



Inflationary universe from higher derivative quantum gravity coupled with scalar electrodynamics

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Abstract

We study inflation for a quantum scalar electrodynamics model in curved space–time and for higher-derivative quantum gravity (QG) coupled with scalar electrodynamics. The corresponding renormalization-group (RG) improved potential is evaluated for both theories in Jordan frame where non-minimal scalar-gravitational coupling sector is explicitly kept. The role of one-loop quantum corrections is investigated by showing how these corrections enter in the expressions for the slow-roll parameters, the spectral index and the tensor-to-scalar ratio and how they influence the bound of the Hubble parameter at the beginning of the primordial acceleration. We demonstrate that the viable inflation may be successfully realized, so that it turns out to be consistent with last Planck and BICEP2/Keck Array data.

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1. Introduction

Recent corrected Planck data as well as latest BICEP2/Keck/Array data propose better quantitative description of the inflationary universe. In its own turn, this increases the interest to theoretical models of inflation (for the reviews, see Ref. [1]) because they maybe better confronted against observational data.

During last years, there were many attempts to take into account quantum effects in order to construct viable inflation in perturbative Einstein QG (for some review, see Ref. [2]). It is quite natural to go beyond semi-classical General Relativity and to investigate the inflationary scenario for multiplicatively-renormalizable higher derivative gravity as well as for string-inspired gravities. The explicit calculation in this direction at strong gravity regime of higher-derivative QG was done in Ref. [3] where possibility of viable QG-induced inflation was proved. Of course, being the multiplicatively-renormalizable theory what gives the chance to evaluate QG corrections, higher-derivative QG represents merely the effective theory. It is known, that in such theory the unitarity problem which is related with the Ostrogradski instability [4] remains to be the open issue. Eventually, in higher-derivative gravity the unitarity maybe restored at the non-perturbative level. Thus, this theory could be considered as good approximation for the effective theory of quantum gravity. One can expect to account for QG effects at least qualitatively within such theory.

The purpose of this work is to study higher-derivative QG effects for Higgs-like inflation. As simplified model we take first massless scalar electrodynamics and investigate RG-improved inflation in such theory. At the next stage, we consider higher-derivative QG coupled to scalar electrodynamics and evaluate the corresponding RG-improved effective potential. The occurrence of viable inflation which is realized thanks to such RG-improved effective potential with account of QG effects is proved.

The paper is organized in the following way. In Section 2 we consider the multiplicatively-renormalizable massless scalar electrodynamics in curved space–time. The form of the renormalization-group improved scalar effective potential is derived in this theory, paying special attention to the non-minimal scalar-gravitational sector. In Section 3 we analyze inflation in frames of above scalar quantum electrodynamics in Jordan frame. We explicitly derive the slow-roll parameters, the spectral index and the tensor-to-scalar ratio showing how the quantum corrections enter in these expressions. We compute the e -folds number and we demonstrate that the model leads to a viable inflationary scenario according with the last Planck and BICEP2/Keck Array data. In Section 4 we consider multiplicatively-renormalizable higher-derivative gravity coupled with scalar electrodynamics. The complicated expression for RG improved effective potential in such theory (with account of QG corrections) is obtained. Section 5 is devoted to the study of QG-induced inflation in comparison with the simplified case of scalar electrodynamics analyzed before. QG does support the realization of inflation. Also in this case, we carefully investigated how the QG corrections enter in the expressions for the slow-roll parameters, the spectral index and the tensor-to-scalar ratio. It is found that the bound of the Hubble parameter describing the quasi-de Sitter solution of inflation is influenced by the correction of the mass scale of the theory. As a consequence, in order to obtain a realistic scenario, the early-time acceleration results to be weaker when the mass decreases. Conclusions and final remarks are given in Section 6.

2. Effective potential in quantum scalar electrodynamics in curved space–time

In this section, we present the renormalization-group (RG) improved effective potential for a massless scalar electrodynamics in curved space–time [6,8]. The general action for multiplicatively-renormalizable higher-derivative gravity can be written as [5,7]

$$I = \int_{\mathcal{M}} d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \Lambda + a_1 R^2 + a_2 C_{\mu\nu\xi\sigma} C^{\mu\nu\xi\sigma} + a_3 G + a_4 \square R + \mathcal{L}_m \right], \quad (2.1)$$

where g is the determinant of the metric tensor $g_{\mu\nu}$, \mathcal{M} represents the space–time manifold, R is the Ricci scalar, Λ a (positive) cosmological constant, \mathcal{L}_m encodes the matter contributions and $\square \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$ is the covariant d’Alembertian, with ∇_μ being the covariant derivative operator associated with the metric. Moreover, G is the Gauss–Bonnet four-dimensional topological invariant and $C_{\mu\nu\xi\sigma} C^{\mu\nu\xi\sigma}$ is the “square” of the Weyl tensor,

$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\xi\sigma}R^{\mu\nu\xi\sigma}, \quad C_{\mu\nu\xi\sigma}C^{\mu\nu\xi\sigma} = \frac{1}{3}R^2 - 2R_{\mu\nu}R^{\mu\nu} + R_{\xi\sigma\mu\nu}R^{\xi\sigma\mu\nu}, \quad (2.2)$$

$R_{\mu\nu}$, $R_{\mu\nu\xi\sigma}$ being the Ricci tensor and the Riemann tensor, respectively.

In the above expression, $a_{1,2,3,4}$ are dimensionless parameters, while $1/\kappa^2$ has the dimension of the square of a mass. At present epoch we know that it has to be $1/\kappa^2 = M_{Pl}^2/8\pi$, M_{Pl} being the Planck mass. As usually we assume the parameters κ^2 , Λ , $a_{1,2,3,4}$ to be constant, then the contribution of the Gauss–Bonnet and of the surface term $\square R$ drop down, and the action takes the simplified form,

$$I = \int_{\mathcal{M}} d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \Lambda + a_1 R^2 + a_2 C_{\mu\nu\xi\sigma} C^{\mu\nu\xi\sigma} + \mathcal{L}_m \right]. \quad (2.3)$$

At the early-time universe, the matter Lagrangian contains gauge fields, scalar multiplets and spinors and the related interactions typical of any Grand Unified Theory (GUT). In what follows, we consider massless scalar quantum electrodynamics (QED), whose Lagrangian in curved space–time reads [9–11],

$$\mathcal{L}_m = -D_\mu \phi D^\mu \phi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \xi R \phi^2 - \frac{1}{4!} f \phi^4. \quad (2.4)$$

Here, $D_\mu = \partial_\mu - eA_\mu$ is the covariant derivative, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic tensor, ξ , f are dimensionless coupling constants, and ϕ is a complex scalar field. The effective Lagrangian reads

$$\mathcal{L}_m = -\frac{\partial_\mu \phi \partial^\mu \phi}{2} - V_{\text{eff}}(\phi, R), \quad (2.5)$$

where $\phi = \sqrt{|\phi|}$, while the effective potential $V_{\text{eff}} \equiv V_{\text{eff}}(\phi, R)$ has to be evaluated in one-loop approximation in the background where ϕ and R are almost constants. It satisfies the standard RG equation,

$$\left[\mu \frac{\partial}{\partial \mu} + \beta_{e^2}(t') \frac{\partial}{\partial e^2(t')} + \beta_f(t') \frac{\partial}{\partial f(t')} + \beta_\xi(t') \frac{\partial}{\partial \xi(t')} - \gamma(t') \phi(t') \frac{\partial}{\partial \phi(t')} \right] V_{\text{eff}} = 0.$$

(2.6)

In this expression, couplings $e^2(t')$, $f(t')$, $\xi(t')$ and $\phi(t')$ are the functions of the renormalization parameter t' given by

$$t' = \frac{1}{2} \log \left[\frac{\phi^2}{\mu^2} \right], \tag{2.7}$$

where μ is a mass parameter in the range $\mu \sim \mu_{GUT} = 10^{15}$ GeV. We point out that $\mu < M_{Pl} \simeq 1.2 \times 10^{19}$ GeV, and during inflation $1 < \phi^2/\mu^2$. Moreover, $\beta_{e^2, f, \xi}(t')$ and $\gamma(t')$ are the corresponding beta-functions, namely (see works on RG-improved effective potential in flat and curved spacetime [12,6])

$$\begin{aligned} \beta_{e^2}(t') &= \frac{2e^4(t')}{3(4\pi)^2}, & \beta_f(t') &= \frac{1}{(4\pi)^2} \left(\frac{10}{3} f(t')^2 - 12e(t')^2 f(t') + 36e(t')^4 \right), \\ \beta_\xi(t') &= \frac{\left(\xi(t') - \frac{1}{6} \right)}{(4\pi)^2} \left(\frac{4}{3} f(t') - 6e(t')^2 \right), & \gamma(t') &= -\frac{3e^2(t')}{(4\pi)^2}. \end{aligned} \tag{2.8}$$

