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Traveling wave solutions for the (3+1)-dimensional Davey-Stewartson equations

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Abstract. In this work, the extended tanh method is used to construct wave solutions for the Davey–Stewartson equations. The extended tanh method is a powerful solution method for obtaining different kind solutions of nonlinear evolution equations. This method can be applied to nonintegrable equations as well as to integrable ones.

1. Introduction

Integrable equations are quite interesting and a lot of their properties such as integrability, exact solutions are well studied in 1+1 dimensions [1-3], in 2+1 dimensions [4-10]. Two well-known examples of integrable equations in two space dimensions, which arise as higher dimensional generalizations the nonlinear Schrodinger (NLS) equation, is the classical Davey-Stewartson (DS) [11]. The solutions for the DS equation were studied in various aspects previously. The bifurcation method was used to study the exact traveling wave solutions of the generalized DS equations [12], explode-decay dromions through Hirota method was found in [13], the extended Weierstrass transformation method was applied in [14], approximate analytical solutions for the fractional DS equations using the variational iteration method were found in [15], the rational expansion method was used to construct periodic and solitary wave solutions of the DS equations [16].

In this paper, we study the (3+1)-dimensional Davey-Stewartson equations as

$$i\psi_t + \psi_{xx} + \alpha_1\psi_{yy} + \psi_{zz} - \alpha_2|\psi|^2\psi - \psi w = 0, \quad (1)$$
$$w_{xx} + \beta_1 w_{yy} + w_{zz} - \beta_2(|\psi|^2)_{yy} = 0, \quad (2)$$

where ψ is complex while w is real, $\alpha_1, \alpha_2, \beta_1, \beta_2$ are nonzero real constants. Lie symmetry algebra of the (3+1)-dimensional DS system (1)-(2) was studied in [17] and multiple scales asymptotic expansion method was applied in [18].

The main focus of the present work is to study equations (1)-(2) by extended tanh method that is considered to be the most effective and direct algebraic method for solving nonlinear equations [19-22].

Our paper is organized as follows: in Section II, we consider the (3+1)-dimensional DS equations and obtain traveling wave solutions through the extended tanh method. Sect. III gives out the summary of this paper.

2. Traveling wave solutions

In this part, we construct exact traveling wave solutions of the (3+1)-dimensional DS using the extended tanh method [19]. For applying this method, we ought to reduce the system (1)-(2) to system of ordinary differential equation. We use the transformation

$$\psi(x,y,z,t) = e^{i(ax+by+sz+dt)}\Psi(x,y,z,t), \tag{3}$$

where a, b, s, d are constants, $\Psi(x,y,z,t)$ is real valued function, then the system (1)-(2) reduced to following system of differential equations

$$\Psi(-d-a^2-\alpha_1 b^2-s^2)+\Psi_{xx}+\alpha_1\Psi_{yy}+\Psi_{zz}-\alpha_2\Psi^3-\Psi w = 0, \tag{4}$$

$$\Psi_t+2a\Psi_x+2b\Psi_y+2s\Psi_z = 0, \tag{5}$$

$$w_{xx}+\beta_1 w_{yy}+w_{zz}-\beta_2(\Psi^2)_{yy} = 0. \tag{6}$$

Substituting wave transformation

$$\Psi(x,y,z,t) = \Psi(\xi) = \Psi(x+y+z-ct), \tag{7}$$

$$w(x,y,z,t) = W(\xi) = W(x+y+z-ct), \tag{8}$$

into system (4)-(6), we obtain that

$$\Psi(-d-a^2-\alpha_1 b^2-s^2)+\Psi^{''}(2+\alpha_1)-\alpha_2\Psi^3-\Psi W = 0, \tag{9}$$

$$\Psi^{'}(-c+2a+2b+2s) = 0, \tag{10}$$

$$W^{''}(2+\beta_1)-\beta_2(\Psi^2)'' = 0. \tag{11}$$

From equation (10) we can obtain that

$$c = 2(a+b+s). \tag{12}$$

Integration twice equation (11) with respect to ξ and taking integration constants are zero for simplicity, we find

$$W = \frac{\beta_2\Psi^2}{2+b_1}. \tag{13}$$

By substituting equation (13) into the equation (9), we obtain following ordinary differential equation

