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Traveling wave solutions of the two-dimensional Konopelchenko-Dubrovsky equation

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Abstract. In this study, the (2+1)-dimensional system of Konopelchenko-Dubrovsky is studied. This equation extends the Korteweg-de Vries(KdV) equation, that describes the evolution of small amplitude dispersion waves generated in shallow water. The tan-cot method is applied to receive the traveling wave solutions. It is presented that the tan-cot method is a useful mathematical tool for obtaining analytical solution to the equations of mathematical physics. In the figures, the graphs of the obtained solutions are shown

Keywords: traveling wave solution, tan-cot method, nonlinearity, Konopelchenko-Dubrovsky equation.

1. Introduction

Several branches of mathematics, physics, biology, and chemistry use partial differential equations to model nonlinear processes [1-4]. For example, dispersive waves of small amplitude on shallow water are described by the (1+1)-dimensional Korteweg-de Vries(KdV) equations [5] and the (1+1)-dimensional modified Korteweg-de Vries(mKdV) equation. The(1+1)-dimensional complex modified Korteweg-de Vries equation [6-8] is used to model the nonlinear evolution of plasma waves. The fundamental model in optics, the nonlinear Schrodinger equation, explains how optical waves move through Kerr media [9-10]. Consequently, a variety of analytical methods have been created in light of the significance of these issues: Darboux transformation [11-12], exp-function approach [13], Kudryashov method [14], Jacobi elliptic method [15], sine-cosine method [16], and others.

In work, we study the (2+1)-dimensional Konopelchenko-Dubrovsky system [17] in form as

$$u_t - u_{xxx} - 6buu_x + \frac{2}{3}a^2u^2u_x - 3\nu_y + 3au_x\nu = 0, \quad (1)$$

$$u_y = \nu_x, \quad (2)$$

where a, b a real parameters. If in Eqs. (1)-(2) the parameters are $a = 0$, $b = -1$, $\nu = 0$, then we can get the (1+1)-dimensional KdV equation. If in Eqs. (1)-(2) the parameters are $a = 3$, $b = 0$, $\nu = 0$, then we receive the (1+1)-dimensional mKdV equation. Therefore, the (2+1)-dimensional Konopelchenko-Dubrovsky system is referred to as an extension of the KdV equation. The Eqs. (1)-(2) were studied in [18-20].

We use the tan-cot method to get some analytical solutions for the (2+1)-dimensional Konopelchenko-Dubrovsky esystem (1)-(2). This method is a very efficient and powerful modern

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tool for dealing with various classes of nonlinear equations arising in the mathematical physics [21-22].

2. Description of the tan-cot function method.

In this section, the method of tangent and cotangent is examined. Using this method, the wave transformation takes form as follows. A partial differential equation

$$P_1(u_t, u_x, u_y, u_{xx}, u_{yy}, u_{xy}, ...) = 0, \tag{3}$$

can be transformed by the wave variables

$$u(x, y, t) = u(\xi), \xi = (x + y - ct). \tag{4}$$

to the ordinary differential equation(ODE)

$$P_2(u, u^{'}, u^{''}, u^{'''}, ...) = 0. \tag{5}$$

The solution of Eq. (5) can be found by:

$$u(x, y, t) = \lambda \tan^{\beta}(\mu\xi), \tag{6}$$

or

$$u(x, y, t) = \lambda \cot^{\beta}(\mu\xi), \tag{7}$$

where $\xi = x+y-ct$, the coefisients μ, β and λ will be defined. There are forms for the derivatives of (6)

$$u'(\mu\xi) = \lambda\beta\mu \tan^{\beta-1}(\mu\xi) + \lambda\beta\mu \tan^{\beta+1}(\mu\xi), \tag{8}$$

$$u''(\mu\xi) = \lambda\mu^2\beta(\beta-1) \tan^{\beta-2}(\mu\xi) + 2\lambda\mu^2\beta^2 \tan^{\beta}(\mu\xi) + \lambda\mu^2\beta(\beta+1) \tan^{\beta+2}(\mu\xi), \tag{9}$$

and there are forms for the derivatives of (7)

$$u'(\mu\xi) = -\lambda\beta\mu \cot^{\beta-1}(\mu\xi) - \lambda\beta\mu \cot^{\beta+1}(\mu\xi), \tag{10}$$

$$u''(\mu\xi) = \lambda\mu^2\beta(\beta-1) \cot^{\beta-2}(\mu\xi) + 2\lambda\mu^2\beta^2 \cot^{\beta}(\mu\xi) + \lambda\mu^2\beta(\beta+1) \cot^{\beta+2}(\mu\xi). \tag{11}$$

Using (6)-(11) into the ODE we can get a trigonomical equation of $\tan^r(\mu\xi)$ or $\cot^r(\mu\xi)$ terms. Next, to find β we equate the degrees of the tangent or cotangent pairs and establish the parameters. After that, we gather all coefficients of the same degree for $\tan^r(\mu\xi)$ or $\cot^r(\mu\xi)$. We then create a system of algebraic equations involving the unknowns λ and μ to solve for the coefficients.