One finds that Eq. (2.6) can be recasted in the form

$$V_{\text{eff}} \equiv V_{\text{eff}}(\mu e^{t'}, e^2(t'), f(t'), \xi(t'), \phi(t')), \tag{2.9}$$

such that

$$\frac{de^2(t')}{dt'} = \beta_{e^2}(t'), \quad \frac{df(t')}{dt'} = \beta_f(t'), \quad \frac{d\xi(t')}{dt'} = \beta_\xi(t'), \quad \frac{d\phi(t')}{dt'} = -\gamma(t')\phi(t'). \tag{2.10}$$

Thus, one derives

$$\begin{aligned} e(t')^2 &= e^2 \left(1 - \frac{2e^2 t'}{3(4\pi)^2} \right)^{-1}, \\ f(t') &= \frac{e(t')^2}{10} \left[\sqrt{719} \tan \left[\frac{\sqrt{719}}{2} \log e(t')^2 + C \right] + 19 \right], \\ \xi(t') &= \frac{1}{6} + \left(\xi - \frac{1}{6} \right) \left(\frac{e(t')^2}{e^2} \right)^{-26/5} \frac{\cos^{2/5}[\sqrt{719}(\log e^2)/2 + C]}{\sqrt{719}(\log e^2(t'))/2 + C}, \\ \phi^2(t') &= \phi^2 \left(1 - \frac{2e^2 t'}{3(4\pi)^2} \right)^{-9}, \end{aligned} \tag{2.11}$$

where we set $e \equiv e(t' = 0)$, $f \equiv f(t' = 0)$, $\xi \equiv \xi(t' = 0)$, $\phi \equiv \phi(t' = 0)$ and

$$C = \arctan \left[\frac{1}{\sqrt{719}} \left(\frac{10f}{e^2} - 19 \right) - \frac{1}{2} \sqrt{719} \log e^2 \right].$$

Finally, one rewrites the effective potential V_{eff} in the form

$$V_{\text{eff}} = -\frac{1}{4!} f(t') \phi^4(t') + \frac{1}{2} \xi(t') R \phi^2(t'). \tag{2.12}$$

By plugging the corresponding expressions for the effective coupling constants, one gets for small t' and weak coupling the following one-loop effective potential,

$$V_{\text{eff}} = -\tilde{f}\phi^4 - A\phi^4 \left[\log \frac{\phi^2}{\mu^2} - \frac{25}{6} \right] + \tilde{\xi}R\phi^2 - BR\phi^2 \left[\log \frac{\phi^2}{\mu^2} - 3 \right], \tag{2.13}$$

with

$$\begin{aligned} \tilde{f} &= \frac{f}{4!}, \quad \tilde{\xi} = \frac{\xi}{2}, \quad A = \frac{1}{48(4\pi)^2} \left(\frac{10}{3}f^2 + 36e^4 \right), \\ B &= \frac{1}{12(4\pi)^2} \left[\left(\xi - \frac{1}{6} \right) \left(\frac{4f}{3} - 6e^2 \right) + 6\xi e^2 \right]. \end{aligned} \tag{2.14}$$

This result is valid for ϕ and therefore R almost constants. Moreover, μ^2 represents the scale of inflation (we assume that when $\phi^2 = \mu^2$ inflation ends). In the next section, we use the Lagrangian (2.3) with $\Lambda = 0$ and constant coefficients in the gravitational sector. Note that we work in Jordan frame through this paper.

3. Inflation in scalar quantum electrodynamics

It is interesting to see how the model can reproduce the early-time inflation at the GUT scale. Note that RG-improved effective potential has been applied for the study of inflation in Refs. [6, 13,14]. Actually, the inflation due to scalar QED has been already studied in Ref. [14] in the Einstein frame, but here we work in the Jordan frame. This is due to the fact that account of quantum corrections breaks the mathematical equivalence between Einstein and Jordan frames[15]. Hence, the inflationary predictions from QFT like the case under consideration maybe significantly different. Furthermore, generally speaking there is no even classical equivalence between Jordan and Einstein frames in the presence of Weyl-squared term. We also mention that the study of RG improved inflationary scalar electrodynamics and $SU(5)$ scenarios confronted with Planck 2013 and BICEP2 results can be found in Ref. [14].

Let us consider the flat Friedmann–Robertson–Walker (FRW) space–time described by the metric

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2, \tag{3.1}$$

$a \equiv a(t)$ being the scale factor of the universe. We immediately note that the square of the Weyl tensor in (2.3) is identically null and does not give any contribution to the dynamics of the model. We will also set the cosmological constant term $\Lambda = 0$. If the field $\phi \equiv \phi(t)$ depends on the cosmological time only, the equations of motion (EOMs) are derived as

$$\frac{3H^2}{\kappa^2} + 12a_1H^2R = a_1R^2 + \frac{\dot{\phi}^2}{2} + \left[V_{\text{eff}} - R \frac{dV_{\text{eff}}}{dR} \right] + 6H^2 \frac{dV_{\text{eff}}}{dR} - 3H\dot{F}, \tag{3.2}$$

$$-2F\dot{H} = \dot{\phi}^2 + \ddot{F} - H\dot{F}. \tag{3.3}$$

Here, $H = \dot{a}/a$ is the Hubble parameter, the dot denotes the time derivative, V_{eff} is given by (2.13)–(2.14) and we introduced the following notation,

$$F \equiv F(R, \phi) = \frac{1}{\kappa^2} + 4a_1R - 2 \frac{dV_{\text{eff}}}{dR}. \tag{3.4}$$

From (3.2)–(3.3) we also infer the continuity equation of the scalar field,

$$\ddot{\phi} + 3H\dot{\phi} = -V'_{\text{eff}}, \tag{3.5}$$

with

$$V'_{\text{eff}} \equiv \frac{dV_{\text{eff}}}{d\phi}. \tag{3.6}$$

Inflation is commonly described by a (quasi) de Sitter solution in slow-roll approximation regime ($\dot{\phi}^2 \ll V_{\text{eff}}$, $0 < V_{\text{eff}}$, and $|\ddot{\phi}| \ll |H\dot{\phi}|$), when Eq. (3.2) and Eq. (3.5) take the form

$$\frac{3H^2}{\kappa^2} \simeq \left[V_{\text{eff}} - 6H^2 \frac{dV_{\text{eff}}}{dR} \right], \quad 3H\dot{\phi} \simeq -V'_{\text{eff}}, \tag{3.7}$$

where $R \simeq 12H^2$. In the limit $1 \ll a_1 \kappa^2 R$ one recovers the chaotic inflation of the Starobinsky-like models [16–18] in the Jordan frame with Eq. (3.2) asymptotically satisfied for a given boundary value of the Hubble parameter. Here, we assume that $a_1 R \kappa^2$ is not asymptotically dominant. Thus, from the first equation above, one derives the de Sitter solution,

$$H_{\text{dS}}^2 \simeq \frac{\left[\tilde{f} + A \left[\log \left[\frac{\phi^2}{\mu^2} \right] - \frac{25}{6} \right] \right] \kappa^2 \phi^4}{-3 + 6 \left[\tilde{\xi} - B \left[\log \left[\frac{\phi^2}{\mu^2} \right] - 3 \right] \right] \kappa^2 \phi^2}. \tag{3.8}$$

We immediately see that H_{dS}^2 is large as long as,

$$1 \ll \tilde{\xi} \kappa^2 \phi^2 \rightarrow \frac{M_{\text{Pl}}^2}{\tilde{\xi}} \ll \phi^2. \tag{3.9}$$

In general, since the field exceeds the Planck mass during inflation, we must also require that $\tilde{f}/\tilde{\xi} < 1$. From the second equation in (3.7) we obtain

$$\dot{\phi} \simeq \frac{2\phi \left[12H^2 \left[-2B - \tilde{\xi} + B \log \left[\frac{\phi^2}{\mu^2} \right] \right] + \left[-22A/3 + 2\tilde{f} + 2A \log \left[\frac{\phi^2}{\mu^2} \right] \right] \phi^2 \right]}{3H}. \tag{3.10}$$

