PAPER • OPEN ACCESS

Travelling wave solutions for the generalized Schrödinger equation

View the article online for updates and enhancements.

Travelling wave solutions for the generalized Schrödinger equation

G.N. Shaikhova[1]**, B.K. Rakhimzhanov**[2]**, and Zh.K. Zhanbosinova**[3]

[1] L.N.Gumilyov Eurasian National University, Department of General and Theoretical Physics, Nur-Sultan, 010008, Kazakhstan
[2] JSC "National company "Kazakhstan Gharysh Sapary", Nur-Sultan, 010008, Kazakhstan
[3] D. Serikbayev East Kazakhstan Technical University, Ust-Kamenogorsk, 070004, Kazakhstan

E-mail: g.shaikhova@gmail.com

Abstract. In this work, the generalized nonlinear Schrödinger equation is investigated. This equation is integrable and admits Lax pair. To obtain travelling wave solutions the extended tanh method is applied. This method is effective to obtain the exact solutions for different types of nonlinear partial differential equations. Graphs of obtained solutions are presented. The derived solutions are found to be important for the explanation of some practical physical problems.

1. Introduction

Nonlinear evolution equations can be used to describe many essential phenomena in physics and other fields [1-3]. As mathematical models of these phenomena, it is very necessary to search for exact solutions for nonlinear evolution equations in mathematical physics. There are various methods to define exact solutions of nonlinear evolution equations including the Hirota method [4-6], extended tanh method [7-9], Darboux transformation [10-12], sine-cosine method [13-14], Kudryashov method [15-18], and so on.

The well-known soliton equation is the nonlinear Schrödinger (NLS) equation which is one of the most important and universal nonlinear models in modern science. Various NLS equations are widely used to describe different nonlinear physical problems, such as nonlinear optics [19-20], Bose-Einstein condensates [21], and others.

One of the simplest extensions of the NLS equation is the so-called generalized nonlinear Schrodinger (GNLS) equation [22-23], which is

$$q_z = i(q_{tt} + 2|q|^2 q + \alpha q) + \gamma q_t, \tag{1}$$

where q is a complex-valued function of the spatial coordinate z and the time t, the subscripts denote the partial derivatives with respect to the variables z, t. The GNLS equation (1) is a typical soliton equation with rich physical and mathematical applications where α denotes the amplification or absorption and γ relates to the group velocity [22-23]. In the case $\alpha = 0, \gamma = 0$ in Eq. (1) we can get classical NLS equation. The Eq. (1) jointly with Maxwell–Bloch equations were studied in [24] where researchers found bound solitons and two types of breathers by the Darboux transformation.

The aim of this paper is to construct some new exact solutions for the Eq. (1). We study the Eq. (1) by the extended tanh method that has been widely applied for a wide variety of nonlinear problems.

2. Travelling wave solutions

In this section we apply the extended tanh method to GNLS equation (1). By transformation

$$q = e^{i(az+dt)}u(z,t), \tag{2}$$

the Eq. (1) can be converted to

$$iau + u_z + id^2u + 2du_t - iu_{tt} - 2iu^3 - i\alpha u - id\gamma u - \gamma u_t = 0. \tag{3}$$

Separating real and imaginary parts in the Eq. (3) we obtain next system

$$u_z + 2du_t - \gamma u_t = 0, \tag{4}$$
$$u(a + d^2 - \alpha - \gamma d) - u_{tt} - 2u^3 = 0. \tag{5}$$

Substituting the wave transformation

$$u(z,t) = u(\xi) = u(z - ct), \tag{6}$$

into system of Eqs. (4)-(5) we get

$$u'(1 - 2dc + \gamma c) = 0, \tag{7}$$
$$u(a + d^2 - \alpha - \gamma d) - c^2 u'' - 2u^3 = 0. \tag{8}$$

From Eq. (7) we can derive

$$c = \frac{1}{2d - \gamma}. \tag{9}$$

Next, we study ordinary differential equation (8)

$$u(a + d^2 - \alpha - \gamma d) - c^2 u'' - 2u^3 = 0, \tag{10}$$

where prime denotes the derivation with respect to ξ and c is expressed by (9).