3. Application of the tan-cot method.

To apply the tan-cot method to an independent derivative differential equation, the following transformation is used

$$u(x, y, t) = u(\xi) = u(x + y - ct) \tag{12}$$

where c is a constant. We substitute (12) into (1)and (2),get differential equations as follows:

$$-cu' - u''' - 6buu' + \frac{2}{3}a^2u^2u' - 3\nu' + 3au'\nu = 0, \tag{13}$$

$$u' = \nu'. \tag{14}$$

By integrating Eqs. (13)-(14) once, considering the integrated constant zero and we obtain the following equation

$$-cu - u'' - 3bu^2 + \frac{2}{9}a^2u^3 - 3u + \frac{3}{2}au^2 = 0. \tag{15}$$

If $b = \frac{a}{2}$ in equation (15) then we find

$$(c+3)u + u'' - \frac{2}{9}a^2u^3 = 0, a \neq 0. \tag{16}$$

3.1. The tangent solution.
The approach states that transfornation can be used to get the solution to equation (16)

$$u(\mu\xi) = \lambda \tan^{\beta}(\mu\xi), \tag{17}$$

and 2nd order derivative is

$$u''(\mu\xi) = \lambda\mu^2\beta(\beta-1)\tan^{\beta-2}(\mu\xi) + 2\lambda\mu^2\beta^2\tan^{\beta}(\mu\xi) + \lambda\mu^2\beta(\beta+1)\tan^{\beta+2}(\mu\xi) \tag{18}$$

Substituting (17) and (18) into (16) we get

$$\begin{aligned}(c+3)\lambda\tan^{\beta}(\mu\xi) + \lambda\mu^2\beta(\beta-1)\tan^{(\beta-2)}(\mu\xi) + \\ +2\lambda\mu^2\beta^2\tan^{\beta}(\mu\xi) + \lambda\mu^2\beta(\beta+1)\tan^{(\beta+2)}(\mu\xi) - \frac{2}{9}a^2\lambda^3\tan^{3\beta}(\mu\xi) = 0.\end{aligned} \tag{19}$$

Applying the balance method, by equating the exponents of $\tan^j$, from (19) we define β:

$$3\beta = \beta + 2, \Rightarrow \beta = 1. \tag{20}$$

Substitute (20) in (19) we get the next equation:

$$(c+3)\lambda\tan(\mu\xi) + 2\lambda\mu^2\tan(\mu\xi) + 2\lambda\mu^2\tan^3(\mu\xi) - \frac{2}{9}a^2\lambda^3\tan^3(\mu\xi) = 0. \tag{21}$$

From (21) we have the next system

$$\tan^1(\mu\xi) \; : \; (c+3)\lambda + 2\lambda\mu^2 = 0, \tag{22}$$

$$\tan^3(\mu\xi) \; : \; 2\lambda\mu^2 - \frac{2}{9}a^2\lambda^3 = 0, \tag{23}$$

We calculate the values of the following coefficients using the system of equations (22) and (23)

$$\mu = \sqrt{-\frac{(c+3)}{2}}, \lambda = \frac{3}{a}\sqrt{-\frac{(c+3)}{2}}, \quad with \quad (c+3) < 0. \tag{24}$$

By substituting (24) into Eq. (17), and then obtained expression into (12), we find the traveling wave solutions of the (2+1)-dimensional Konopelchenko-Dubrovsky system

$$u_1(x,y,t) = \frac{3}{a}\sqrt{-\frac{(c+3)}{2}}\tan(\sqrt{-\frac{(c+3)}{2}}(x+y-ct)), \quad with \quad (c+3) < 0, \tag{25}$$

$$\nu_1(x,y,t) = \frac{3}{a}\sqrt{-\frac{(c+3)}{2}}\tan(\sqrt{-\frac{(c+3)}{2}}(x+y-ct)), \quad with \quad (c+3) < 0, \tag{26}$$

The dynamics of the solution $u_1(x,y,t)$ is shown in Fig. 1.

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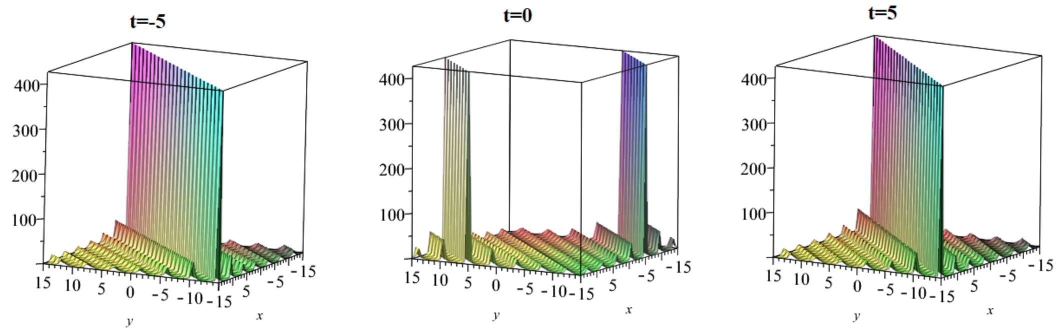