This result is valid when the slow-roll approximation $\dot{\phi}^2/V_{\text{eff}} \ll 1$ holds true, namely,

$$\frac{\dot{\phi}^2}{V_{\text{eff}}} \simeq - \frac{4 \left[2(\tilde{f} - 25A/6) - 4B(\tilde{f} - 25A/6)\kappa^2 \phi^2 - 2A(\tilde{\xi} + 3B)\kappa^2 \phi^2 - 2A \left[-1 + B\kappa^2 \phi^2 \right] \log \left[\frac{\phi^2}{\mu^2} \right] \right]^2}{3\kappa^2 \phi^2 \left[\tilde{f} + A \left[\log \left[\frac{\phi^2}{\mu^2} \right] - \frac{25}{6} \right] \right]^2 \left[-1 - 2\tilde{\xi} \kappa^2 \phi^2 + 2B \left[\log \left[\frac{\phi^2}{\mu^2} \right] - 3 \right] \kappa^2 \phi^2 \right]} \ll 1. \tag{3.11}$$

Since the quantum corrections encoded in A, B are small,

$$\frac{\dot{\phi}^2}{V_{\text{eff}}} \sim \frac{16}{3\kappa^2 \phi^2 + 6\tilde{\xi} \kappa^4 \phi^4}, \tag{3.12}$$

and (3.11) is well satisfied by taking into account (3.9).

To study perturbations left at the end of inflation, one needs the “slow-roll” parameters [19, 20],

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_2 = \frac{\ddot{\phi}}{H\dot{\phi}}, \quad \epsilon_3 = \frac{\dot{F}}{2HF}, \quad \epsilon_4 = \frac{\dot{E}}{2HE}, \tag{3.13}$$

where

$$E = F + \frac{3\dot{F}^2}{2\dot{\phi}^2}. \tag{3.14}$$

The slow-roll parameters at the first order in A and B are obtained¹ under the condition (3.9),

$$\begin{aligned} \epsilon_1 &\simeq \frac{4}{\kappa^2\phi^2} + \frac{4A(2 - \tilde{\xi}\kappa^2\phi^2)}{\tilde{f}\kappa^2\phi^2} + 8B \left(-1 + \frac{1}{\tilde{\xi}\kappa^2\phi^2} \right), \\ \epsilon_2 &\simeq \frac{2}{\tilde{\xi}\kappa^4\phi^4} + \frac{2A(-3 + 4\tilde{\xi}\kappa^2\phi^2)}{\tilde{f}\kappa^2\phi^2} + 8B \left(2 - \frac{1}{\tilde{\xi}\kappa^2\phi^2} \right), \\ \epsilon_3 &\simeq -\frac{4}{\kappa^2\phi^2} - \frac{4A(8a_1\tilde{f} - \tilde{\xi}(4a_1\tilde{f} - \tilde{\xi}^2)\kappa^2\phi^2)}{\tilde{f}\kappa^2\phi^2(4a_1\tilde{f} - \tilde{\xi}^2)} + 8B \left(1 + \frac{(4a_1\tilde{f} + \tilde{\xi}^2)}{\kappa^2\phi^2(\tilde{\xi}^3 - 4a_1\tilde{f}\tilde{\xi})} \right), \\ \epsilon_4 &\simeq -\frac{4}{\kappa^2\phi^2} + 2A \left(\frac{2\tilde{\xi}}{\tilde{f}} - \frac{4(240a_1^2\tilde{f}^2 + 4a_1\tilde{f}(1 - 18\tilde{\xi})\tilde{\xi} + 3\tilde{\xi}^4)}{\tilde{f}(48a_1\tilde{f} + \tilde{\xi} - 12\tilde{\xi}^2)(4a_1\tilde{f} - \tilde{\xi}^2)\kappa^2\phi^2} \right) \\ &\quad + 8B \left(1 + \frac{(4a_1\tilde{f} + \tilde{\xi}^2)}{\kappa^2\phi^2(\tilde{\xi}^3 - 4a_1\tilde{f}\tilde{\xi})} \right). \end{aligned} \tag{3.15}$$

We see that in the first approximation $\epsilon_1 \simeq -\epsilon_3$ like in pure modified gravity. It is also interesting to note that the R^2 -term contributes only in the one-loop corrections. This fact is not surprising. The R^2 -higher derivative term in the gravitational action may support the de Sitter expansion if it is dominant (otherwise, like in our case, its contribution disappears from the Friedmann-like equations with constant Hubble parameter), but does not drive the exit from inflation (for example, in the Jordan frame of the Starobinsky model this role is played by the Einstein’s term).

The amount of inflation is measured by the e -folds number,

$$N := \log \left[\frac{a(t_f)}{a(t_i)} \right] = \int_{t_i}^{t_f} H dt, \tag{3.16}$$

where $t_{i, f}$ are the time at the beginning and at the end of inflation, respectively. In our case we derive

$$N = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi \simeq \frac{1}{8}\kappa^2\phi_1^2, \tag{3.17}$$

¹ Note that, by using (3.3), asymptotically one must find [21,22],

$$\epsilon_4 = \frac{\left[\frac{\dot{\phi}^2}{H\dot{F}(R, \phi)} (-4\epsilon_3) + 6\epsilon_1 + 6\epsilon_3(1 - \epsilon_2) \right]}{2 \left[\frac{\dot{\phi}^2}{H\dot{F}(R, \phi)} + 3\epsilon_3 \right]}.$$

However, in our model

$$\frac{H\dot{F}(R, \phi)}{\dot{\phi}^2} \simeq \frac{\kappa^2\phi^2(\tilde{\xi}^2 - 4a_1\tilde{f})}{\tilde{\xi}} + \frac{50Aa_1\kappa^2\phi^2}{3\tilde{\xi}} + \frac{6B\kappa^2\phi^2}{\tilde{\xi}} - \frac{3B\kappa^2\phi^2(\tilde{\xi}^2 - 4a_1\tilde{f})}{\tilde{\xi}^2},$$

diverges as $\sim \kappa^2\phi^2$ like ϵ_1, ϵ_3 , rendering $\epsilon_1 \simeq -\epsilon_3$ in the limit $1 \ll \tilde{\xi}\kappa^2\phi^2$ (otherwise, $\epsilon_4 \simeq (\epsilon_1/\epsilon_3 + 1)$ results to be large) and the expression above for ϵ_4 is useless (it holds true only at the zero order respect to $\epsilon_{1,2,3}$).

where $\phi_{i,f}$ are the values of the field at the beginning and at the end of inflation and we considered $\kappa^2\phi_e^2 \ll \kappa^2\phi_i^2$. In order to obtain the thermalization of observable universe, it must be $55 < N < 65$.

The spectral index n_s and the tensor-to-scalar ratio r take into account the cosmological scalar and tensorial perturbations left at the end of inflation and are given by [20],

$$n_s = 1 - 4\epsilon_1 - 2\epsilon_2 + 2\epsilon_3 - 2\epsilon_4, \quad r = 16(\epsilon_1 + \epsilon_3), \tag{3.18}$$

where $\epsilon_{1,2,3,4}$ must be evaluated in the limit $\phi = \phi_i$. Since in our case in first approximation $\epsilon_1 \simeq -\epsilon_3$, we write the whole formula for the tensor-to-scalar ratio r as,

$$r = -8(3 - \sqrt{4n_T + 1}), \quad n_T = \frac{(1 + \epsilon_3)(2 - \epsilon_1 + \epsilon_3)}{(1 - \epsilon_1)^2}, \tag{3.19}$$

which leads to (at the second order in the slow-roll parameters),

$$r \simeq 16(\epsilon_1 + \epsilon_3) + 16\epsilon_1(\epsilon_1 + \epsilon_3). \tag{3.20}$$