To apply the extended tanh method we have to define a value M [7-9]. For this purpose, we balance the highest order derivative u'', which has the exponent $M + 2$, with the nonlinear term u^3, which has the exponent $3M$ in (10). It gives $3M = M + 2$ that yields $M = 1$. Then, the extended tanh method let to apply the substitution

$$u(\xi) = a_0 + a_1 Y + \frac{b_1}{Y}, \tag{11}$$

where $Y = \tanh(\mu\xi)$, $\xi = z - ct$, and μ is the wave number. Substituting (11) into (10) and collecting the coefficients of Y^n, then we have a system of algebraic equations for μ, a_0, a_1, b_1. By solving obtained system with the aid of Maple, we get the next results:

Case 1:

$$a_0 = 0, \quad a_1 = \pm\frac{1}{2}\sqrt{\frac{1}{2}(a + d^2 - \alpha - \gamma d)}, \tag{12}$$

$$b_1 = \pm\frac{1}{2}\sqrt{\frac{1}{2}(a + d^2 - \alpha - \gamma d)}, \quad \mu = \pm\frac{1}{2c}\sqrt{-\frac{1}{2}(a + d^2 - \alpha - \gamma d)}. \tag{13}$$

Case 2:

$$a_0 = 0, \quad a_1 = \mp\frac{1}{2}\sqrt{-(a + d^2 - \alpha - \gamma d)}, \tag{14}$$

$$b_1 = \pm\frac{1}{2}\sqrt{-(a + d^2 - \alpha - \gamma d)}, \quad \mu = \pm\frac{1}{2c}\sqrt{(a + d^2 - \alpha - \gamma d)}. \tag{15}$$

Case 3:

$$a_0 = 0, \quad a_1 = 0, \tag{16}$$

$$b_1 = \pm\sqrt{\frac{1}{2}(a + d^2 - \alpha - \gamma d)}, \quad \mu = \pm\frac{1}{c}\sqrt{-\frac{1}{2}(a + d^2 - \alpha - \gamma d)}. \tag{17}$$

Case 4:

$$a_0 = 0, \quad a_1 = \pm\sqrt{\frac{1}{2}(a + d^2 - \alpha - \gamma d)}, \tag{18}$$

$$b_1 = 0, \quad \mu = \pm\frac{1}{c}\sqrt{-\frac{1}{2}(a + d^2 - \alpha - \gamma d)}. \tag{19}$$

By substituting Eq. (11) with (6) into Eq. (2) we have general solutions as

$$q(z,t) = e^{i(az+dt)}(a_0 + a_1 \tanh(\mu\xi) + b_1 \coth(\mu\xi)), \tag{20}$$

where $\xi = z - ct$.

Finally, applying the coefficients (12)-(19) into Eq. (20), we derive travelling wave solutions for Eq.(1) in the next forms

Case 1:

$$
\begin{aligned}
q_1(z,t) &= e^{i(az+dt)}(\pm\frac{1}{2}\sqrt{\frac{1}{2}(a + d^2 - \alpha - \gamma d)} \tanh(\frac{1}{2c}\sqrt{-\frac{1}{2}(a + d^2 - \alpha - \gamma d)}\xi) + \\
&\quad \frac{1}{2}\sqrt{\frac{1}{2}(a + d^2 - \alpha - \gamma d)} \coth(\frac{1}{2c}\sqrt{-\frac{1}{2}(a + d^2 - \alpha - \gamma d)}\xi)),
\end{aligned} \tag{21}
$$

