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THE BOCHKAREV TYPE INEQUALITY IN THE REGULAR SYSTEM

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For a measurable function f, defined by $[0,1]$ introduce the distribution function $m(\sigma, f)$ by the formula

$$m(\sigma, f) = \mu\{x \in [0,1] : |f(x)| > \sigma\}.$$

Function $f^*(t) = \inf\{\sigma : m(\sigma, f) \Box t\}$ it is called a non-increasing permutation of the function f.

Definition A1 [1]. *Let* $1 \leq p < \infty,\quad 1 \leq q \leq \infty.$ *Will say that function* f *belongs to the Lorentz space* $L_{p,q}[0,1],$ *if* $1 \leq q < \infty,$

$$\|f\|_{L_{p,q}} = \left(\int_0^1 (t^{1/p} f^*(t))^q \frac{dt}{t}\right)^{1/q} < \infty,$$

by $q = \infty$

$$\|f\|_{L_{p\infty}} = \sup_t t^{1/p} f^*(t) < \infty.$$

For $p = q$ we get that $L_{p,p} = L_p$

In the paper "Estimates of the Fourier coefficients of functions from Lorentz spaces" Reports of the Academy of Sciences, 1998, T. 360, P.730-733. **Ошибка! Источник ссылки не найден.** Bochkarev S. V. Exact upper and lower bounds for the Fourier coefficients of functions from Lorentz spaces are established L_{2r}, $2 < r \leq \infty,$ and also the question of the strengthened Carleman singularity for general orthonormal systems is considered.

The question of estimating the Fourier coefficients of functions from Lorentz spaces $L_{2,r}$ $2 < r \leq \infty,$ remained open.

From the estimates of the Fourier coefficients of functions from Lorentz spaces proved in this report, it follows that the Hausdorff-Young-Riesz theorem does not apply to space $L_{2,r}$, if $r \neq 2$

Definition A2 [2]. *Let* $\Phi = \{\phi_k\}_{k \in N}$ *be an orthonormal system of complex, on* $[0,1]$*, call it regular, if a constant exists* B*, such that*

1) for any segment e *from* $[0,1]$ *and* $k \in \mathbb{N}$ *correctly*

$$\left|\int_e \phi_k(x)dx\right| \leq B \min(\mu e, 1/k),$$

2) for any segment w *(finite arithmetic progression in increments of 1) from* $\mathbb{N}$ *and* $t \in (0,1]$ *made*

$$\left(\sum_{k \in w} \phi_k\right)^*(t) \leq B \min(|w|, 1/t),$$

where $\left(\sum_{k \in w} \phi_k(f)\right)^*(t)$ -- *non-increasing permutation of the function* $\sum_{k \in w} \phi_k(x)$, $|w|$ *- the number of elements in the set* w.

All trigonometric systems, the Walsh system, and multiplicative systems with bounded generators are regular. The Haar system is not regular.

Note that if the system is regular, then it is bounded in the aggregate, that is, there is a constant P, for everyone $k \in \mathbb{N}$ and everyone $x \in [0,1]$ $|\phi_k(x)| \leq P.$

Theorem B[3]. Let $\{\varphi_n\}_{n=1}^{\infty}$ - *be an orthonormal system of complex-valued functions on* $[0,1]$ *such that*

$$\|\varphi_n\|_{\infty} \leq M, \qquad n = 1,2,...,$$

let $f \in L_{2,r}$, $2 < r \leq \infty$, *then the equality*

$$\sup_{n\in N} \frac{1}{|n|^{\frac{1}{2}} (\log(n+1))^{\frac{1}{2}-\frac{1}{r}}} \sum_{m=1}^{n} a_m^* \leq C\|f\|_{L_{2,r}}, \qquad (1)$$

holds, where a_n *are the Fourier coefficients of the system* $\{\varphi_n\}_{n=1}^{\infty}$.

On the coefficients of Fourier series in trigonometry systems in spaces $L_{2,r}$. In the article [4], in the case of a trigonometric system, was received the inequality (1) in a certain sense.

Let M --- the set of all arithmetic progressions of $\mathbb{Z}$. For a finite set A of $\mathbb{Z}$ identified the number $[A]_M$ as follows:

$$[A]_M = \min\{l : A = \bigcup_{k=1}^{l} I_k, I_k \in M\}.$$

Theorem C. *Let* M *--- the set of all segments of* $\mathbb{Z}$. Φ -- *trigonometric system,* $f \sim \sum_{m=0}^{\infty} \hat{f}(m)\cos mx.$ *For any function* $f \in L_{2,r}$, $2 < r < \infty$ ⇓

$$\sup_{A\subset \mathbb{Z}} \frac{1}{|A|^{1/2} \log_2(1+[A]_M)^{1/2-1/r}} \left|\sum_{m\in A} \hat{f}(m)\right| \leq 20\|f\|_{L_{2,r}}. \qquad (2)$$

The methods of proof of Bochkarev S. V. were based on the specifics of trigonometric series.

The main result of this paragraph is the following statement. The main result of the work is to obtain a Bochkarev type theorem for a function from the Lorentz space $L_{2,r}$ in the regular system.

Lemma. Let $1 < q < 2$, $\{\varphi_n\}_{n=1}^{\infty}$ - regular system, $\|\varphi_n\| \leq B$, $\forall n = 1,2,3,...$ $f \sim \sum_{m\in\mathbb{N}} \hat{f}(m)\varphi_m$, then for any measurable set A of finite measure of $\mathbb{N}$ the inequality

$$\frac{1}{|A|^{1/q}}\left|\sum_{m\in A}\hat{f}(m)\right| \le B\left(\frac{q}{q-1}\right)^{\frac{1}{q}-\frac{1}{2}} \| f \|_{L_{q,2}} .$$

Theorem 1. Let $\{\varphi_n\}_{n=1}^{\infty}$- regular system, $\|\varphi_n\| \le B$, $\forall n = 1,2,3,...$ then for any $f \in L_{2,r}[0,1]$, $2 < r \le \infty$ we have the inequality:

$$\sup_{N\ge 8}\frac{1}{N^{1/2}(\log_2(N+1))^{1/2-1/r}}\sum_{m=1}^{N}\hat{f}^*(m) \le B\| f \|_{L_{2r}} .$$

Theorem 2. Let $\{\varphi_n\}_{n=1}^{\infty}$ -regular system, $\|\varphi_n\| \le B$, $n = 1,2,3,...$

If $\sum_{k=1}^{\infty}\hat{f}^*(k)k^{-\frac{1}{2}}(\log_2(k+1))^{\frac{1}{2}-\frac{1}{r}} < \infty$,so row $\sum_{k=1}^{\infty}\hat{f}(k)\varphi_k$ converges to some function $f \in L_{2,r}[0,1]$, for any $1 < r \le 2$ the inequality:

$$\|f\|_{L_{2,r}} \le C\sum_{k=1}^{\infty}\hat{f}^*(k)k^{-\frac{1}{2}}(\log_2(k+1))^{\frac{1}{2}-\frac{1}{r}}$$

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