We get²

$$\begin{aligned} (1 - n_s) &\simeq \frac{16}{\kappa^2\phi^2} + \frac{4A(192a_1\tilde{f} + (5 - 48\tilde{\xi})\tilde{\xi})}{\tilde{f}(48a_1\tilde{f} + \tilde{\xi} - 12\tilde{\xi}^2)\kappa^2\phi^2} + \frac{16B}{\tilde{\xi}\kappa^2\phi^2}, \\ r &\simeq \frac{64\tilde{\xi}}{(4a_1\tilde{f} - \tilde{\xi}^2)\kappa^4\phi^4} - \frac{128A\tilde{\xi}^2}{\tilde{f}(4a_1\tilde{f} - \tilde{\xi}^2)\kappa^2\phi^2} - \frac{256B\tilde{\xi}}{(4a_1\tilde{f} - \tilde{\xi}^2)\kappa^2\phi^2}. \end{aligned} \tag{3.21}$$

By using the limit $\phi \simeq \phi_i$ and by plugging the e -folds number (3.17) one has

$$\begin{aligned} (1 - n_s) &\simeq \frac{2(1 + B/\tilde{\xi})}{N} + \frac{A(192a_1\tilde{f} + (5 - 48\tilde{\xi})\tilde{\xi})}{2\tilde{f}(48a_1\tilde{f} + \tilde{\xi} - 12\tilde{\xi}^2)N}, \\ r &\simeq \frac{\tilde{\xi}}{(4a_1\tilde{f} - \tilde{\xi}^2)N^2} - \frac{16A\tilde{\xi}^2}{\tilde{f}(4a_1\tilde{f} - \tilde{\xi}^2)N} - \frac{32B\tilde{\xi}}{(4a_1\tilde{f} - \tilde{\xi}^2)N}. \end{aligned} \tag{3.22}$$

The recent Planck satellite results [23,24] constraint these quantities as $n_s = 0.968 \pm 0.006$ (68% CL) and $r < 0.11$ (95% CL). Moreover, the last BICEP2/Keck Array data [25] yield a (combined) upper limit for the tensor-to-scalar ratio as $r < 0.07$ (95% CL). If one takes $N \sim 55$ – 65 , in the limit $A = B = 0$, the tensor-to-scalar ratio is small enough to satisfy the Planck and the BICEP2/Keck Array data, while the spectral index is in agreement with the Planck results inside the given range. Thus, the one-loop potential slightly changes these indexes, and the model is viable as long as $|B/\tilde{\xi}|, |A/\tilde{f}| \ll 1$.

4. The one-loop effective potential in quantum scalar electrodynamics with higher-derivative quantum gravity

Let us now generalize the results of above section when quantum gravity (QG) coupled with massless QED is taken into account. This theory is known to be multiplicatively renormalizable but the question with its unitarity remains to be open. In this work we consider such theory as kind of effective QG model in order to estimate its possible influence to inflationary universe. QG corrections to the QED beta-functions can be found in Ref. [5], but the derivation of the

² In the computation of the tensor-to-scalar ratio we have taken into account the contribution from $1/(\kappa^4\phi^4)$ also.

effective potential is quite complicated and can be given only in an implicit form applying linear curvature approximation, due to the complexity of the one-loop RG equations.

Higher derivative quantum corrections enter in (2.10) as

$$\begin{aligned} \frac{de^2(t')}{dt'} &= \beta_{e^2}(t'), & \frac{df(t')}{dt'} &= \beta_f(t') + \Delta\beta_f(t'), & \frac{d\xi(t')}{dt'} &= \beta_\xi(t') + \Delta\beta_\xi(t'), \\ \frac{d\phi(t')}{dt'} &= -(\gamma(t') + \Delta\gamma(t'))\phi(t'), \end{aligned} \tag{4.1}$$

where $\beta_{e^2, f, \xi}(t')$ and $\gamma(t')$ are given by (2.8) and the QG corrections read

$$\begin{aligned} \Delta\beta_f(t') &= \frac{1}{(4\pi)^2} \left[\lambda(t')^2 \xi(t')^2 \left(15 + \frac{3}{4\omega(t')^2} - \frac{9\xi(t')}{\omega(t')^2} + \frac{27\xi(t')^2}{\omega(t')^2} \right) \right. \\ &\quad \left. - \lambda(t')f(t') \left(5 + 3\xi(t')^2 + \frac{33\xi(t')^2}{2\omega(t')} - \frac{6\xi(t')}{\omega(t')} + \frac{1}{2\omega(t')} \right) \right], \\ \Delta\beta_\xi(t') &= \frac{1}{(4\pi)^2} \lambda(t')\xi(t') \left[-\frac{3}{2}\xi(t')^2 + 4\xi(t') + 3 + \frac{10}{3}\omega(t') \right. \\ &\quad \left. + \frac{1}{\omega(t')} \left(-\frac{9}{4}\xi(t')^2 + 5\xi(t') + 1 \right) \right], \\ \Delta\gamma(t') &= \frac{\lambda(t')}{4(4\pi)^2} \left[\frac{13}{3} - 8\xi(t') - 3\xi(t')^2 - \frac{1}{6\omega(t')} - \frac{2\xi(t')}{\omega(t')} + \frac{3\xi(t')^2}{2\omega(t')} \right]. \end{aligned} \tag{4.2}$$

Here, $\lambda(t')$ and $\omega(t')$, where only $\lambda(t')$ has an explicit formulation, correspond to the running coupling constants $a_1 \equiv a_1(t')$ and $a_2 \equiv a_2(t')$ in (2.3), which interact with the matter sector and are given by

$$a_1(t') = -\frac{\omega(t')}{3\lambda(t')}, \quad a_2(t') = \frac{1}{\lambda(t')}, \tag{4.3}$$

with

$$\begin{aligned} \lambda(t') &= \frac{\lambda}{1 + \frac{203\lambda t'}{15(4\pi)^2}}, \\ \frac{d\omega(t')}{dt'} &= \beta_\omega(t') = -\frac{\lambda(t')}{(4\pi)^2} \left[\frac{10}{3}\omega(t')^2 + \left(5 + \frac{203}{15} \right) \omega(t') + \frac{5}{12} + 3 \left(\xi(t') - \frac{1}{6} \right)^2 \right], \end{aligned} \tag{4.4}$$

where $\lambda \equiv \lambda(t' = 0)$ and in general $0 < \lambda$. The local gauge invariance prohibits the QG correction to $e^2(t')$, which has the same form of (2.11). Now it is possible to find the effective potential (2.12) for higher-derivative QG with scalar QED, and for small t' and small couplings one derives [8]

$$V_{\text{eff}} = -\tilde{f}\phi^4 - A\phi^4 \left[\log \frac{\phi^2}{\mu^2} - \frac{25}{6} \right] + \tilde{\xi}R\phi^2 - BR\phi^2 \left[\log \frac{\phi^2}{\mu^2} - 3 \right], \tag{4.5}$$

with

$$\begin{aligned}
 \tilde{f} &= \frac{f}{4!}, \quad \tilde{\xi} = \frac{\xi}{2}, \\
 A &= \frac{1}{48(4\pi)^2} \left[\frac{10}{3} f^2 + 36e^4 + \lambda^2 \xi^2 \left(15 + \frac{3}{4\omega^2} - \frac{9\xi}{\omega^2} + \frac{27\xi^2}{\omega^2} \right) \right. \\
 &\quad \left. - \lambda f \left(\frac{28}{3} + 18 \frac{\xi^2}{\omega} - \frac{8\xi}{\omega} - 8\xi + \frac{1}{3\omega} \right) \right], \\
 B &= -\frac{1}{4(4\pi)^2} \left[\left(\xi - \frac{1}{6} \right) \left(\frac{4f}{3} - 6e^2 \right) + 6\xi e^2 \right. \\
 &\quad \left. + \lambda \xi \left[8\xi + \frac{5}{6} + \frac{10}{3}\omega + \frac{1}{\omega} \left(-3\xi^2 + 6\xi + \frac{13}{12} \right) \right] \right], \tag{4.6}
 \end{aligned}$$

where, as usually, $\omega \equiv \omega(t' = 0)$, $e \equiv e(t' = 0)$, $f \equiv f(t' = 0)$, $\xi \equiv \xi(t' = 0)$ and $\phi \equiv \phi(t' = 0)$. In the next section, this expression for the effective potential is applied to study inflation in higher-derivative QG with scalar QED.