Case 2:

$$
\begin{aligned}
q_2(z,t) &= e^{i(az+dt)}(\mp\frac{1}{2}\sqrt{-(a + d^2 - \alpha - \gamma d)} \tanh(\frac{1}{2c}\sqrt{(a + d^2 - \alpha - \gamma d)}\xi) + \\
&\quad \frac{1}{2}\sqrt{-(a + d^2 - \alpha - \gamma d)} \coth(\frac{1}{2c}\sqrt{(a + d^2 - \alpha - \gamma d)}\xi)),
\end{aligned} \tag{22}
$$

Case 3:

$$q_3(z,t) = e^{i(az+dt)}(\sqrt{\frac{1}{2}(a + d^2 - \alpha - \gamma d)} \coth(\frac{1}{c}\sqrt{-\frac{1}{2}(a + d^2 - \alpha - \gamma d)}\xi)), \tag{23}$$

Case 4:

$$q_4(z,t) = e^{i(az+dt)}(\pm\sqrt{\frac{1}{2}(a + d^2 - \alpha - \gamma d)} \tanh(\frac{1}{c}\sqrt{-\frac{1}{2}(a + d^2 - \alpha - \gamma d)}\xi), \tag{24}$$

where $\xi = z - ct$, with $c = \frac{1}{2d-\gamma}$.

Graphical presentation of the solutions (21)-(24) are shown in Figure 1-Figure 4 on the $z - t$ plane with the parameters: $a = 1; \ d = 1; \alpha = 1; \gamma = 3$. As we notice from 3D plots and density plots the solutions q_1, q_3 give the bright solitons in contrast to q_4 that shows dark soliton. Moreover, periodic type solution q_2 is presented in Figure 2. Analyzing the graphs of obtained solutions we notice that the extended tanh method can yield various types of travelling wave solutions.
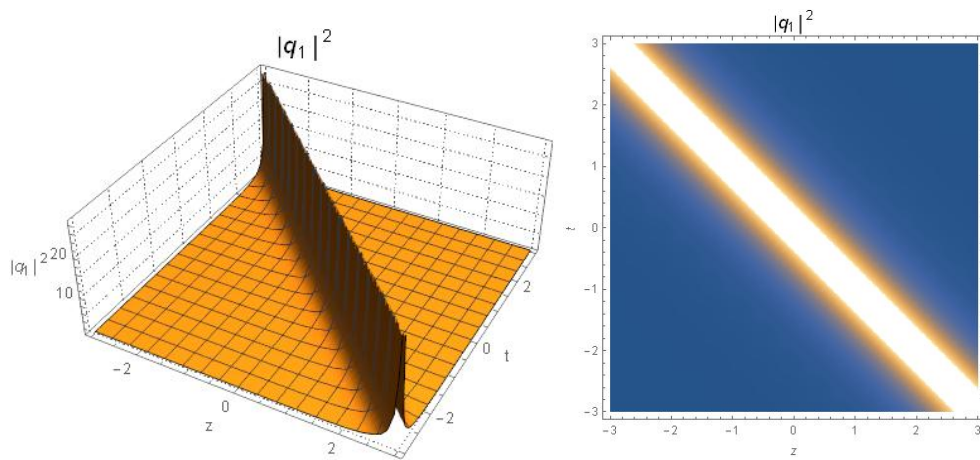
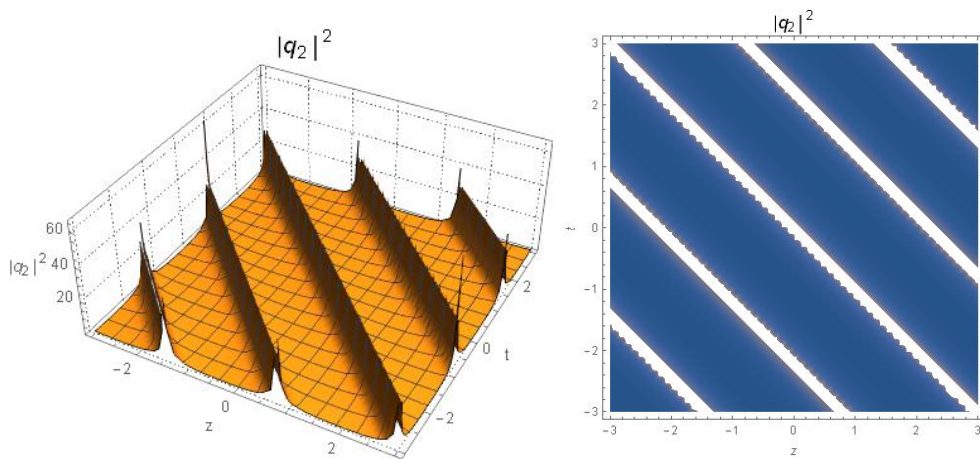Figure 1: Graph of solutions (21) with the parameters: $a = 1; \ d = 1; \alpha = 1; \gamma = 3$.

