu\overline{\psi})\Gamma^\mu \psi\Big] - V(u) \Big\}, \tag{14}$$

where ψ and $\bar{\psi} = \psi^+\gamma^0$ is a fermion function and its adjoint, respectively. Generally, Pauli matrices are replaced by $\Gamma^\mu = e_a^\mu \gamma^a$, where $\mu = e_a^\mu$ are tetrad fields. The generalized Pauli matrices obey the Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$. $h(u)$ and $V(u)$ is the function that describes the coupling and self-interaction potential density of the fermionic field which is depend on only functions of the bilinear $u = \bar{\psi}\psi$.

The covariant derivatives in (14) written as

$$D_\mu \psi = \partial_\mu \psi - \Omega_\mu \psi, \qquad D_\mu \overline{\psi} = \partial_\mu \overline{\psi} + \overline{\psi}\Omega_\mu, \qquad \Omega_\mu = -\frac{1}{4}g_{\rho\sigma}\Big[\Gamma_{\mu\delta}^\rho - e_b^\rho\Big(\partial_\mu e_\delta^b\Big)\Big]\Gamma^\delta\Gamma^\sigma. \tag{15}$$

Here Ω_μ and $\Gamma_{\sigma\lambda}^\nu$ is the denotes the spin connection and Christoffel symbols, respectively.

The Dirac matrices of curved spacetime Γ^μ are

$$\Gamma^0 = \gamma^0, \quad \Gamma^j = a^{-1}\gamma^j, \quad \Gamma^5 = -i\sqrt{-g}\Gamma^0\Gamma^1\Gamma^2\Gamma^3 = \gamma^5, \quad \Gamma_0 = \gamma^0, \quad \Gamma_j = a\gamma^j(i = 1, 2, 3). \tag{16}$$

For FRW metric in (6) we obtain

$$\Gamma^0 = \gamma^0, \qquad \Gamma^i = \frac{1}{a(t)}\gamma^i, \qquad \Omega_0 = 0, \qquad \Omega_i = \frac{1}{2}\dot{a}(t)\gamma^i\gamma^0, \tag{17}$$

with the dot denoting time derivative.

The Lagrange multipliers is a useful method for solving problems without the need to explicitly solve the conditions and use them to eliminate extra variables. We can use the method of Lagrange multipliers to set T as a constraint of the dynamics.

In our case the action (14) written as

$$\mathcal{S} = \int d^4x \sqrt{-g} \left\{ hF - \lambda \left[T + 6 \left(\frac{\dot{a}^2}{a^2} \right) \right] + \frac{\imath}{2} \left(\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi \right) - V \right\}, \tag{18}$$

where λ is a Lagrange multiplier. The variation of this action with respect to T gives

$$\lambda = hF_T \tag{19}$$

Thus, the action (18) rewritten as

$$\mathcal{S} = \int d^4x \sqrt{-g} \left\{ hF - hF_T \left[T + 6 \left(\frac{\dot{a}^2}{a^2} \right) \right] + \frac{\imath}{2} \left(\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi \right) - V \right\}, \tag{20}$$

Considering the background (6), it is possible to obtain the point-like Lagrangian from action (18)

$$\mathcal{L} = a^3 hF - hF_T a^3 \left[T + 6 \left(\frac{\dot{a}^2}{a^2} \right) \right] + \frac{\imath a^3}{2} \left(\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi \right) - a^3 V, \tag{21}$$

here, because of homogeneity and isotropy of the metric it is assumed that the spinor field depends only on time, i.e. $\psi = \psi(t)$.

To obtain equation of motion for a, T, ψ and $\bar{\psi}$, we use Euler-Lagrange equation

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right) - \frac{\partial \mathcal{L}}{\partial q_i} = 0 \tag{22}$$

Equation of motion for fermion fields and its adjoint ψ and $\bar{\psi}$, respectively as

$$\dot{\psi} + \frac{3}{2} H\psi - \imath(Fh' - F_T Th' + 6F_T H^2 h' - V')\gamma^0\psi = 0 \tag{23}$$

$$\dot{\bar{\psi}} + \frac{3}{2} H\bar{\psi} + \imath(Fh' - F_T Th' + 6F_T H^2 h' - V')\bar{\psi}\gamma^0 = 0 \tag{24}$$

where $H = \dot{a}/a$ denotes the Hubble parameter and the dot denotes a derivative with respect to the time t.