5. Inflation in quantum gravity with scalar quantum electrodynamics

In this section, we will analyze the inflation for the effective potential (4.5) with running coupling constants for the gravitational Lagrangian in (2.3). The general formalism of a RG-improved theory requires an explicit dependence on the renormalization scale of $\kappa^2 \equiv \kappa^2(t')$ and $\Lambda \equiv \Lambda(t')$ in (2.3). In particular, $\kappa^2(t')$ obeys to the differential equation [5],

$$\frac{d\kappa^2(t')}{dt'} = \frac{\kappa^2 \lambda(t')}{(4\pi)^2} \left[\frac{10\omega(t')}{3} - \frac{13}{6} - \frac{1}{4\omega(t')} \right]. \tag{5.1}$$

Despite to the fact that it is not possible to solve explicitly the equation for $\omega(t')$ in (4.4), we will try to estimate the gravitational running coupling constants by using the fixed points of this equation, which correspond to³

$$\omega_{1,2} = \frac{1}{50} \left[-139 \pm \sqrt{2 \left(9473 + 750\tilde{\xi} - 4500\tilde{\xi}^2 \right)} \right], \tag{5.2}$$

where $\xi(t') \simeq \xi$ and we have introduced the notation in (4.6). By perturbing the solution of $\omega(t')$ around the fixed points as $\omega(t') \simeq \omega_{1,2} + \delta\omega(t')$ with $|\delta\omega(t')| \ll 1$, from (4.4) one has,

$$\frac{d\omega(t')}{dt'} \simeq -\frac{\lambda}{(4\pi)^2 \left(1 + \frac{203\lambda t'}{15(4\pi)^2} \right)} \left[\frac{20}{3}\omega_{1,2} + \left(5 + \frac{203}{15} \right) \right] \delta\omega(t'), \tag{5.3}$$

whose solution reads

$$\omega(t') \simeq \omega_{1,2} + \frac{c_0}{\left(1 + \frac{203\lambda t'}{15(4\pi)^2} \right)^q}, \quad q = \frac{15}{203} \left[\frac{20}{3}\omega_{1,2} + \left(5 + \frac{203}{15} \right) \right], \tag{5.4}$$

c_0 being a constant. The solution does not diverge only if $0 < q$ and we may assume a stable fixed point for $\omega(t') \simeq \omega_1$ (i.e., with the sign plus inside (5.2)). In this case, from equation (5.1) we obtain

³ A derivation of the adimensional quantity $\kappa(t')^4 \Lambda(t')$ can be found in Ref. [5].

$$\kappa^2(t') \simeq \tilde{\kappa}^2 \left(1 + \frac{203\lambda t'}{15(4\pi)^2} \right)^{15z/203}, \quad z = \left[\frac{10\omega_1}{3} - \frac{13}{6} - \frac{1}{4\omega_1} \right], \tag{5.5}$$

with $\tilde{\kappa}^2 \equiv \kappa^2(t' = 0)$. We must pose $\tilde{\kappa}^2 = 8\pi/M_{Pl}^2$, namely we would like to recover the Planck mass when quantum effects disappear, and we require that $0 < z$, such that during inflation the mass scale of the theory decreases.

By taking t' small, one can work with the following forms of $\kappa^2(t')$, $a_1(t')$ inside (2.3),

$$\frac{1}{\kappa^2(t')} = \frac{1}{\tilde{\kappa}^2} - 2m^2 t', \quad a_1(t') \equiv \tilde{a}_1 + 2b_1 t', \tag{5.6}$$

where $\tilde{a}_1 = a_1(t' = 0)$, b_1 is an adimensional parameter and m^2 a mass constant such that (during inflation),

$$m^2 < \frac{1}{2\tilde{\kappa}^2 t'}. \tag{5.7}$$

Specifically, it is easy to verify that

$$m^2 = \frac{1}{2\tilde{\kappa}^2} \frac{z\lambda}{(4\pi)^2}, \quad \tilde{a}_1 = -\frac{\omega}{3\lambda}, \quad b_1 = -\frac{1}{2} \left[\frac{203\omega}{45(4\pi^2)} \right]. \tag{5.8}$$

Finally, by using (2.7), we have

$$\frac{1}{\kappa^2} = \frac{1}{\tilde{\kappa}^2} - m^2 \log \left[\frac{\phi^2}{\mu^2} \right], \quad a_1 = \tilde{a}_1 + b_1 \log \left[\frac{\phi^2}{\mu^2} \right]. \tag{5.9}$$

As in the previous section, we will set $\Lambda \equiv \Lambda(t') = 0$ in (2.3) and observe that the variation of the square of the Weyl tensor on FRW metric when $a_2 \equiv a_2(t')$ reads

$$\delta I_{C^2} = a_2(t') \delta \left(\sqrt{-g} C^2 \right) + \left(\sqrt{-g} C^2 \right) \delta a_2(t') = 0, \tag{5.10}$$

due to the fact that the square of the Weyl tensor is identically null in homogeneous and isotropic space–time. We must note that in the presence of running coupling constants also the Gauss–Bonnet G and the $\square R$ -terms in the general formulation of the action (2.1) give contribution, but here, for the sake of simplicity, we will omit such terms.

On flat FRW space–time the first Friedmann equation of the model is derived as

$$3H^2 \left[\frac{1}{\tilde{\kappa}^2} - m^2 \log \left[\frac{\phi^2}{\mu^2} \right] \right] + 12 \left[\tilde{a}_1 + b_1 \log \left[\frac{\phi^2}{\mu^2} \right] \right] H^2 R = \left[\tilde{a}_1 + b_1 \log \left[\frac{\phi^2}{\mu^2} \right] \right] R^2 \frac{\dot{\phi}^2}{2} + \left[V_{\text{eff}} - R \frac{dV_{\text{eff}}}{dR} \right] + 6H^2 \frac{dV_{\text{eff}}}{dR} - 3H \dot{F}, \tag{5.11}$$

with

$$F \equiv F(R, \phi) = \left[\frac{1}{\tilde{\kappa}^2} - m^2 \log \left[\frac{\phi^2}{\mu^2} \right] \right] + 4 \left[\tilde{a}_1 + b_1 \log \left[\frac{\phi^2}{\mu^2} \right] \right] R - 2 \frac{dV_{\text{eff}}}{dR}. \tag{5.12}$$

Moreover, the continuity equation of the scalar field is given by

$$\ddot{\phi} + 3H\dot{\phi} = -V'_{\text{eff}} + \frac{1}{\phi} \left[-m^2 R + 2b_1 R^2 \right]. \tag{5.13}$$

In the slow-roll approximation with $R \simeq 12H^2$ the equations (5.11) and (5.13) assume the form

$$\begin{aligned}
 3H^2 \left[\frac{1}{\tilde{\kappa}^2} - m^2 \log \left[\frac{\phi^2}{\mu^2} \right] \right] &\simeq \left[V_{\text{eff}} - 6H^2 \frac{dV_{\text{eff}}}{dR} \right], \\
 3H\dot{\phi} &\simeq -V'_{\text{eff}} + \frac{1}{\phi} \left[-12H^2 m^2 + 288b_1 H^4 \right].
 \end{aligned}
 \tag{5.14}$$

Now the de Sitter solution for the effective potential (2.13) is given by,

$$H_{\text{dS}}^2 \simeq \frac{\left[\tilde{f} + A \left[\log \left[\frac{\phi^2}{\mu^2} \right] - \frac{25}{6} \right] \right] \tilde{\kappa}^2 \phi^4}{-3 \left[1 - m^2 \tilde{\kappa}^2 \log \left[\frac{\phi^2}{\mu^2} \right] \right] + 6 \left[\tilde{\xi} - B \left[\log \left[\frac{\phi^2}{\mu^2} \right] - 3 \right] \right] \tilde{\kappa}^2 \phi^2},
 \tag{5.15}$$

and it is large under the condition

$$\frac{\left[1 - m^2 \tilde{\kappa}^2 \log \left[\frac{\phi^2}{\mu^2} \right] \right]}{\tilde{\kappa}^2 \tilde{\xi}} \ll \phi^2.
 \tag{5.16}$$

If we identify $\tilde{\kappa}^2 = 8\pi/M_{Pl}^2$, since the field may be larger than the Planck mass during inflation, we must also require $\tilde{f}/\tilde{\xi} < 1$. From the second equation in (5.14) we get

$$\dot{\phi} \simeq \frac{288b_1 H^4 - 12H^2 m^2 \phi^2 + 2\phi^2 \left[12H^2 \left[-2B - \tilde{\xi} + B \log \left[\frac{\phi^2}{\mu^2} \right] \right] + \left[-22A/3 + 2\tilde{f} + 2A \log \left[\frac{\phi^2}{\mu^2} \right] \right] \phi^2 \right]}{3H\phi}.
 \tag{5.17}$$