$$\Psi(-d-a^2-\alpha_1 b^2-s^2)(2+\beta_1)+\Psi^{''}(2+\alpha_1)(2+\beta_1)-\alpha_2\Psi^3(2+\beta_1)-\beta_2\Psi^3 = 0, \tag{14}$$

where prime denotes the derivation with respect to ξ. The extended tanh method consists of using the new independent variable $Y = tanh(\mu\xi)$, that leads to the following changes of variable:

$$\frac{d\Psi}{d\xi} = \mu(1-Y^2)\frac{d\Psi}{dY}, \tag{15}$$

$$\frac{d^2\Psi}{d\xi^2} = -2\mu^2 Y(1-Y^2)\frac{d\Psi}{dY}+\mu^2(1-Y^2)^2\frac{d^2\Psi}{dY^2}, \tag{16}$$

where μ is the wave number, $\xi = x+y+z-ct$. Assume that the solution is expressed in the form

$$\Psi(\xi) = \sum_{i=0}^{m} a_i Y^i + \sum_{i=1}^{m} b_i Y^{-i}, \tag{17}$$

where the parametrs m can be found from (14) by balancing the nonlinear term Ψ^3, that has the exponent $3m$, with the highest order derivative Ψ'', that has the exponent $m+2$, in (14) yields $3m = m+2$ that gives $m = 1$. Than the extended tanh method admits the use of the finite expansion for

$$\Psi(\xi) = a_0 + a_1 Y + \frac{b_1}{Y}. \tag{18}$$

Coefficients a_0, a_1, b_1, μ are to be determined. Substituting (18) into (14) and equating expressions at $Y^{-3}, Y^{-2}, Y^{-1}, Y^0, Y^1, Y^2, Y^3$ to zero we have the following system of equations:

$$-a_1^3\alpha_2\beta_1 + 2a_1\alpha_1\beta_1\mu^2 - 2a_1^3\alpha_2 - a_1^3\beta_2 + 4a_1\alpha_1\mu^2 + 4a_1\beta_1\mu^2 + 8a_1\mu^2 = 0, \tag{19}$$

$$-3a_0a_1^2\alpha_2\beta_1 - 6a_0a_1^2\alpha_2 - 3a_0a_1^2\beta_2 = 0, \tag{20}$$

$$\begin{aligned}-3a_0^2a_1\alpha_2\beta_1 - 3a_1^2\alpha_2b_1\beta_1 - a_1\alpha_1b^2\beta_1 - 2a_1\alpha_1\beta_1\mu^2 - a^2a_1\beta_1 - 6a_0^2a_1\alpha_2 -\\ -3a_0^2a_1\beta_2 - 6a_1^2\alpha_2b_1 - 3a_1^2b_1\beta_2 - 2a_1\alpha_1b^2 - 4a_1\alpha_1\mu^2 - 4a_1\beta_1\mu^2 -\\ -a_1\beta_1s^2 - 2a^2a_1 - a_1\beta_1d - 8a_1\mu^2 - 2a_1s^2 - 2a_1d = 0,\end{aligned} \tag{21}$$

$$\begin{aligned}-a_0^3\alpha_2\beta_1 - 6a_0a_1b_1\alpha_2\beta_1 - a_0\alpha_1b^2\beta_1 - a_0a^2\beta_1 - 2a_0^3\alpha_2 - a_0^3\beta_2 - 12a_0a_1b_1\alpha_2 -\\ -6a_0a_1b_1\beta_2 - 2a_0\alpha_1b^2 - a_0s^2\beta_1 - 2a_0a^2 - a_0d\beta_1 - 2a_0s^2 - 2a_0d = 0,\end{aligned} \tag{22}$$

$$\begin{aligned}-3a_0^2\alpha_2b_1\beta_1 - 3a_1\alpha_2b_1^2\beta_1 - \alpha_1b^2b_1\beta_1 - 2\alpha_1b_1\beta_1\mu^2 - a^2b_1\beta_1 -\\ -6a_0^2\alpha_2b_1 - 3a_0^2b_1\beta_2 - 6a_1\alpha_2b_1^2 - 3a_1b_1^2\beta_2 - 2\alpha_1b^2b_1 - 4\alpha_1b_1^2\mu^2 -\\ -4b_1\beta_1\mu^2 - b_1\beta_1s^2 - 2a^2b_1 - b_1\beta_1d - 8b_1\mu^2 - 2b_1s^2 - 2b_1d = 0,\end{aligned} \tag{23}$$