Equation of motion for torsion scalar T written as

$$F_{TT} \left(T + 6\frac{\dot{a}^2}{a^2} \right) = 0 \tag{25}$$

If $F_{TT} \neq 0$, we recovered expression (9).

On the other hand, from the point-like Lagrangian (21) and by considering the Dirac's equations, we find the acceleration equation for a from the Euler-Lagrange equation as

$$\frac{\ddot{a}}{a} = -\frac{\rho_f + 3p_f}{12hF_T}. \tag{26}$$

and Hamiltonian constraint (energy condition) corresponding to Lagrangian $\mathcal{L}$ defined by

$$E_{\mathcal{L}} = \frac{\partial \mathcal{L}}{\partial \dot{a}}\dot{a} + \frac{\partial \mathcal{L}}{\partial \dot{T}}\dot{T} + \frac{\partial \mathcal{L}}{\partial \dot{\psi}}\dot{\psi} + \dot{\bar{\psi}}\frac{\partial \mathcal{L}}{\partial \dot{\bar{\psi}}} - \mathcal{L}, \tag{27}$$

By substituting (21) into (27) with the energy condition, we obtain

$$H^2 = \frac{\rho_f}{6hF_T}. \tag{28}$$

In the acceleration and Friedmann equations, ρ_f and p_f are the effective energy density and pressure of the fermion (spinor) field, respectively, so that they have the following forms

$$\rho_f = hF_T T - hF + V, \tag{29}$$

$$p_f = 4h'\dot{u}HF_T - 48hF_{TT}H^2\dot{H} - (Fh' - F_T Th' + 6F_T h'H^2 - V')u + hF - hF_T T - V. \tag{30}$$

It is very hard to find solution for the equations (23)-(26) since these are highly non-linear systems. In order to solve the field equations we have to determine a form for the coupling function and the potential density of the theory. To do this, in the following section we will use the Noether symmetry approach.

4. Noether symmetries analysis

In this section we will search the Noether symmetry for model with a fermionic field that is non-minimally coupled to gravity in the framework of $F(T)$ gravity. In terms of the components of the fermion field $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)^T$ and its adjoint $\bar{\psi} = (\psi_1{}^\dagger, \psi_2{}^\dagger, -\psi_3{}^\dagger, -\psi_4{}^\dagger)$, the Lagrangian (21) can be rewritten as

$$\mathcal{L} = a^3 hF - hF_T a^3 \left[T + 6\left(\frac{\dot{a}^2}{a^2}\right)\right] + \frac{\imath a^3}{2}\sum_{i=1}^{4}(\psi_i^\dagger \dot{\psi}_i - \dot{\psi}_i^\dagger \psi_i) - a^3 V, \tag{31}$$

Noether symmetry approach tells us that Lie derivative of the Lagrangian with respect to a given vector field $\mathbf{X}$ vanishes, i.e.

$$L_{\mathbf{X}}\mathcal{L} = 0. \tag{32}$$

The Noether symmetry generator is a vector field defined by

$$\mathbf{X} = \alpha\frac{\partial}{\partial a} + \beta\frac{\partial}{\partial T} + \dot{\alpha}\frac{\partial}{\partial \dot{a}} + \dot{\beta}\frac{\partial}{\partial \dot{T}} + \sum_{j=1}^{4}\left(\nu_j\frac{\partial}{\partial \psi_j} + \dot{\nu}_j\frac{\partial}{\partial \dot{\psi}_j} + \delta_j\frac{\partial}{\partial \psi_j^\dagger} + \dot{\delta}_j\frac{\partial}{\partial \dot{\psi}_j^\dagger}\right) \tag{33}$$