Thus, by taking A, B and b_1 small, we obtain

$$\frac{\dot{\phi}^2}{V_{\text{eff}}} \sim \frac{16 \left[1 - m^2 \tilde{\kappa}^2 \left[\log \left[\frac{\phi^2}{\mu^2} \right] - 1 \right] \right]^2}{3\tilde{\kappa}^2 \phi^2 \left[1 + m^2 \tilde{\kappa}^2 \log \left[\frac{\phi^2}{\mu^2} \right] \right] + 6\tilde{\xi} \tilde{\kappa}^4 \phi^4},
 \tag{5.18}$$

which goes to zero when (5.16) is satisfied (the corrections to $\dot{\phi}^2/V_{\text{eff}}$ are at the second order in b_1). The slow-roll parameters, at the first order in A, B and b_1 , read

$$\begin{aligned}
 \epsilon_1 &\simeq \frac{4 \left[1 - m^2 \tilde{\kappa}^2 \left[\log \left[\frac{\phi^2}{\mu^2} \right] - 1 \right] \right]}{\tilde{\kappa}^2 \phi^2} + \frac{2A \left[4 - m^2 \tilde{\kappa}^2 \left[4 \log \left[\frac{\phi^2}{\mu^2} \right] - 3 \right] - 2\tilde{\xi} \tilde{\kappa}^2 \phi^2 \right]}{\tilde{f} \tilde{\kappa}^2 \phi^2} \\
 &+ \frac{8B \left[1 - m^2 \tilde{\kappa}^2 \left[\log \left[\frac{\phi^2}{\mu^2} \right] - 1 \right] - \tilde{\xi} \tilde{\kappa}^2 \phi^2 \right]}{\tilde{\xi} \tilde{\kappa}^2 \phi^2} - \frac{8b_1 \tilde{f} \left[-m^2 \tilde{\kappa}^2 + 2\tilde{\xi} \tilde{\kappa}^2 \phi^2 \right]}{\tilde{\xi}^2 \tilde{\kappa}^2 \phi^2}, \\
 \epsilon_2 &\simeq \frac{8m^2}{\phi^2} + \frac{2A \left[-3 - m^2 \tilde{\kappa}^2 \left[1 - 3 \log \left[\frac{\phi^2}{\mu^2} \right] \right] + 4\tilde{\xi} \tilde{\kappa}^2 \phi^2 \right]}{\tilde{f} \tilde{\kappa}^2 \phi^2} \\
 &- \frac{8B \left[1 - m^2 \tilde{\kappa}^2 \left[\log \left[\frac{\phi^2}{\mu^2} \right] - 1 \right] - 2\tilde{\xi} \tilde{\kappa}^2 \phi^2 \right]}{\tilde{\xi} \tilde{\kappa}^2 \phi^2} \\
 &+ \frac{8b_1 \tilde{f} \left[-1 - m^2 \tilde{\kappa}^2 \left[3 - \log \left[\frac{\phi^2}{\mu^2} \right] \right] + 4\tilde{\xi} \tilde{\kappa}^2 \phi^2 \right]}{\tilde{\xi}^2 \tilde{\kappa}^2 \phi^2},
 \end{aligned}$$

$$\begin{aligned}
 \epsilon_3 \simeq & -\frac{4\left[1-m^2\tilde{\kappa}^2\left[\log\left[\frac{\phi^2}{\mu^2}\right]-1\right]\right]}{\tilde{\kappa}^2\phi^2} \\
 & +\frac{2A\left[2\tilde{\xi}(4a_1\tilde{f}-\tilde{\xi}^2)\tilde{\kappa}^2\phi^2+4a_1\tilde{f}(-4-3m^2\tilde{\kappa}^2)-m^2\tilde{\kappa}^2\tilde{\xi}^2+16a_1\tilde{f}m^2\tilde{\kappa}^2\log\left[\frac{\phi^2}{\mu^2}\right]\right]}{\tilde{f}(4a_1\tilde{f}-\tilde{\xi}^2)\tilde{\kappa}^2\phi^2} \\
 & +\frac{8B\left[(\tilde{\xi}^3-4a_1\tilde{f}\tilde{\xi})\tilde{\kappa}^2\phi^2+(4a_1\tilde{f}+\tilde{\xi}^2)(1-m^2\tilde{\kappa}^2)\left[\log\left[\frac{\phi^2}{\mu^2}\right]-1\right]\right]}{(\tilde{\xi}^3-4a_1\tilde{f}\tilde{\xi})\tilde{\kappa}^2\phi^2} \\
 & -\frac{8b_1\tilde{f}\left[-2\tilde{\xi}(-4a_1\tilde{f}+\tilde{\xi}^2)\tilde{\kappa}^2\phi^2-4a_1\tilde{f}m^2\tilde{\kappa}^2+\tilde{\xi}^2(-3-2m^2\tilde{\kappa}^2)+3m^2\tilde{\kappa}^2\tilde{\xi}^2\log\left[\frac{\phi^2}{\mu^2}\right]\right]}{\tilde{\xi}^2(-4a_1\tilde{f}+\tilde{\xi}^2)\tilde{\kappa}^2\phi^2}, \\
 \epsilon_4 \simeq & -\frac{4\left[1-m^2\tilde{\kappa}^2\left[\log\left[\frac{\phi^2}{\mu^2}\right]-1\right]\right]}{\tilde{\kappa}^2\phi^2}+2A\left[\frac{2\tilde{\xi}}{\tilde{f}}\right. \\
 & -\frac{\left[-192a_1^2\tilde{f}^2(-5-2m^2\tilde{\kappa}^2)+4a_1\tilde{f}\tilde{\xi}(4-72\tilde{\xi}-3m^2\tilde{\kappa}^2(-1+16\tilde{\xi}))-\tilde{\xi}^3(-12\tilde{\xi}-m^2\tilde{\kappa}^2(1+24\tilde{\xi}))\right]}{\tilde{f}(192a_1^2\tilde{f}^2+4a_1\tilde{f}(1-24\tilde{\xi})\tilde{\xi}+\tilde{\xi}^3(-1+12\tilde{\xi}))\tilde{\kappa}^2\phi^2} \\
 & \left.+\frac{4m^2\tilde{\kappa}^2(240a_1^2\tilde{f}^2+4a_1\tilde{f}(1-18\tilde{\xi})\tilde{\xi}+3\tilde{\xi}^4)\log\left[\frac{\phi^2}{\mu^2}\right]}{\tilde{f}(192a_1^2\tilde{f}^2+4a_1\tilde{f}(1-24\tilde{\xi})\tilde{\xi}+\tilde{\xi}^3(-1+12\tilde{\xi}))\tilde{\kappa}^2\phi^2}\right] \\
 & +8B\left[1-\frac{(4a_1\tilde{f}+\tilde{\xi}^2)\left[48a_1\tilde{f}-\tilde{\xi}(-1-m^2\tilde{\kappa}^2+12\tilde{\xi})-m^2\tilde{\kappa}^2(48a_1\tilde{f}+\tilde{\xi}-12\tilde{\xi}^2)\log\left[\frac{\phi^2}{\mu^2}\right]\right]}{\tilde{\xi}\left(192a_1^2\tilde{f}^2+4a_1\tilde{f}(1-24\tilde{\xi})\tilde{\xi}+\tilde{\xi}^3(-1+12\tilde{\xi})\right)\tilde{\kappa}^2\phi^2}\right] \\
 & +8b_1\tilde{f}\left[\frac{2}{\tilde{\xi}}+\frac{\left[-192a_1^2\tilde{f}^2(-1-2m^2\tilde{\kappa}^2)+\tilde{\xi}^3(-3-2m^2\tilde{\kappa}^2+36\tilde{\xi})+4a_1\tilde{f}\tilde{\xi}(-48\tilde{\xi}-m^2\tilde{\kappa}^2(1+24\tilde{\xi}))\right]}{\tilde{\xi}^2(192a_1^2\tilde{f}^2+4a_1\tilde{f}(1-24\tilde{\xi})\tilde{\xi}+\tilde{\xi}^3(-1+12\tilde{\xi}))\tilde{\kappa}^2\phi^2}\right. \\
 & \left.-\frac{3m^2\tilde{\kappa}^2(64a_1^2\tilde{f}^2-64a_1\tilde{f}\tilde{\xi}^2+\tilde{\xi}^3(-1+12\tilde{\xi}))\log\left[\frac{\phi^2}{\mu^2}\right]}{\tilde{\xi}^2(192a_1^2\tilde{f}^2+4a_1\tilde{f}(1-24\tilde{\xi})\tilde{\xi}+\tilde{\xi}^3(-1+12\tilde{\xi}))\tilde{\kappa}^2\phi^2}\right]. \tag{5.19}
 \end{aligned}$$

