

PAPER • OPEN ACCESS

## gravity with scalar field viscous fluid

To cite this article: S R Myrzakul *et al* 2021 *J. Phys.: Conf. Ser.* **1730** 012021

View the [article online](#) for updates and enhancements.

### You may also like

- [On the question of numerical modeling of the flow stability of bodies](#)  
A.N. Maksimov, L.A. Maksimova, N.N. Arinina et al.
- [Generalized optimality criteria of energy-efficient composites](#)  
V Kersh, A Kolesnikov, N Khlytsov et al.
- [About control of convection intensity in chemical reacting equilibrium gas](#)  
I B Palymskiy, V I Palymskiy, P A Fomin et al.



**ECS** The Electrochemical Society  
Advancing solid state & electrochemical science & technology

## 242nd ECS Meeting

Oct 9 – 13, 2022 • Atlanta, GA, US

Early hotel & registration pricing ends September 12

Presenting more than 2,400 technical abstracts in 50 symposia

The meeting for industry & researchers in

**BATTERIES**  
**ENERGY TECHNOLOGY**  
**SENSORS AND MORE!**

 Register now!

  **ECS Plenary Lecture featuring M. Stanley Whittingham,**  
Binghamton University  
Nobel Laureate –  
2019 Nobel Prize in Chemistry



## $f(T)$ gravity with scalar field viscous fluid

S R Myrzakul<sup>1,2,3</sup>, M Imankul<sup>3,4</sup>, M Arzimbetova<sup>4</sup>, T R Myrzakul<sup>5</sup>

<sup>1</sup>Department of General and Theoretical Physics, L.N. Gumilyov Eurasian National University, Nur-Sultan, 010008, Kazakhstan <sup>2</sup>Department of Physics, School of Science and Technology,

<sup>2</sup>Nazarbayev University, Nur-Sultan, 010000, Kazakhstan

<sup>3</sup>Ratbay Myrzakulov Eurasian International Centre for Theoretical Physics, Nur-Sultan, 010008, Kazakhstan

<sup>4</sup>Department of Solid State Physics And Nolinean Physics, al-Farabi Kazakh National University, Almaty, 050040, Kazakhstan

<sup>5</sup>Department of physics, Kazakh National Women Teacher Training University Almaty 050000, Kazakhstan  
srmyrzakul@gmail.com

**Abstract.** We focus on a viable teleparallel cosmological model with a scalar field in a flat Friedman-Robertson-Walker universe. The Lagrangian and equations of motion are obtained. The case of an inhomogeneous viscous dark energy is considered and the cosmological parameters associated with the viscosity parameter are determined. It is shown that when considering constant viscosity and state parameter, the Universe expands exponentially

### 1. Introduction

With the discovery of the accelerated expansion of the Universe at the end of the last millennium [1,2], it became necessary to revise the entire picture of the universe. The generally accepted standard theory, which has successfully described the evolution of the universe for many years, has become unsuitable on a cosmological scale. For about thirty years now, scientists have been trying to create a theory describing the current state of the universe. The first attempts are to add some super-energy to the composition of the Universe, which is called dark energy [3], since its nature is not yet known and it manifests itself only in gravitational interaction. Another attempt is all kinds of modification of Einstein's general theory of relativity [3] or alternative theories [4]. The modification of Einstein's theory began with the introduction of a high order of curvature into the Hilbert-Einstein action, for example, the Starobinsky model [5], which he proposed even before the discovery of the fact of accelerated expansion in 1981. This model is widely used in cosmology. No less well-known modification is taking into account the torsion of space-time or teleparallel gravity (for example Rev. [6]).