with Lagrangian

$$L_{\mathbf{X}}\mathcal{L} = \alpha\frac{\partial\mathcal{L}}{\partial a} + \beta\frac{\partial\mathcal{L}}{\partial T} + \dot{\alpha}\frac{\partial\mathcal{L}}{\partial \dot{a}} + \dot{\beta}\frac{\partial\mathcal{L}}{\partial \dot{T}} + \sum_{j=1}^{4}\left(\nu_j\frac{\partial\mathcal{L}}{\partial \psi_j} + \dot{\nu}_j\frac{\partial\mathcal{L}}{\partial \dot{\psi}_j} + \delta_j\frac{\partial\mathcal{L}}{\partial \psi_j^\dagger} + \dot{\delta}_j\frac{\partial\mathcal{L}}{\partial \dot{\psi}_j^\dagger}\right) \tag{34}$$

where

$$\dot{\alpha} = \frac{\partial\alpha}{\partial a}\dot{a} + \frac{\partial\alpha}{\partial T}\dot{T} + \frac{\partial\alpha}{\partial \psi_j}\dot{\psi}_j + \frac{\partial\alpha}{\partial \psi_j^\dagger}\dot{\psi}_j^\dagger \tag{35}$$

$$\dot{\beta} = \frac{\partial\beta}{\partial a}\dot{a} + \frac{\partial\beta}{\partial T}\dot{T} + \frac{\partial\beta}{\partial \psi_j}\dot{\psi}_j + \frac{\partial\beta}{\partial \psi_j^\dagger}\dot{\psi}_j^\dagger \tag{36}$$

$$\dot{\nu} = \frac{\partial\nu}{\partial a}\dot{a} + \frac{\partial\nu}{\partial T}\dot{T} + \frac{\partial\nu}{\partial \psi_j}\dot{\psi}_j + \frac{\partial\nu}{\partial \psi_j^\dagger}\dot{\psi}_j^\dagger \tag{37}$$

$$\dot{\delta} = \frac{\partial\delta}{\partial a}\dot{a} + \frac{\partial\delta}{\partial T}\dot{T} + \frac{\partial\delta}{\partial \psi_j}\dot{\psi}_j + \frac{\partial\delta}{\partial \psi_j^\dagger}\dot{\psi}_j^\dagger \tag{38}$$

By substituting (31) into (34) and using relations (35)-(38) and requiring the coefficients of $\dot{a}^2, \dot{a}\dot{T}, \dot{a}\dot{\psi}_j, \dot{a}\dot{\psi}_j^\dagger, \dot{a}, \dot{T}, \dot{\psi}_j$ and $\dot{\psi}_j^\dagger$ to be zero, we find that system of equations as following

$$\dot{a}^2: \qquad \alpha F_T + 2aF_T \frac{\partial \alpha}{\partial a} + \beta a F_{TT} + \frac{h'}{h} aF_T \sum_{i=1}^{4} \left(\epsilon_i \nu_i \psi_i^\dagger + \epsilon_i \delta_i \psi_i \right) = 0, \tag{39}$$

$$\dot{a}\dot{T}: \qquad hF_T \frac{\partial \alpha}{\partial T} = 0, \tag{40}$$

$$\dot{a}\dot{\psi}_j: \qquad hF_T \frac{\partial \alpha}{\partial \psi_j} = 0, \tag{41}$$

$$\dot{a}\dot{\psi}_j^\dagger: \qquad hF_T \frac{\partial \alpha}{\partial \psi_j^\dagger} = 0, \tag{42}$$

$$\dot{a}: \qquad \sum_{i=1}^{4} \left(\frac{\partial \nu_i}{\partial a} \psi_i^\dagger - \frac{\partial \delta_i}{\partial a} \psi_i \right) = 0, \tag{43}$$

$$\dot{T}: \qquad \sum_{i=1}^{4} \left(\frac{\partial \nu_i}{\partial T} \psi_i^\dagger - \frac{\partial \delta_i}{\partial T} \psi_i \right) = 0, \tag{44}$$

$$\dot{\psi}_j: \qquad 3\alpha \psi_j^\dagger + a\delta_j + a \sum_{i=1}^{4} \left(\frac{\partial \nu_i}{\partial \psi_j} \psi_i^\dagger - \frac{\partial \delta_i}{\partial \psi_j} \psi_i \right) = 0, \tag{45}$$