The e -folds is given by

$$\begin{aligned}
 N \simeq & -\int_{\phi_f}^{\phi_i} \frac{\tilde{\kappa}^2\phi}{4-4m^2\tilde{\kappa}^2\left[\log\left[\frac{\phi^2}{\mu^2}\right]-1\right]} \\
 \simeq & \frac{\tilde{\kappa}^2\phi_i^2}{8-8m^2\tilde{\kappa}^2\left[\log\left[\frac{\phi_i^2}{\mu^2}\right]-1\right]}\sum_{n=0}^{\tilde{n}} \frac{n!(-4m^2\tilde{\kappa}^2)^n}{\left[4-4m^2\tilde{\kappa}^2\left[\log\left[\frac{\phi_i^2}{\mu^2}\right]-1\right]\right]^n}, \tag{5.20}
 \end{aligned}$$

where we used the fact $\phi_f \ll \phi_i$ and we must cut the series at some $n = \tilde{n}$. For example, a simple estimation of \tilde{n} may be given by

$$\frac{\phi_i}{\phi_f} \simeq \left[\frac{4-4m^2\tilde{\kappa}^2\left[\log\left[\frac{\phi_i^2}{\mu^2}\right]-1\right]}{4-4m^2\tilde{\kappa}^2\left[\log\left[\frac{\phi_f^2}{\mu^2}\right]-1\right]} \right]^{\tilde{n}+1}, \tag{5.21}$$

namely when we cannot ignore the contributions from ϕ_f due to the large number of n . In the limit $m^2 = 0$, one recovers (3.17). We observe that in general, when ϕ_i is large enough with respect to the mass scale μ , the computation of the e -folds simply leads to

$$N \simeq \frac{\kappa^2 \phi_1^2}{8 - 8m^2 \tilde{\kappa}^2 \left[\log \left[\frac{\phi_1^2}{\mu^2} \right] - 1 \right]}. \tag{5.22}$$

By using (3.18)–(3.19) we are ready to calculate the spectral index and the tensor-to-scalar ratio of the theory as,

$$\begin{aligned} (1 - n_s) &\simeq \frac{16 \left[1 - m^2 \tilde{\kappa}^2 \left[\log \left[\frac{\phi^2}{\mu^2} \right] - 2 \right] \right]}{\tilde{\kappa}^2 \phi^2} \\ &+ \frac{4A \left[-96a_1 \tilde{f} (-2 - 3m^2 \tilde{\kappa}^2) + \tilde{\xi} (5 - 48\tilde{\xi} - m^2 \tilde{\kappa}^2 (-5 + 24\tilde{\xi})) - m^2 \tilde{\kappa}^2 (192a_1 \tilde{f} + 5\tilde{\xi} - 48\tilde{\xi}^2) \log \left[\frac{\phi^2}{\mu^2} \right] \right]}{\tilde{f} (48a_1 \tilde{f} + \tilde{\xi} - 12\tilde{\xi}^2) \tilde{\kappa}^2 \phi^2} \\ &- \frac{16B \left[48a_1 \tilde{f} (1 + 2m^2 \tilde{\kappa}^2) - \tilde{\xi} (-1 + 12\tilde{\xi} - m^2 \tilde{\kappa}^2) - m^2 \tilde{\kappa}^2 (48a_1 \tilde{f} + \tilde{\xi} - 12\tilde{\xi}^2) \log \left[\frac{\phi^2}{\mu^2} \right] \right]}{\tilde{\xi} (-48a_1 \tilde{f} - \tilde{\xi} + 12\tilde{\xi}^2) \tilde{\kappa}^2 \phi^2} \\ &+ \frac{16b_1 \tilde{f} \left[-96a_1 \tilde{f} m^2 \tilde{\kappa}^2 + \tilde{\xi} (1 - 12\tilde{\xi} - m^2 \tilde{\kappa}^2 (-1 + 36\tilde{\xi})) - m^2 \tilde{\kappa}^2 (\tilde{\xi} - 12\tilde{\xi}^2) \log \left[\frac{\phi^2}{\mu^2} \right] \right]}{\tilde{\xi}^2 (-48a_1 \tilde{f} - \tilde{\xi} + 12\tilde{\xi}^2) \tilde{\kappa}^2 \phi^2}, \\ r &\simeq \frac{64\tilde{\xi} \left[1 - m^2 \tilde{\kappa}^2 \left[\log \left[\frac{\phi^2}{\mu^2} \right] - 1 \right] \right]^2}{(4a_1 \tilde{f} - \tilde{\xi}^2) \tilde{\kappa}^4 \phi^4} \\ &- \frac{\left[128A\tilde{\xi}^2 / \tilde{f} + 256B\tilde{\xi} \right] \left[1 - m^2 \tilde{\kappa}^2 \left[\log \left[\frac{\phi^2}{\mu^2} \right] \right] - 1 \right]}{(4a_1 \tilde{f} - \tilde{\xi}^2) \tilde{\kappa}^2 \phi^2} \\ &- \frac{384b_1 \tilde{f} \left[1 - m^2 \tilde{\kappa}^2 \left[\log \left[\frac{\phi^2}{\mu^2} \right] - 1 \right] \right]}{(4a_1 \tilde{f} - \tilde{\xi}^2) \tilde{\kappa}^2 \phi^2}. \end{aligned} \tag{5.23}$$

In the limit $m^2 = 0$, these indexes with (5.20) read

$$\begin{aligned} (1 - n_s) &\simeq \frac{2(1 + B/\tilde{\xi})}{N} + \frac{A(192a_1 \tilde{f} + (5 - 48\tilde{\xi})\tilde{\xi})}{2\tilde{f}(48a_1 \tilde{f} + \tilde{\xi} - 12\tilde{\xi}^2)N} + \frac{2b_1 \tilde{f} \left[\tilde{\xi} (1 - 12\tilde{\xi}) \right]}{\tilde{\xi}^2 (-48a_1 \tilde{f} - \tilde{\xi} + 12\tilde{\xi}^2)N}, \\ r &\simeq \frac{\tilde{\xi}}{(4a_1 \tilde{f} - \tilde{\xi}^2)N^2} - \frac{16A\tilde{\xi}^2}{\tilde{f}(4a_1 \tilde{f} - \tilde{\xi}^2)N} - \frac{32B\tilde{\xi}}{(4a_1 \tilde{f} - \tilde{\xi}^2)N} - \frac{194b_1 \tilde{f}}{4(4a_1 \tilde{f} - \tilde{\xi}^2)N}, \end{aligned} \tag{5.24}$$

and we recover (3.21) with the contribution of the log-correction to R^2 . On the other hand, when $m^2 \neq 0$, in the limit $A = B = 0$ one gets

$$(1 - n_s) \simeq \frac{2 \left[8 - 8m^2 \tilde{\kappa}^2 \left[-2 + \log \left[\frac{\phi^2}{\mu^2} \right] \right] \right]}{\tilde{\kappa}^2 \phi^2}, \quad r \simeq \frac{64\tilde{\xi} \left[1 - m^2 \tilde{\kappa}^2 \left[\log \left[\frac{\phi^2}{\mu^2} \right] - 1 \right] \right]^2}{(4a_1 \tilde{f} - \tilde{\xi}^2) \tilde{\kappa}^4 \phi^4}, \tag{5.25}$$

and in order to satisfy the last Planck satellite results it must be

$$\frac{2}{(1 - n_s)} = \frac{\tilde{\kappa}^2 \phi^2}{\left[8 - 8m^2 \tilde{\kappa}^2 \left[\log \left[\frac{\phi^2}{\mu^2} \right] - 2 \right] \right]} \simeq 60. \tag{5.26}$$