Figure 1: Dynamics of the solutions of $u_1(x, y, t)$ in case: $a = 1, b = 0.5, c = -4$

3.2. The cotangent solution

To find the cotangent solution, we will use the transformation in (7).

$$u(\mu\xi) = \lambda \cot^{\beta}(\mu\xi), \tag{27}$$

And the second order derivative is

$$u''(\mu\xi) = \lambda\mu^2\beta(\beta-1)\cot^{\beta-2}(\mu\xi) + 2\lambda\mu^2\beta^2\cot^{\beta}(\mu\xi) + \lambda\mu^2\beta(\beta+1)\cot^{\beta+2}(\mu\xi). \tag{28}$$

By substituting Eqs. (27) and (28) into Eq. (16), we obtain the following equation:

$$(c+3)\lambda\cot^{\beta}(\mu\xi) + \lambda\mu^2\beta(\beta-1)\cot^{(\beta-2)}(\mu\xi) + \tag{29}$$

$$+2\lambda\mu^2\beta^2\cot^{\beta}(\mu\xi) + \lambda\mu^2\beta(\beta+1)\cot^{(\beta+2)}(\mu\xi) - \frac{2}{9}a^2\lambda^3\cot^{3\beta}(\mu\xi) = 0. \tag{30}$$

Applying the balance method, by equating the exponents of $\cot^j$, from (29) we define β

$$3\beta = \beta + 2, \Rightarrow \beta = 1. \tag{31}$$

Substitute (30) into (29) we get the next equation

$$(c+3)\lambda\cot(\mu\xi) + 2\lambda\mu^2\cot(\mu\xi) + 2\lambda\mu^2\cot^3(\mu\xi) - \frac{2}{9}a^2\lambda^3\cot^3(\mu\xi) = 0. \tag{32}$$

From (31) we have the next system

$$\cot^1(\mu\xi) \ : \ (c+3)\lambda + 2\lambda\mu^2 = 0, \tag{33}$$

$$\cot^3(\mu\xi) \ : \ 2\lambda\mu^2 - \frac{2}{9}a^2\lambda^3 = 0. \tag{34}$$

The system of the equations (32)-(33) give us

$$\mu = \sqrt{-\frac{(c+3)}{2}}, \lambda = \frac{3}{a}\sqrt{-\frac{(c+3)}{2}}, \quad with \quad (c+3) < 0. \tag{35}$$

Substituting (35) into (27) and then obtained expression into (12) we get the cotangent solutions

$$u_2(x,y,t) = \frac{3}{a}\sqrt{-\frac{(c+3)}{2}}\cot\left(\sqrt{-\frac{(c+3)}{2}}(x+y-ct)\right), \quad with \quad (c+3)<0, \tag{36}$$

$$\nu_2(x,y,t) = \frac{3}{a}\sqrt{-\frac{(c+3)}{2}}\cot\left(\sqrt{-\frac{(c+3)}{2}}(x+y-ct)\right), \quad with \quad (c+3)<0. \tag{37}$$

The dynamics of solution $u_2(x,y,t)$ is shown in Fig. 2.

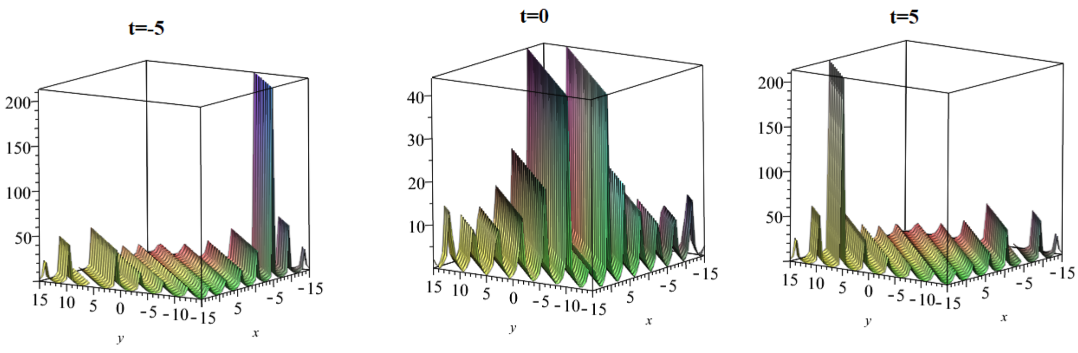

Figure 2: Dynamics of the solutions of $u_2(x,y,t)$ in the case: $a = 1, b = 0.5, c = -4$

4. Conclusions

This work investigated the (2+1)-dimensional Konopelchenko-Dubrovsky system, in case $a = 0$, $b = -1$, $\nu = 0$ becomes the (1+1)-dimensional KdV equation, if $a = 3$, $b = 0$, $\nu = 0$, it becomes the (1+1)-dimensional mKdV equation. The tan-cot method was used to get the explicit solutions. During the study, traveling wave solutions were identified. Graphs of the obtained solutions were plotted.

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