$$\dot{\psi}_j^\dagger: \qquad 3\alpha \psi_j + a\nu_j - a \sum_{i=1}^{4} \left(\frac{\partial \nu_i}{\partial \psi_j^\dagger} \psi_i^\dagger - \frac{\partial \delta_i}{\partial \psi_j^\dagger} \psi_i \right) = 0, \tag{46}$$

$$3\alpha V + aV' \sum_{i=1}^{4} \left(\epsilon_i \nu_i \psi_i^\dagger + \epsilon_i \delta_i \psi_i \right) = 0, \tag{47}$$

$$3\alpha F - 3\alpha F_T T - \beta a F_{TT} T + aF \frac{h'}{h} \sum_{i=1}^{4} \left(\epsilon_i \nu_i \psi_i^\dagger + \epsilon_i \delta_i \psi_i \right) -$$

$$-aF_T T \frac{h'}{h} \sum_{i=1}^{4} \left(\epsilon_i \nu_i \psi_i^\dagger + \epsilon_i \delta_i \psi_i \right) = 0 \tag{48}$$

where $\epsilon_i = \begin{cases} 1 & for \quad i = 1, 2 \\ -1 & for \quad i = 3, 4 \end{cases}$

For Noether symmetry the first integral or conserved quantity (Noether charge) defined as

$$Q = \xi^j \frac{\partial \mathcal{L}}{\partial \dot{q}^j} = \alpha \frac{\partial \mathcal{L}}{\partial \dot{a}} + \beta \frac{\partial \mathcal{L}}{\partial \dot{T}} + \sum_{i=1}^{4} \left(\frac{\partial \mathcal{L}}{\partial \dot{\psi}_i} \nu_j + \delta_j \frac{\partial \mathcal{L}}{\partial \dot{\psi}_i^\dagger} \right) = const \tag{49}$$

Here ξ^j is some symmetry generators, q^j is variables.

Further, we will be investigate system of equations from (39) to (48). From (40), (41) and (42) one can see that α is only a function depends of $a(t)$ i.e scale factor.

$$\alpha = \alpha(a) \tag{50}$$

From the equation (47) we rewrite as

$$\frac{3\alpha V}{aV'} = -\sum_{i=1}^{4} \left(\epsilon_i \nu_i \psi_i^\dagger + \epsilon_i \delta_i \psi_i \right) \tag{51}$$

On the other hand, from (48) we have

$$\beta a F_{TT} T = 3\alpha F - 3\alpha F_T T + aF\frac{h'}{h}\sum_{i=1}^{4}\left(\epsilon_i \nu_i \psi_i^\dagger + \epsilon_i \delta_i \psi_i\right) - aF_T T \frac{h'}{h}\sum_{i=1}^{4}\left(\epsilon_i \nu_i \psi_i^\dagger + \epsilon_i \delta_i \psi_i\right) \quad (52)$$

If we will multiply T in both sides of (39) and substituting equalities (52) into it with (51), we find

$$\frac{a}{\alpha}\frac{d\alpha}{da} - 1 + \frac{3F}{2F_T T} - \frac{3F}{2F_T T}\left(\frac{h'}{h}\frac{V}{V'}\right) = 0 \quad (53)$$

In (53) we denote

$$\frac{h'}{h}\frac{V}{V'} = m \quad (54)$$

So we have

$$\frac{a}{\alpha}\frac{d\alpha}{da} - 1 + \frac{3F}{2F_T T} - \frac{3Fm}{2F_T T} = n \quad (55)$$

For convenience to solve, we can rewrite above equation as

$$\frac{2a}{3\alpha}\frac{d\alpha}{da} - \frac{2}{3} = \frac{F}{F_T T} - \frac{Fm}{F_T T} = n \quad (56)$$