Figure 2: Graph of solutions (22) with the parameters: $a = 1; d = 1; \alpha = 1; \gamma = 3$.
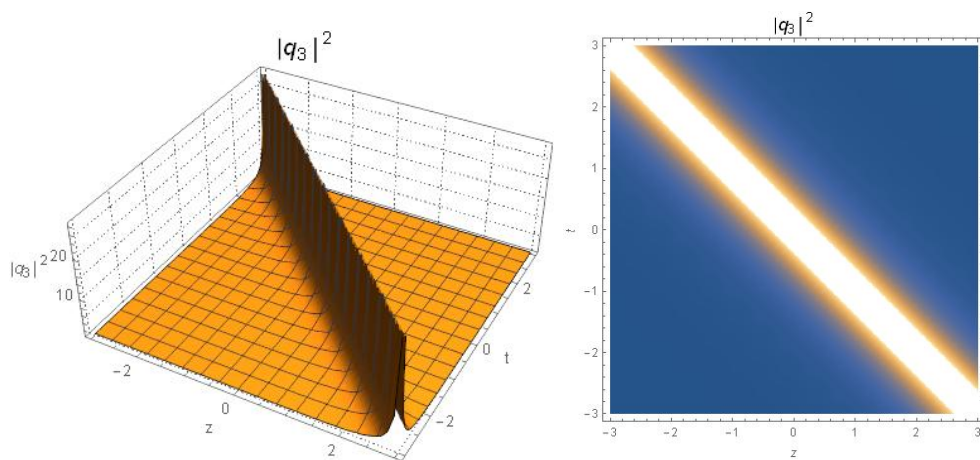

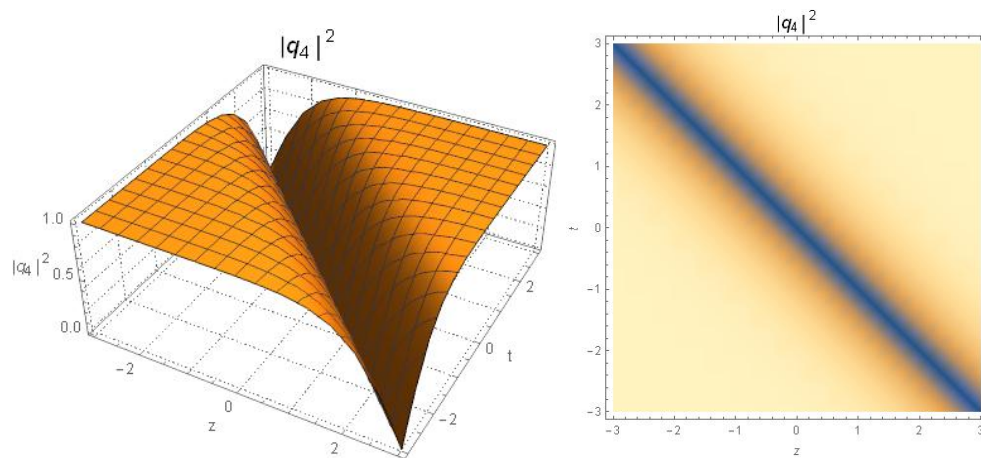Figure 3: Graph of solutions (23) with the parameters: $a = 1; \ d = 1; \alpha = 1; \gamma = 3$.

