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THE CAMASSA-HOLM FAMILY OF EQUATIONS AND THE TANH-COTH METHOD

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In this paper, we consider the shallow water equation of the family of Camassa – Holm equations (CH), particularly the Fornberg – Whitham (FW) equation [1]. The CH equation models just like the Korteweg-de Vries (KdV) equation the propagation of two-dimensional shallow water waves over a flat bed. Camassa – Holm equation was introduced in 1993 by R. Camassa and D.Holm as a bi-Hamiltonian model for waves in shallow water [2]. The CH equation is the integrable, dimensionless and non-linear partial differential equation.

The family of Camassa – Holm equations is

$$u_t - u_{xxt} + au_x + buu_x = ku_xu_{xx} + uu_{xxx}, \quad (1)$$

where *a*,*b*and k are constants, and *u(x,t)* is the unknown function depending on temporal variable *t* and spatial variable *x*.

The Camassa-Holm family of equations consist of four different equations [1] which changes dependently on coefficients *a*,*b* and *k* listed below

$$u_{1t} - u_{xxt} + au_x + 3uu_x = 2u_xu_{xx} + uu_{xxx}, \quad (2)$$

$$u_{2t} - u_{xxt} + au_x + 4uu_x = 3u_xu_{xx} + uu_{xxx}, \quad (3)$$

$$u_{3t} - u_{xxt} + u_x + uu_x = 3u_xu_{xx} + uu_{xxx}, \quad (4)$$

$$u_{4t} - u_{xxt} + 2u_x + 3uu_x = 2u_xu_{xx} + uu_{xxx}. \quad (5)$$

The equations (2)-(5) are known as the Camassa-Holm (CH) that is (2), the Degasperis-Procesi (DP) that is (3, the Fornberg – Whitham (FW) (4), and the Fuchssteiner – Fokas – Camassa – Holm equation (5) respectively.

For *b=1*, *k=3*, *a=1* inequation (1)we have Fornberg – Whitham (FW) equation in the next form [3]

$$u_t - u_{xxt} + u_x + uu_x = 3u_xu_{xx} + uu_{xxx}, \quad (6)$$

The purpose of this research is to study equation (6) bythe tanh-coth method [1].

The tanh-coth method

For using this method, first of all we need convert out PDE (1) to the ODE in the form

$$(a-c)u + cu'' + \frac{b}{2}u^2 - \frac{k-1}{2}\left(u'\right)^2 - uu'' = 0 \quad (7)$$

in the case of Fornberg-Whitham we got equation in the form

$$(1-c)u + cu'' + \frac{1}{2}u^2 - (u')^2 - uu'' = 0. \tag{8}$$

obtained upon using the transformation $z = x - ct$ and adding a new independent variable $Y = \tanh(\mu z)$, where μ called wave number. Next, we need to change our derivatives according to the tanh-coth method [1] as a

$$\frac{d}{dz} = \mu(1-Y^2)\frac{d}{dY}, \tag{9}$$

$$\frac{d^2}{dz^2} = -2\mu^2 Y(1-Y^2)\frac{d}{dY} + \mu^2(1-Y^2)^2\frac{d^2}{dY^2}. \tag{10}$$

The tanh-coth method admits the use of the finite expansion

$$u(\mu z) = S(Y) = \sum_{k=0}^{M} a_k Y^k + \sum_{k=1}^{M} b_k Y^{k-1}, \tag{11}$$

where M is usually positive integer, but in the case of nonlinear CH equation the balance between uu'' and $(u')^2$ will work if $M = -1$. So, by putting our value of M in equation (6) we have

$$u(x,t) = (a_0 + a_1 Y + b_1 Y^{-1})^{-1}, \tag{12}$$

$$\frac{d}{dz} = \frac{\mu(1-Y^2)(a_1 - \frac{b_1}{Y^2})}{(a_0 + a_1 Y + b_1 Y^{-1})} \tag{13}$$

$$\frac{d^2}{dz^2} = \frac{2\mu^2 Y(1-Y^2)(a_1 - \frac{b_1}{Y^2})}{(a_0+a_1Y+b_1Y^{-1})^2} + \mu^2(1-Y^2)^2\left(\frac{2(a_1-\frac{b_1}{Y^2})^2}{(a_0+a_1Y+b_1Y^{-1})^2} - \frac{2b_1}{(a_0+a_1Y+b_1Y^{-1})^2}Y^3\right) \tag{14}$$

By substituting equations (12)-(14) into (8), we can derive value of constants which correspond to

Case I:

$$a_0 = \frac{bk}{2[(k+1)(c-a)-bc]}, a_1 = \pm\frac{bk}{2[(k+1)(c-a)-bc} \tag{15}$$

$$b_1 = 0, \mu = \frac{1}{2}\sqrt{\frac{b}{k+1}}, k \neq -1.$$

Case II:

$$a_0 = \frac{bk}{2[(k+1)(c-a)-bc]}, \quad a_1 = 0,$$

$$b_1 = \pm\frac{bk}{2[(k+1)(c-a)-bc]}, \mu = \frac{1}{2}\sqrt{\frac{b}{k+1}}, \quad k \neq -1, \tag{16}$$