A modified teleparallel theory of gravity with a torsion scalar has recently received a lot of attention as a possible explanation for dark energy. The role of torsion in gravity has been extensively studied in the main direction - the approximation of gravity to its gauge formulation and the inclusion of rotation in the geometric description. For example, work [7] considered various torsion structures, from teleparallel to Einstein-Cartan theory and metric-affine gauge theories, leading to the expansion



of torsion gravity in the paradigm of  $f(T)$  gravity, where  $f(T)$  is an arbitrary function of the torsion scalar. Based on this theory, the authors considered the corresponding cosmological and astrophysical applications and showed that the cosmological solutions arising from gravity, both at the background level and at the level of perturbations, in different epochs of cosmic expansion. The gravitational construction can provide a theoretical interpretation of the acceleration of the universe in late time, an alternative to the cosmological constant, and it can easily adapt to a regular history of thermal expansion, including phases dominated by radiation and cold dark matter. A new class of  $f(T)$  models with k-essence is presented in [8]. In [9], the authors obtained the equations of motion for the Lemaitre-Tolman-Bondi metric in the cases of diagonal and off-diagonal tetrads. It was shown that, for diagonal tetrads, the equations of motion of the  $f(T)$  theory impose constant torsion or the same equations of general relativity, while in the case of an off-diagonal system, the equations are very different from the equations, and in the off-diagonal case shows, as it were, an increase in comparison with the diagonal mass of matter at its predominance in the universe. Also, power, exponential and quadratic exponential  $f(T)$  dependences on torsion were investigated in [10].

The modification of the Hilbert-Einstein action is performed not only due to the spatial Lagrangian, but also due to the matter Lagrangian. In [11], fluids with viscosity are investigated and various types of f-essences are considered that can lead to accelerated expansion [12]. Fluid representation of dark energy has many benefits. For example, apart from the fact that we can still use the formalism of general relativity through Friedman's equations, almost any modification of general relativity can be encoded in a fluid-like form, so studying inhomogeneous viscous fluids is one of the easiest ways to understand some of the general features of such an alternative theory. In this work, we considered a gravity model with an inhomogeneous viscous fluid. The aim of this work is to study the behavior of an inhomogeneous viscous fluid in  $f(T)$  gravity.

We use units of  $8\pi G = 1 = c$  and denote the gravitational constant,  $G_N$ , by  $k^2 = 8\pi G_N$ , such that  $G_N^{-1/2} = M_{Pl}$ ,  $M_{Pl} = 1.2 \times 10^{19} GeV$  being the Planck mass.

## 2. $f(T)$ gravity with scalar field theories

At first, the action for  $f(T)$  modified theory of gravity:

$$S = \int d^4x e \left[ K(\phi) f(T) + \frac{1}{2} \varepsilon_t \dot{\phi}^2 - V(\phi) \right] \quad (1)$$

where  $e = \det(e_\mu^a) = \sqrt{-g}$ ,  $e_\mu^a$  is the vierbein or tetrad fields (Greek indices run over the coordinate space-time and Latin indices run over the tangent space-time), which is used as a dynamical object in teleparallel gravity and  $g = \det(g_{\mu\nu})$  is the space-time metric tensor,  $K(\phi)$  is the function dependent on  $\phi$ -scalar field,  $f(T)$  is a function dependent on  $T$ ,  $T$  is torsion,  $V(\phi)$  is a potential energy of the scalar field. As usual the 'dot' over the symbol means derivative with respect to cosmological time  $t$ . Here  $\varepsilon = +1, -1$  represent quintessence and phantom (or ghost) dark energies, respectively.

We will deal with the well-known Friedman-Robertson-Walker (FRW) isotropic and homogeneous models in spatial flat metric

$$ds^2 = -dt^2 + a(t)^2 (dx^2 + dy^2 + dz^2) \quad (2)$$

where  $a(t)$  is the cosmological scale factor. In order to derive the cosmological equations in a FRW metric, one can define a canonical Lagrangian  $L = L(a, \dot{a}, \phi, \dot{\phi}, T)$ , where  $Q = \{a, \phi, T\}$  is the

configuration space and  $GQ = \{a, \dot{a}, \phi, \dot{\phi}, T\}$  is the related tangent bundle on which  $L$  is defined. Next, we will omit the function dependencies to keep things simple. One can use the method of the Lagrange multipliers to set  $T$  as a constraint of the dynamics. Selecting the suitable Lagrange multiplier and integrating by parts, the Lagrangian  $L$  becomes canonical. In our case, we have

$$S = 2\pi^2 \int dt a^3 \left[ K(\phi)f(T) - \lambda \left( T + 6 \frac{\dot{a}^2}{a^2} \right) + \frac{1}{2} \dot{\phi}^2 - V(\phi) \right] \quad (3)$$

here the Lagrange multiplier is  $\lambda = K \frac{df(T)}{dT}$ .