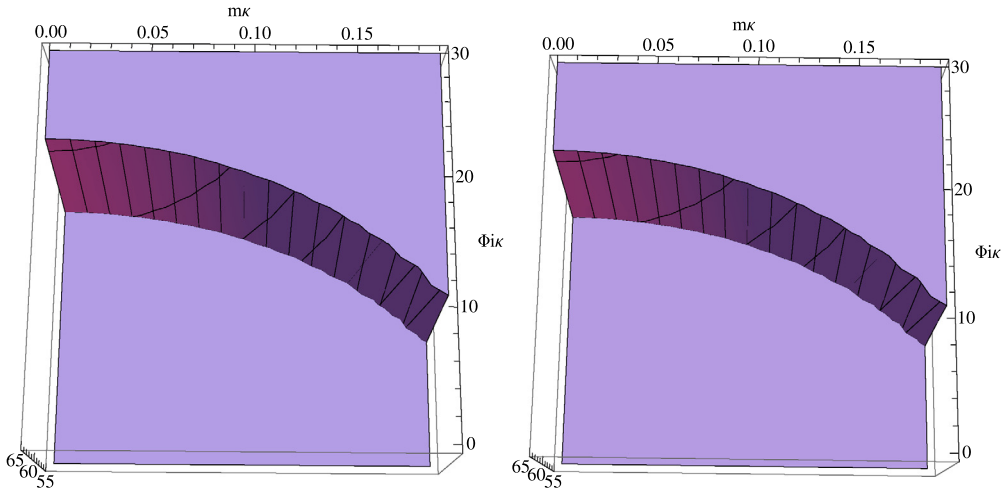


Fig. 1. The e -folds number N (left) and the quantity $2/(1 - n_s)$ (right) as functions of $\phi_i \bar{\kappa}$ and $m \bar{\kappa}$ for the quantum scalar electro-dynamics with higher-derivative quantum gravity corrections. The dark zones correspond to $55 < N < 65$ and $55 < 2/(1 - n_s) < 65$, respectively. We can observe that the values of ϕ_i and m which lead to a correct amount of inflation (N), also lead to a spectral index consistent with Planck data.

In this case, the spectral index lies inside the observed range, while the tensor-to-scalar ratio is small enough to be in agreement with Planck and BICEP2/Keck Array data. When $\phi \simeq \phi_i$ and ϕ_i is large enough, this condition is satisfied for $55 < N < 65$. In Fig. 1 we plot the e -folds number N and the quantity $2/(1 - n_s)$ as functions of $\phi_i \bar{\kappa}$ and $m \bar{\kappa}$. The calculation of N has been carried out in numerical way⁴ by using the integral in (5.20). Since at the end of inflation the quantum gravity corrections disappear, we posed $\bar{\kappa} = \kappa \equiv \sqrt{8\pi}/M_{Pl}$ and $\mu = 10^{-4}/\bar{\kappa}$ ($\sim \mu_{GUT}$). The final value of ϕ has been set as $\phi_f = \mu$, while the range of ϕ_f and m have been chosen as $\mu < \phi_f < 10^2/\kappa$ (we remember that the field can exceed the Planck scale) and $0 \leq m < [\bar{\kappa} \sqrt{\log[10^4/(\mu^2 \bar{\kappa}^2)}]^{-1}$ (see condition (5.7)), respectively. The dark zones in the graphics correspond to $55 < N < 65$ and $55 < 2/(1 - n_s) < 65$ and confirm that the values of ϕ_i and m which lead to a correct amount of inflation, also lead to a spectral index according with Planck results. Thus, in order to have a viable inflationary scenario, $10/\kappa \leq \phi_f \leq 20/\kappa$ has to match $0 \leq m < 0.20/\kappa$. For $m = 0$, one obtains $\phi_f \simeq 20/\kappa$ (in this limit, we have $N \simeq 60$ in (3.17)).

We note that in terms of the e -folds number, the QG corrections to the tensor-to-scalar ratio in (5.23) assume the same form of (5.24), namely, given A , B and b_1 with a correct amount of inflation, the model leads to the same corrections to the tensor spectral index. On the other hand, the much more involved expression for the spectral index brings it to have a different form with respect to (5.24), namely, by plugging in the spectral index the expression for the e -folds number, it remains an explicit dependence on the mass scale m^2 . In this sense, given A , B and b_1 with a correct amount of inflation, the QG effects lead to different corrections in the spectral index if compare with the case of pure scalar QED.

⁴ Mathematica ©.

The running mass scale of the model influences the bound of the field and therefore the de Sitter solution of inflation, since from (5.15), for large values of the field, we get

$$H_{\text{dS}}^2 \sim \frac{\tilde{f}\phi^2}{6\tilde{\xi}}, \tag{5.27}$$

which is the same expression of (3.8). Given \tilde{f} and $\tilde{\xi}$, when $m = 0$, in order to have $N \simeq 60$, the field must be $\phi \simeq 22/\tilde{\kappa}^2$, but when $0 < m$, to obtain the same amount of inflation, the field and the Hubble parameter must be smaller. In this sense, the quantum corrections to the Planck mass bring to a weaker acceleration during inflation.

As in the previous case, the R^2 -term does not play a significant role for the exit from inflation. However, an important remark is in order. If the mass scale of theory essentially decreases at the early-time epoch due to the quantum corrections, the following condition may be realized for a subplanckian value of the curvature,

$$1 \ll a_1(t')\kappa(t')^2 R. \tag{5.28}$$

In this case, equation (5.11) is asymptotically satisfied for some boundary value of the de Sitter Hubble parameter, and one recovers inflation from R^2 -gravity with log-corrections (see Ref. [18]).

From the expression of ϵ_1 in (3.15) or (5.19) we have, in terms of $N = \log(a_t f/a(t))$,

$$\epsilon_1 \simeq \frac{1}{2N}. \tag{5.29}$$

By taking into account that $d/dt = -H(N)d/dN$ together with the definition of ϵ_1 , one easily derives the behaviour of the Hubble parameter during inflation,

$$H(N)^2 = H_0^2 N, \tag{5.30}$$

where $H_0^2 \ll H_{\text{dS}}^2 = H_0^2 N|_{N \simeq 60}$ gives the value of the Hubble parameter at the end of inflation. Graceful exit occurs when $N \simeq 0$ and ϵ_1 exceeds the unit. Thus, the quantum gravity effects will disappear ($\phi \simeq \mu$) and our gravitational Lagrangian will turn out to be General Relativity plus a quadratic correction of the Ricci scalar. The behaviour of this model at the end of inflation has been well investigated in literature, and it has been demonstrated that it is compatible with the reheating process for particle production at the beginning of the Friedmann expansion predicted by General Relativity.

6. Conclusions

In this paper, we have analyzed inflation for a quantum scalar electrodynamics model in curved space–time and for higher-derivative quantum gravity with scalar electrodynamics. The RG improved effective potential is calculated for both theories (i.e. without and with QG corrections) in Jordan frame. At the FRW universe, the gravitational action contains R^2 -term beyond the Hilber–Einstein term R . Our analysis has been carried out in the Jordan frame, due to non-equivalence of quantum corrected Jordan and Einstein frames.

The resulting inflationary scenarios are in agreement with the Planck and the last BICEP2/Keck Array data and bring to an amount of inflation compatible with the thermalization of the observable universe. Note that as it is clearly seen from the explicit expressions for slow-roll parameters the analysis of Jordan frame inflation seems to be much more complicated than the corresponding analysis in convenient Einstein frame.

When the quadratic R^2 -term is not asymptotically dominant in the gravitational action, its contribution appears only via log-corrections in the spectral index and the tensor-to-scalar ratio, namely it does not play a significant role in the exit from inflation, like in the Jordan-frame representation of the Starobinsky-like models. However, we note that, due to the running mass scale of the theory, the R^2 -term may be dominant for a large subplanckian value of the curvature: in this case we obtain a pure R^2 -gravitational model with log-corrections.

Our analysis shows how one-loop QED and QG corrections enter in the spectral index and in the tensor-to-scalar ratio of the model under discussion. The most interesting corrections in the coupling constants of the gravitational action from QG effects are related to the running gravitational constant. Here, we stress that the viability of the inflationary scenario does not directly require that the QG correction to R is small, like in the case of the log-quantum correction to R^2 or the one-loop corrections in the effective potential of the field. If at the early-time epoch the Planck mass of the theory decreases, the bound of the field must be smaller to get a realistic inflationary scenario. As a consequence, also the Hubble parameter of the (quasi) de Sitter solution describing inflation is smaller leading to a weaker acceleration. It is interesting to note that it is straightforward to generalize this study for Standard Model with higher-derivative QG. However, the corresponding expressions turn out to be much more involved.

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