ҚАЗАҚСТАН РЕСПУБЛИКАСЫ БІЛІМ ЖӘНЕ ҒЫЛЫМ МИНИСТРЛІГІ
Л.Н. ГУМИЛЕВ АТЫНДАҒЫ ЕУРАЗИЯ ҰЛТТЫҚ УНИВЕРСИТЕТІ

ЕУРАЗИЯ ҰЛТТЫҚ УНИВЕРСИТЕТІ

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The proceedings are the papers of students, undergraduates, doctoral students and young researchers on topical issues of natural and technical sciences and humanities.

В сборник вошли доклады студентов, магистрантов, докторантов и молодых ученых по актуальным вопросам естественно-технических и гуманитарных наук.

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предпочтение часто отдавалось витопарным кабелям ввиду дороговизны оптического сырья и оборудования, то сейчас по капитальным затратам и трудоемкости монтажа системы различаются незначительно. По-прежнему популярным является строительство совмещенного вида сетей - FTTH, где медная пара используется только на участке от коммутатора к абоненту. Однако динамика все больше смещается в сторону PON, в том числе и благодаря тому, что установка пассивной сети допускает модификацию без вмешательства в архитектуру системы и перекладки кабеля. По этому оно являются залогом успешного будущего оптических сетей доступа.

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ONE SOLITON SOLUTIONS FOR (1+1)-DIMENSIONAL INHOMOGENEOUS COMPLEX MODIFIED KORTEWEG-DE VRIES AND MAXWELL-BLOCH EQUATIONS

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Introduction

The theory of nonlinear science has attracted more interest and is fundamentally linked to several basic developments in the area of soliton theory. It is well-known that the Korteweg-de Vries (KdV) equation, modified Korteweg-de Vries (mKdV) equation, sine Gordon equation and the nonlinear Schrodinger (NLS) equation are the most typical and well-studied integrable evolution equations which describe nonlinear wave phenomena for a range of dispersive physical systems. The first in [1], homogeneous complex modified Korteweg-de Vries and Maxwell-Bloch (hcmKdV-MB) equations were presented. At present inhomogeneous integrable equations become more and more attractive [2], in the same paper the author considered the positons for the inhomogeneous Hirota and Maxwell-Bloch equations, which describes propagation through inhomogeneous doped fiber.

The purpose of this paper is to construct one-fold Darboux transformation for inhomogeneous complex modified Korteweg-de Vries and Maxwell-Bloch (icmKdV-MB) equations.

The paper is organized as follows. In Section 2, the Lax representation of icmKdV-MB equations will be introduced firstly. In Section 3, we derived the one-fold Darboux transformation of the icmKdV-MB equations. In Section 4, assuming trivial seed solutions for one-soliton solutions of icmKdV-MB equations were obtained. In Section 5, we discuss the results which obtained from Section 3 and 4.

Lax representation of the icmKdV-MB system

In this paper, we will concentrate on the inhomogeneous complex modified KdV and the Maxwell-Bloch system as following specific [2] form

$$b_{1t}q + b_1 q_t + \alpha_1 q_{xxx} + 6\alpha_1 b_1^2 |q|^2 q_x - b_1\alpha_2 p = 0, \quad (2.1)$$

$$p_x - 2b_1 q\eta + 2ib_2\omega p = 0, \quad (2.2)$$

$$\eta_x + b_1 qp^* + b_1 q^* p = 0, \quad (2.3)$$

if we set $\alpha_1 = b_1 = 1, \alpha_2 = 2, b_2 = -1,$ the icmKdV-MB equations will be reduced to hcmKdV-MB equations as following [1],

$$q_t - q_{xxx} + 6|q|^2 q_x - 2p = 0, \quad (2.4)$$

$$p_x - 2q\eta - 2i\omega p = 0, \quad (2.5)$$

$$\eta_x + qp^* + q^* p = 0. \quad (2.6)$$

The linear eigenvalue problem of icmKdV-MB takes the form [3]

$$\Phi_x = A\Phi, \quad (2.7)$$

$$\Phi_t = B\Phi, \quad (2.8)$$

where A and B can be expressed in following polynomials about complex constant eigenvalue parameter λ

$$A = i\lambda\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 0 & b_1 q \\ -b_1 q^* & 0 \end{pmatrix} = i\lambda\sigma_3 + A_0, \quad (2.9)$$