$$-3a_0\alpha_2b_1^2\beta_1 - 6a_0\alpha_2b_1^2 - 3a_0b_1^2\beta_2 = 0, \tag{24}$$

$$2\alpha_1b_1\beta_1\mu^2 - \alpha_2b_1^3\beta_1 + 4\alpha_1b_1\mu^2 - 2\alpha_2b_1^3 - b_1^3\beta_2 + 4b_1\beta_1\mu^2 + 8b_1\mu^2 = 0. \tag{25}$$

Solving system (19)-(25) with the aid of Maple, we obtain the following results:

Case 1:

$$a_0 = 0, \quad a_1 = 0, \quad c = 2(a+b+s), \tag{26}$$

$$b_1 = \pm\sqrt{-\frac{(d+a^2+\alpha_1b^2+s^2)(2+\beta_1)}{\alpha_2(\beta_1+2)+\beta_2}}, \quad \mu = \pm\sqrt{-\frac{\alpha_1b^2+a^2+s^2+d}{2\alpha_1+4}}. \tag{27}$$

Case 2:

$$a_0 = 0, \quad b_1 = 0, \quad c = 2(a+b+s), \tag{28}$$

$$a_1 = \pm\sqrt{-\frac{(d+a^2+\alpha_1b^2+s^2)(2+\beta_1)}{\alpha_2(\beta_1+2)+\beta_2}}, \quad \mu = \pm\sqrt{-\frac{\alpha_1b^2+a^2+s^2+d}{2\alpha_1+4}}. \tag{29}$$

Case 3:

$$a_0 = 0, \quad c = 2(a+b+s), \quad a_1 = \pm\sqrt{\frac{(d+a^2+\alpha_1b^2+s^2)(2+\beta_1)}{2\alpha_2(\beta_1+2)+2\beta_2}}, \tag{30}$$

$$b_1 = \mp\frac{1}{2}\frac{(d+a^2+\alpha_1b^2+s^2)(2+\beta_1)}{\sqrt{\frac{(d+a^2+\alpha_1b^2+s^2)(2+\beta_1)}{2\alpha_2(\beta_1+2)+\beta_2}}(\alpha_2(\beta_1+2)+\beta_2)}, \quad \mu = \pm\sqrt{\frac{\alpha_1b^2+a^2+s^2+d}{4\alpha_1+8}}. \tag{31}$$

Case 4:

$$a_0 = 0, \quad c = 2(a+b+s), \quad a_1 = \pm\sqrt{-\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{4\alpha_2(\beta_1+2)+4\beta_2}}, \tag{32}$$

$$b_1 = \mp\frac{1}{4}\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{-\sqrt{\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{2\alpha_2(\beta_1+2)+\beta_2}}(\alpha_2(\beta_1+2)+\beta_2)}, \quad \mu = \pm\sqrt{-\frac{\alpha_1 b^2+a^2+s^2+d}{8\alpha_1+16}}. \tag{33}$$

Corresponding expressions for $\psi(x,y,z,t), w(x,y,z,t)$ are

$$\psi(x,y,z,t) = e^{i(ax+by+dt+sz)}(a_0 + a_1\tanh(\mu\xi) + b_1\coth(\mu\xi)), \tag{34}$$

$$w(x,y,z,t) = \frac{\beta_2}{2+b_1}(a_0 + a_1\tanh(\mu\xi) + b_1\coth(\mu\xi))^2, \tag{35}$$

where $\xi = x+y+z-ct$.