Then the point-like Lagrangian reads (ignoring a constant  $2\pi^2$ )

$$L = a^3 Kf - a^3 Kf_T T - 6\dot{a}^2 a Kf_T + \frac{1}{2} a^3 \dot{\phi}^2 - a^3 V. \quad (4)$$

After that, using the Euler-Lagrange equations for this Lagrangian function and zero condition energy we obtain following equations of motion

$$3H^2 = \rho_{eff} \quad (5)$$

$$-(2\dot{H} + 3H^2) = p_{eff} \quad (6)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = K_\phi [f - f_T (T + 6H^2)] \quad (7)$$

$$-a^3 Kf_{TT} (T + 6H^2) = 0 \quad (8)$$

where  $H = \frac{\dot{a}(t)}{a(t)}$  denotes the Hubble parameter and indices mean derivatives on given variables,  $\rho_{eff}$

and  $p_{eff}$  are the effective energy density and pressure of the modified gravity model

$$\rho_{eff} = \frac{1}{2} \left[ -\frac{f}{f_T} + T + \left( \frac{1}{2} \dot{\phi}^2 + V \right) \frac{1}{Kf_T} \right] \quad (9)$$

$$p_{eff} = \frac{1}{2} \left[ \frac{f}{f_T} - T + \left( \frac{1}{2} \dot{\phi}^2 - V \right) \frac{1}{Kf_T} + 4H \left( \frac{K_\phi}{K} \dot{\phi} + \frac{f_{TT}}{f_T} \dot{T} \right) \right]. \quad (10)$$

### 3. Inhomogeneous viscous fluids in flat FRW space-time

Thus, if an equation of state (EoS) connecting the pressure  $p$  and energy density  $\rho$  is chosen in the form

$$p = \omega(\rho)\rho - 3H\xi(H), \quad (11)$$

where  $\xi(H)$  is the bulk viscosity and in our ansatz, it depends only on the Hubble parameter,  $H(t)$ .

For the realistic model, we take viscosity term which introduces the effective pressure in the energy-momentum tensor [13,14],

$$p = \omega\rho - 3H\xi_0, \quad (12)$$

where  $\xi = \xi_0$  is the constant bulk viscosity.

Substituting (10) and (11) into (12) we obtain,

$$H(t) = \frac{\xi_0}{1 + e^{\frac{3}{2}\xi_0 t} \xi_0 + \omega} \quad (13)$$

Then using parameter of Hubble, we find time-dependent scale factor

$$a(t) = a_0 \left( (1 + \omega)e^{\frac{3}{2}\xi_0 t} + \xi_0 \right)^{\frac{2}{3(\omega+1)}} \quad (14)$$

Let us start from an inhomogeneous fluid with EoS parameter equal -1.

$$H(t) = e^{\frac{3}{2}\xi_0 t}; \quad a(t) = a_0 \exp\left(\frac{2}{3\xi_0} e^{\frac{3}{2}\xi_0 t}\right); \quad T(t) = -6e^{3\xi_0 t} \quad (15)$$

and let's assume that  $f(T) = e^{\beta T} = \exp[-6\beta e^{3\xi_0 t}]$ ,  $\beta$  is a constant. Consider a massless scalar field with potential  $V(\phi) \sim \phi^2$  and a similar dependence of the coupling function  $K(\phi) \sim \phi^2$ . From the Dirac equation (7) taking into account (8), when equating all constants to unity, we obtain