Figure 4: Graph of solutions (24) with the parameters: $a = 1; d = 1; \alpha = 1; \gamma = 3$.

3. Conclusions

In this work, the extended tanh method have been applied to the generalized nonlinear Schrodinger equation. We obtained various type of the travelling wave solutions such as solitons, periodic solution. Moreover, the graphical presentation of the obtained solutions is shown in the figures. The obtained solutions can have an application to some practical physical problems.

References

[1] Yesmakhanova K, Nugmanova G, Shaikhova G, Bekova G, and Myrzakulov R. 2020 *International Journal of Geometric Methods in Modern Physics* **17(7)** 2050104 (1–19).
[2] Nugmanova G, Sagidullayeva Zh and Myrzakulov R 2017 *J. Phys.: Conf. Series* **804** 012035.
[3] Nugmanova G, Sagidullayeva Zh 2017 *Bulletin of the Karaganda University of the-Mathematics* **86 (2)** 91–96.
[4] Kutum BB, Yesmakhanova K, Shaikhova G 2019 *J. Phys.: Conf. Series* **1391** 012122 (1–7).
[5] Kutum BB, Shaikhova G 2019 *Bulletin of the University of Karaganda-Physics* **3(95)** 22–26.
[6] Mukhanmedina KT, Syzdykova AM and Shaikhova GN *Bulletin of the Karaganda university-Mathematics* **80(4)** 103–107.
[7] Wazwaz AM 2009 *Partial differential equations and solitary waves theory* (Publisher: Springer, Berlin).
[8] Wazwaz AM 2007 *Applied Mathematics and Computation* **184**(2) 1002–1014.
[9] Serikbayev NS, Shaikhova G, Yesmakhanova K and Myrzakulov R 2019 *J. Phys.: Conf. Series* **1391** 012166 (1–6).
[10] Bekova G, Yesmakhanova K, Ozat N and Shaikhova G 2018 *J. Phys.: Conf. Series* **965** 012035 (1–10).
[11] Bekova G, Yesmakhanova K, Myrzakulov R and Shaikhova G 2017 *J. Phys.: Conf. Series* **965** 012045 (1–9).
[12] Shaikhova G, Serikbayev N, Yesmakhanova K, and Myrzakulov R. 2020 *Geometry, Integrability and Quantization* **21** 265–271.
[13] Kutum BB and Shaikhova G 2020 *Eurasian Physical Technical Journal* **17** 169–174.
[14] Shaikhova GN, Kutum BB, Altaybaeva AB, and Rakhimzhanov BK 2019 *J. Phys.: Conf. Series* **1416** 012030.
[15] Shaikhova G 2018 *Bulletin of the Karaganda University-Mathematics* **92** 94–98.
[16] Kudryashov NA 2012 *Commun. Nonlinear Sci. Numer. Simul.* **17** 2248–2253.
[17] Kudryashov NA 2004 *Moscow-Izhevsk: Institute of Computer Investigations* 360.
[18] Ryabov PN 2010 *Applied Mathematics and Computation* **217** 3585–3590.

[19] Kivshar Y and Agrawal G P 2003 Optical Solitons: From Fibers to Photonic Crystals (New York: Academic).
[20] Hasegawa A 1989 Optical Solitons in Fibers (Berlin: Springer).
[21] Dalfovo F, Giorgini S, Pitaevskii L P and Stringari S 1999 *Rev. Mod. Phys.* **71** 463–512.
[22] Agrawal G. P Nonlinear Fiber Optics (Academic Press, San Diego, 2001).
[23] Allen L and Eberly J H Optical Resonance and Two-Level Atoms (Wiley Press, New York, 1975).
[24] Guo R, Hao H and Zhang L 2013 *Modern Physics Letters* **27** 1350130.