$$B = \lambda\begin{pmatrix} -2ib_1^2|q|^2 & 2ia_1 b_1 q_x \\ -2ia_1 b_1 q_x^* & 2ia_1 b_1^2 |q|^2 \end{pmatrix} + \begin{pmatrix} a_1 b_1 q^* q_x - a_1 b_1 q_x & -2a_1 b_1^2 |q|^2 q - a_1 q_{xs} \\ -2a_1 b_1^2 |q|^2 q^* - a_1 q_{xx}^* & -a_1 b_1 q_x^* q + a_1 b_1 q^* q_x \end{pmatrix} + V_1\begin{pmatrix} \eta & -p \\ -p^* & -\eta \end{pmatrix}$$

$$= \lambda B_1 + B_0 + V_1 B_{-1}, \quad (2.10)$$

where $V_1 = \dfrac{-b_1 a_2 i}{2(\lambda + \omega b_2)}$. The compatibility condition of (2.7)-(2.8) is

$$A_t - B_x + [A, B] = 0. \quad (2.11)$$

One fold Darboux transformation for the IcmKdV-MB equation

In this section we construct the DT for the (1+1)-dimensional cmKdV-MB equations (2.4)-(2.6), and as example we give in detail the one-fold DT.

Let Φ and $\Phi^{[1]}$ are two solutions of the system (2.7)-(2.8) so that

$$\Phi_x^{[1]} = A^{[1]}\Phi^{[1]}, \quad (1.12)$$

$$\Phi_t^{[1]} = B^{[1]}\Phi^{[1]}, \quad (2.13)$$

We assume that these two solutions are related by the following transformation:

$$\Phi^{[1]} = T\Phi = (\lambda \mathrm{I} - M)\Phi. \quad (2.14)$$

The matrix function T obeys the following equations

$$T_x + TA = A^{[1]}T, \quad (2.15)$$

$$T_t + TB = B^{[1]}T. \quad (2.16)$$

Then the relation between q, p, η and $q^{[1]}, p^{[1]}, \eta^{[1]}$ can be reduced from (2.15)-(2.16). Comparing the coefficient of $\lambda^i\,(i=0,1,2)$ of the sides the equation (2.15), we have

$$\lambda^0 : M_x = A^{[1]}M - MA_0, \tag{2.17}$$

$$\lambda^1 : A_0^{[1]} = A_0 + i[M, \sigma_3], \tag{2.18}$$

$$\lambda^2 : i\mathrm{I}\sigma_3 = i\sigma_3\mathrm{I}. \tag{2.19}$$

The equation (2.18) gives

$$q^{[1]} = q + 2ib_1^{-1}m_{12}, \tag{2.20}$$

$$q^{*[1]} = q^* + 2ib_1^{-1}m_{21}, \tag{2.21}$$

where

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}, \mathrm{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{2.22}$$

and additionally we get $m_{21} = -m_{12}^*$. After comparing the coefficient of $\lambda^i\,(i=0,1,2)$ the equation (2.16) gives us the following relations

$$\lambda^0 : -M_t = -B_0^{[1]}M + MB_0 + \frac{1}{2}ib_1a_2B_{-1} - \frac{1}{2}ib_1a_2B_{-1}^{[1]}, \tag{2.23}$$

$$\lambda^1 : B_0^{[1]} = B_0 - MB_1 + B_1^{[1]}M, \tag{2.24}$$

$$\lambda^2 : \mathrm{I}B = B_1^{[1]}\mathrm{I}, \tag{2.25}$$

$$\frac{-b_1a_2i}{2(\lambda + \omega b_2)} : 0 = -\omega b_2 B_{-1} - MB_{-1} + \omega b_2 B_{-1}^{[1]} + B_{-1}^{[1]}M. \tag{2.26}$$

Hence, we get DT

$$B_0^{[1]} = B_0 - MB_1 + B_1^{[1]}M, \tag{2.27}$$

$$\mathrm{I}B_1 = B_1^{[1]}\mathrm{I}, \tag{2.28}$$

$$B_{-1}^{[1]} = (M + \omega b_2\mathrm{I})B_{-1}(M + \omega b_2\mathrm{I})^{-1}. \tag{2.29}$$