Now, by using methods separation of variables, we easy to find the solutions of these two differential equations, namely

$$\alpha(a) = \alpha_0 a^{\frac{3}{2}n+1} \quad (57)$$

and

$$F(T) = F_0 T^{\frac{m-1}{n}} \quad (58)$$

where α_0 and F_0 is integration constants. Obviously, the form of $\alpha(a)$ and $F(T)$ are power-law. By substitute (51), (57) and (58) into (52), we find

$$\beta(a,T) = \frac{3\alpha_0 n(m-n-1)}{m-1}a^{\frac{3}{2}n}T \quad (59)$$

Now, from Eqs.(43), (44), (45) and (46) we can obtain solution of symmetry generators ν_i and δ_i as follows

$$\nu_j = -(\frac{3}{2}\alpha_0 a^{\frac{3}{2}n} + \epsilon_j \nu_0)\psi_j, \quad (60)$$

$$\delta_j = -(\frac{3}{2}\alpha_0 a^{\frac{3}{2}n} - \epsilon_j \nu_0)\psi_j^\dagger. \quad (61)$$

Now we can derive from (51) and (54) coupling function and potential as
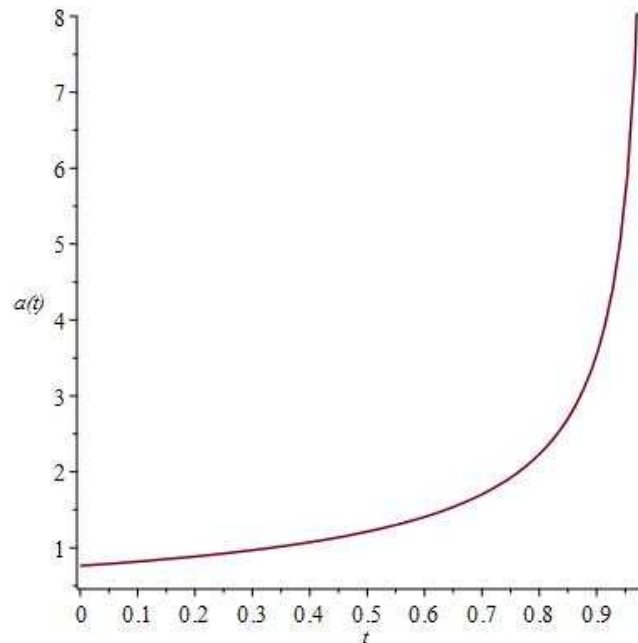
$$V(u) = V_0 u, \quad (62)$$

$$h(u) = h_0 u^m \quad (63)$$

where V_0 and h_0 is constants of integration.

To find first integral or conserved quantity (Noether charge), by substitute the (31), (57), (59), (60) and (61) into (49) we have following expression

$$Q = -12\alpha h F_T \dot{a} a \quad (64)$$

Figure 1. Scale factor $a(t)$ depends on cosmic time t, where $a_0=1$, $c_1=1$, $n=1$

5. Exact cosmological solutions

In this section, we would like to integrate dynamical system, by using equations for fermionic fields (23) and (24). Here, one can write an evolution equation for the bilinear u function which reads

$$\dot{u} + 3Hu = 0, \qquad \text{so that} \qquad u = \frac{u_0}{a^3}, \tag{65}$$

where u_0 is a constant.

In this section we examine general case. By substitute (57), (58), (62), and (63) into (28) with (29) we have

$$\dot{a} = a_0 a^{\frac{3n+2}{2}}, \qquad \text{where} \qquad a_0 = \left(\frac{F_0 h_0 (n - 2m + 2)}{6^{\frac{1-m}{n}} u_0^{1-m} V_0 n} \right)^{\frac{n}{2-2m}} \tag{66}$$

The solution of differential equation

$$a(t) = \left(\frac{2}{3c_1 - 3a_0 n t} \right)^{\frac{2}{3n}} \tag{67}$$

where c_1 is integration constant.

In Fig.1 shown behavior of scale factor of the Universe for our model. In this case we see that in late time of evolution, the parameter $a(t)$ quickly increase.

6. Conclusions

In this paper, we have studied a model with a fermionic field that is non-minimally coupled to gravity in the framework of $F(T)$ gravity. By using the Noether symmetry approach we determined form of the potential and coupled function as function of bilinear u given as $V(u) = V_0 u$ and $h(u) = h_0 u^m$. The cosmological solution of the field equations for FRW

spacetime is presented by using the results obtained from the Noether symmetry approach. Our result shown that fermion field may considered as source of cosmic acceleration in framework $F(T)$ gravity.

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