Finally, we substitute results (26)-(33) into (34)-(35) and obtain new traveling wave solutions for (3+1)-dimensional DS system (1)-(2) in the following forms

$$\psi_1(x,y,z,t) = \pm e^{i(ax+by+dt+sz)}\left(\sqrt{-\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{\alpha_2(\beta_1+2)+\beta_2}}\coth\left(\pm\sqrt{-\frac{\alpha_1 b^2+a^2+s^2+d}{2\alpha_1+4}}\xi\right)\right),$$

$$w_1(x,y,z,t) = \pm\frac{\beta_2}{2+b_1}\left(\sqrt{-\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{\alpha_2(\beta_1+2)+\beta_2}}\coth\left(\pm\sqrt{-\frac{\alpha_1 b^2+a^2+s^2+d}{2\alpha_1+4}}\xi\right)\right)^2,$$

$$\psi_2(x,y,z,t) = \pm e^{i(ax+by+dt+sz)}\left(\sqrt{-\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{\alpha_2(\beta_1+2)+\beta_2}}\tanh\left(\pm\sqrt{-\frac{\alpha_1 b^2+a^2+s^2+d}{2\alpha_1+4}}\xi\right)\right),$$

$$w_2(x,y,z,t) = \pm\frac{\beta_2}{2+b_1}\left(\sqrt{-\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{\alpha_2(\beta_1+2)+\beta_2}}\tanh\left(\pm\sqrt{-\frac{\alpha_1 b^2+a^2+s^2+d}{2\alpha_1+4}}\xi\right)\right)^2,$$

$$\psi_3(x,y,z,t) = e^{i(ax+by+dt+sz)}\left(\pm\sqrt{\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{2\alpha_2(\beta_1+2)+\beta_2}}\tanh\left(\pm\sqrt{\frac{\alpha_1 b^2+a^2+s^2+d}{4\alpha_1+8}}\xi\right)\mp\right.$$
$$\left.\mp\frac{1}{2}\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{\sqrt{\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{2\alpha_2(\beta_1+2)+\beta_2}}(\alpha_2(\beta_1+2)+\beta_2)}\coth\left(\pm\sqrt{\frac{\alpha_1 b^2+a^2+s^2+d}{4\alpha_1+8}}\xi\right)\right),$$

$$w_3(x,y,z,t) = \frac{\beta_2}{2+b_1}\left(\pm\sqrt{\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{2\alpha_2(\beta_1+2)+2\beta_2}}\tanh\left(\pm\sqrt{\frac{\alpha_1 b^2+a^2+s^2+d}{4\alpha_1+8}}\xi\right)\mp\right.$$
$$\left.\mp\frac{1}{2}\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{\sqrt{\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{2\alpha_2(\beta_1+2)+\beta_2}}(\alpha_2(\beta_1+2)+\beta_2)}\coth\left(\pm\sqrt{\frac{\alpha_1 b^2+a^2+s^2+d}{4\alpha_1+8}}\xi\right)\right)^2,$$

$$\psi_4(x,y,z,t) = e^{i(ax+by+dt+sz)}(\pm\sqrt{-\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{4\alpha_2(\beta_1+2)+\beta_2}}tanh(\pm\sqrt{-\frac{\alpha_1 b^2+a^2+s^2+d}{8\alpha_1+16}}\xi)\mp$$

$$\mp\frac{1}{4}\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{-\sqrt{\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{2\alpha_2(\beta_1+2)+\beta_2}}(\alpha_2(\beta_1+2)+\beta_2)}\coth(\pm\sqrt{-\frac{\alpha_1 b^2+a^2+s^2+d}{8\alpha_1+16}}\xi)),$$

$$w_4(x,y,z,t) = \frac{\beta_2}{2+b_1}(\pm\sqrt{-\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{4\alpha_2(\beta_1+2)+\beta_2}}\tanh(\pm\sqrt{-\frac{\alpha_1 b^2+a^2+s^2+d}{8\alpha_1+16}}\xi)\mp$$

$$\mp\frac{1}{4}\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{-\sqrt{\frac{(d+a^2+\alpha_1 b^2+s^2)(2+\beta_1)}{2\alpha_2(\beta_1+2)+\beta_2}}(\alpha_2(\beta_1+2)+\beta_2)}\coth(\pm\sqrt{-\frac{\alpha_1 b^2+a^2+s^2+d}{8\alpha_1+16}}\xi))^2,$$

where $\xi = x+y+z-2(a+b+s)t$.

3. Conclusion

In this paper, the extended tanh method which is direct, standard and computerizable, has been successfully applied to find traveling wave solutions of the (3+1)-dimensional Davey-Stewartson equations. The tedious computations associated with the algebraic calculations are facilitated using symbolic computation software such as Maple.

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