At the same, from the system above we get

$$M = \begin{pmatrix} m_{11} & m_{12} \\ -m_{12}^* & m_{11}^* \end{pmatrix}, M^{-1} = \frac{1}{|m_{11}|^2 + |m_{12}|^2}\begin{pmatrix} m_{11}^* & -m_{12} \\ m_{12}^* & m_{11} \end{pmatrix}, \tag{2.30}$$

$$M + \omega b_2\mathrm{I} = \begin{pmatrix} m_{11} + \omega b_2 & m_{12} \\ -m_{12}^* & m_{11}^* + \omega b_2 \end{pmatrix}, \tag{2.31}$$

$$(M + \omega b_2\mathrm{I})^{-1} = \frac{1}{\nabla}\begin{pmatrix} m^*{}_{11} + \omega b_2 & m_{12} \\ -m_{12}^* & m_{11} + \omega b_2 \end{pmatrix}, \tag{2.32}$$

where $\nabla = \det(M + \omega b_2\mathrm{I}) = \omega^2 b_2^2 + \omega b_2(m_{11} + m_{11}^*) + |m_{11}|^2 + |m_{12}|^2$. We now assume that

$$M = H\Lambda H^{-1}, \tag{2.33}$$

here

$$H = \begin{pmatrix} \Phi_1(\lambda_1, t, x) & \Phi_1(\lambda_2, t, x) \\ \Phi_2(\lambda_1, t, x) & \Phi_2(\lambda_2, t, x) \end{pmatrix} := \begin{pmatrix} \Phi_{1,1} & \Phi_{1,2} \\ \Phi_{2,1} & \Phi_{22} \end{pmatrix}, \Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}. \tag{2.34}$$

And $\det H \neq 0$, where λ_1 and λ_2 are complex constants. In order to satisfy the constraints of M and $B_{-1}^{[1]}$ as mentioned above, we note that

$$\lambda_2 = \lambda_1^*, H = \begin{pmatrix} \Phi_1(\lambda_1, t, x) & -\Phi_2^*(\lambda_1, t, x) \\ \Phi_2(\lambda_1, t, x) & \Phi_1^*(\lambda_1, t, x) \end{pmatrix}, \quad (2.35)$$

so the matrix M has the form

$$M = \frac{1}{\Delta}\begin{pmatrix} \lambda_1|\Phi_1|^2 + \lambda_2|\Phi_2|^2 & (\lambda_1 - \lambda_2)\Phi_1\Phi_2^* \\ (\lambda_1 - \lambda_2)\Phi_2\Phi_1^* & \lambda_1|\Phi_2|^2 + \lambda_2|\Phi_1|^2 \end{pmatrix}. \quad (2.36)$$

Finally we can write the one-fold DT for the (1+1)-dimensional icmKdV-MB equations as:

$$q^{[1]} = q + 2ib_1^{-1}m_{12}, \quad (2.37)$$

$$\eta^{[1]} = \frac{(|\omega b_2 + m_{11}|^2 - |m_{12}|^2)\eta - pm_{12}^*(\omega b_2 + m_{11}) - p^* m_{12}(\omega b_2 + m_{11}^*)}{\nabla}, \quad (2.38)$$

$$p^{[1]} = \frac{(\omega b_2 + m_{11})^2 p - p^* m_{12}^2 + 2\eta m_{12}(\omega b_2 + m_{11})}{\nabla}, \quad (2.39)$$

where $m_{11} = \dfrac{\lambda_1|\Phi_1|^2 + \lambda_2|\Phi_2|^2}{\Delta}, m_{12} = \dfrac{(\lambda_1 - \lambda_2)\Phi_1\Phi_2^*}{\Delta}.$

Soliton solutions of the icmKdV-MB eqautions

Having the explicit form of the one-fold DT, we are ready to construct exact solutions of the (1+1)-dimensional icmKdV-MB equations. To get the one-soliton solution we take the seed solution as $q = p = 0, \eta = 1.$ Then corresponding associated linear system takes the form

$$\Phi_{1x} = i\lambda\Phi_1, \quad (2.40)$$

$$\Phi_{2x} = -i\lambda\Phi_2, \quad (2.41)$$

$$\Phi_{1t} = \frac{-ib_1a_2}{2(\lambda + \omega b_2)}\Phi_1, \quad (2.42)$$

$$\Phi_{2t} = \frac{-ib_1a_2}{2(\lambda + \omega b_2)}\Phi_2. \quad (2.43)$$

Easy calculation can lead to following eigenfunctions

$$\Phi_1 = \exp(i\lambda x - \frac{ib_1a_2}{2(\lambda + \omega b_2)}t + \frac{x_0 + iy_0}{2}), \quad (2.44)$$

$$\Phi_2 = \exp(-i\lambda x + \frac{ib_1a_2}{2(\lambda + \omega b_2)}t - \frac{x_0 + iy_0}{2} + i\theta), \quad (2.45)$$