$$\begin{aligned} \phi(t) = & e^{-\frac{3}{2}\xi_0 e^t + \frac{1}{2}t} \left[ Bessell\left(-\frac{1}{2} + i\sqrt{1 - \exp[-6\beta e^{3\xi_0 t}]}; \frac{3}{2}\xi_0 e^t\right) + Bessell\left(\frac{1}{2} + i\sqrt{1 - \exp[-6\beta e^{3\xi_0 t}]}; \frac{3}{2}\xi_0 e^t\right) \right] + \\ & + e^{\frac{3}{2}\xi_0 e^t + \frac{1}{2}t} \left[ -BesselK\left(-\frac{1}{2} + i\sqrt{1 - \exp[-6\beta e^{3\xi_0 t}]}; \frac{3}{2}\xi_0 e^t\right) + BesselK\left(\frac{1}{2} + i\sqrt{1 - \exp[-6\beta e^{3\xi_0 t}]}; \frac{3}{2}\xi_0 e^t\right) \right]. \end{aligned} \quad (16)$$

$$\rho_{eff} = 3e^{3\xi_0 t}, \quad (17)$$

$$p_{eff} = -3e^{\frac{3}{2}\xi_0 t} \left( e^{\frac{3}{2}\xi_0 t} - \xi_0 \right). \quad (18)$$

#### 4. Conclusion

The study of inhomogeneous viscous fluids in the Friedman-Robertson-Walker Universe is one of the important problems of cosmology. This approach gives a more real and general form of the equation of state. Therefore, it can be used both in describing the current accelerated expansion of the Universe and the inflationary period. Within the framework of modified theories of gravity, describing dark energy, an inhomogeneous viscous fluid has been investigated in many works, but within the

framework of  $f(T)$  torsion gravity it has not yet been fully studied. This article is devoted to the study of a modified model of gravity, namely  $f(T)$  torsion gravity is not minimally coupled with a scalar field  $\phi$ . The equations of motion of this model are obtained and the connection with an inhomogeneous viscous fluid is investigated. A constant viscosity with a constant parameter is considered. All cosmological parameters describing the dynamics of the model are obtained. Analyzing with respect to the de Sitter model in which the Hubble parameter is constant, in our model it depends on the value of the viscosity parameter and changes exponentially with respect to time. Also, all other parameters depend on the value of the viscosity parameter. With a positive viscosity, the universe expands exponentially, see equations (14) and (15).

## 5 References

- [1] Riess A G *et al.* 1998 *Astronomical Journal* **116** 3
- [2] Perlmutter S *et al.* 1998 *Nature* **391** 6662
- [3] Munshi D Porciani C Wang Y 2003 *Monthly notices of the royal astronomical society* MNRAS **349**
- [4] Elizalde E Myrzakulov R Obukhov V V Sáez-Gómez D 2010 *Classical and Quantum Gravity* **27** 9
- [5] Myrzakul S R Belisarova F B Myrzakul T R 2017 *News of The National Academy of Sciences of the Republic of Kazakhstan-Series Physico-mathematical* **5** 315
- [6] Myrzakul S R Yerzhanov K Kenzhalin D Z Myrzakulov K R 2019 *News of The National Academy of Sciences of the Republic of Kazakhstan-Series Physico-mathematical* **5** 327
- [7] Cai Y F Capozziello S De Laurentis M Saridakis E N 2016 *Reports on Progress in Physics* **79** 10
- [8] Myrzakulov R 2012 *General Relativity and Gravitation* **44** 12
- [9] Rodrigues M E Daouda H D Houndjo M J S Myrzakulov R Sharif M 2014 *Gravitation and Cosmology* **20** 2 pp. 80-89
- [10] Rodrigues, M E Houndjo M J S Momeni D Myrzakulov R 2013 *International Journal of Modern Physics D* **22** 8 1350043
- [11] Myrzakul S Arzimbetova M Imankul M Myrzakulov R 2019 *8th international conference on mathematical modeling in physical science* **1391** 012167
- [12] Percival W J 2005 *Astronomy & Astrophysics* A&A **443** 3 pp. 819 -830.
- [13] Capozziello S Cardone V F Elizalde E Nojiri S Odintsov S D 2006 *Physical Review D* **73** 4
- [14] Myrzakulov R Sebastiani L and Zerbini S 2013 *Galaxies* **1** 2

## Acknowledgments

This research is funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP08052197)