where x_0, y_0 and θ are all arbitrary fixed real constant. Substituting these two eigenfunctions into the one-fold DT equations (2.37)-(2.39) and choosing $\lambda = \alpha_1 + \beta_1 i, x_0 = y_0 = \theta = 0$ then the one soliton solutions are obtained:

$$q^{[1]} = 2ib_1^{-1}\exp(i\theta_1)\cosh^{-1}(\theta_2), \quad (2.46)$$

$$p^{[1]} = \frac{2\omega b_2\exp(i\theta_1)\cosh^{-1}(\theta_2) + 2i\exp(i\theta_1)\cosh^{-1}(\theta_2) - 2\exp(i\theta_1)\sinh(\theta_2)\cosh^{-1}(\theta_2)}{\omega^2 b_2^2 - 2\omega b_2\tanh(\theta_2) + 1 + \tanh^2(\theta_2) + \cosh^{-2}(\theta_2)}, \quad (2.47)$$

$$\eta^{[1]} = \frac{\omega^2 b_2^2 - 2\omega b_2 \tanh(\theta_2) + 1 + \tanh^2(\theta_2) - \cosh^{-2}(\theta_2)}{\omega^2 b_2^2 - 2\omega b_2 \tanh(\theta_2) + 1 + \tanh^2(\theta_2) + \cosh^{-2}(\theta_2)}, \tag{2.48}$$

Where $\theta_1 = \frac{c_1 x + c_2 t}{c}, \theta_2 = \frac{d_1 x + d_2 t}{c}$,

$$c_1 = 2(\omega^2 \alpha_1 b_1^2 + 2\omega\alpha_1^2 b_2 + {\beta_1}^2 \alpha_1 + \alpha_1^3), c_2 = 2(-\omega a_2 b_1 b_2 - a_2 \alpha_1 b_1),$$

$$c = \omega^2 b_2^2 + 2\omega\alpha_1 b_2 + {\beta_1}^2 + \alpha_1^2, d_1 = 2\beta_1(\omega^2 b_2^2 + 2\omega\alpha_1 b_2 + {\beta_1}^2 + \alpha_1^2), d_2 = 2\beta_1 a_2 b_1.$$

Below the graphic representation is represented for functions, $p^{[1]}, q^{[1]}, \eta^{[1]}$.

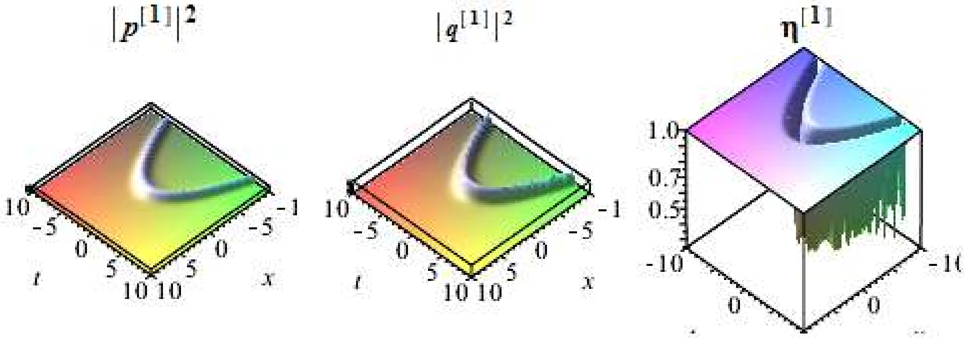

Figure1. One soliton solutions $(p^{[1]}, q^{[1]}, \eta^{[1]})$ of the icmKdV-MB equations where $\alpha_1 = 1, b_1 = 1, b_2 = 1, a_1 = 1, \beta_1 = 1, \omega = 1, a_2 = t$. Here $p^{[1]}$ and $q^{[1]}$ are bright solitons, $\eta^{[1]}$ is dark soliton.

Conclusion

In this paper, we derived the one-fold Darboux transformation of the inhomogeneous complex modified Korteweg-de Vries and Maxwell-Bloch equations. One soliton solutions of the icmKdV-MB equations have been constructed explicity by using Darboux transformation from trivial seed solutions. There are unclear intersting questions such as the higher-order solutions and positon solutions, and their applications in physics. It will be our next topic for study.

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КҮШТІ ГРАВИТАЦИЯЛЫҚ ӨРІС ҮШІН БІРСОЛИТОНДЫ ШЕШІМ

Адылхан Фәриза Жомутбайқызы

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Жалпы салыстырмалық теориясы гравитациялық өрістің негізгі теориясы болып табылады. Бұл теорияның негізінде сызықтық емес Эйнштейн теңдеуі жатыр. Осындай