For $b=1$, $k=3$, $a=1$ in expressions (15)-(16) we obtain parameters for the FW equation in the next forms

1) $a_1 = \frac{3}{4}, a_0 = -\frac{3}{4}, b_1 = 0, \mu = \frac{1}{4}, c = \frac{2}{3}$

2) $a_1 = -\frac{3}{4}, a_0 = -\frac{3}{4}, b_1 = 0, \mu = \frac{1}{4}, c = \frac{2}{3}$

3) $a_1 = 0, a_0 = -\frac{3}{4}, b_1 = \frac{3}{4}, \mu = \frac{1}{4}, c = \frac{2}{3}$

4) $a_1 = 0, a_0 = -\frac{3}{4}, b_1 = -\frac{3}{4}, \mu = \frac{1}{4}, c = \frac{2}{3}$

5) $a_1 = \frac{3}{4}, a_0 = -\frac{3}{4}, b_1 = 0, \mu = -\frac{1}{4}, c = \frac{2}{3}$

6) $a_1 = -\frac{3}{4}, a_0 = -\frac{3}{4}, b_1 = 0, \mu = -\frac{1}{4}, c = \frac{2}{3}$

7) $a_1 = 0, a_0 = -\frac{3}{4}, b_1 = \frac{3}{4}, \mu = -\frac{1}{4}, c = \frac{2}{3}$

8) $a_1 = 0, a_0 = -\frac{3}{4}, b_1 = -\frac{3}{4}, \mu = -\frac{1}{4}, c = \frac{2}{3}$

9) $a_1 = \frac{3}{8}, a_0 = -\frac{3}{4}, b_1 = \frac{3}{8}, \mu = \frac{1}{8}, c = \frac{2}{3}$

10) $a_1 = -\frac{3}{8}, a_0 = -\frac{3}{4}, b_1 = -\frac{3}{8}, \mu = \frac{1}{8}, c = \frac{2}{3}$

11) $a_1 = \frac{3}{8}, a_0 = -\frac{3}{4}, b_1 = \frac{3}{8}, \mu = -\frac{1}{8}, c = \frac{2}{3}$

12) $a_1 = -\frac{3}{8}, a_0 = -\frac{3}{4}, b_1 = -\frac{3}{8}, \mu = -\frac{1}{8}, c = \frac{2}{3}$

where c is left as a free parameter for these two cases.
We substitute above results in to equation (12) and obtain next solutions

1) $u(x,t) = \left(-\frac{3}{4} + \frac{3}{4} \cdot \left(\tanh\left(\frac{1}{4}(x-ct)\right)\right)\right)^{-1}$;

2) $u(x,t) = \left(-\frac{3}{4} - \frac{3}{4} \cdot \left(\tanh\left(\frac{1}{4}(x-ct)\right)\right)\right)^{-1}$;

3) $u(x,t) = \left(-\frac{3}{4} + \frac{3}{4} \cdot \left(\tanh\left(\frac{1}{4}(x-ct)\right)\right)^{-1}\right)^{-1}$;

4) $u(x,t) = \left(-\frac{3}{4} - \frac{3}{4} \cdot \left(\tanh\left(-\frac{1}{4}(x-ct)\right)\right)^{-1}\right)^{-1}$;

5) $u(x,t) = \left(-\frac{3}{4} + \frac{3}{4}\cdot\left(\tanh\left(-\frac{1}{4}(x-ct)\right)\right)\right)^{-1}$;

6) $u(x,t) = \left(-\frac{3}{4} - \frac{3}{4}\cdot\left(\tanh\left(-\frac{1}{4}(x-ct)\right)\right)\right)^{-1}$;

7) $u(x,t) = \left(-\frac{3}{4} + \frac{3}{4}\cdot\left(\tanh\left(-\frac{1}{4}(x-ct)\right)\right)^{-1}\right)^{-1}$;

8) $u(x,t) = \left(-\frac{3}{4} - \frac{3}{4}\cdot\left(\tanh\left(-\frac{1}{4}(x-ct)\right)\right)^{-1}\right)^{-1}$;

9) $u(x,t) = \left(-\frac{3}{4} + \frac{3}{8}\cdot\left(\tanh\left(\frac{1}{8}(x-ct)\right)\right) + \frac{3}{8}\cdot\left(\tanh\left(\frac{1}{8}(x-ct)\right)\right)^{-1}\right)^{-1}$;

10) $u(x,t) = \left(-\frac{3}{4} - \frac{3}{8}\cdot\left(\tanh\left(\frac{1}{8}(x-ct)\right)\right) - \frac{3}{8}\cdot\left(\tanh\left(\frac{1}{8}(x-ct)\right)\right)^{-1}\right)^{-1}$;

11) $u(x,t) = \left(-\frac{3}{4} + \frac{3}{8}\cdot\left(\tanh\left(-\frac{1}{8}(x-ct)\right)\right) + \frac{3}{8}\cdot\left(\tanh\left(-\frac{1}{8}(x-ct)\right)\right)^{-1}\right)^{-1}$;

12) $u(x,t) = \left(-\frac{3}{4} - \frac{3}{8}\cdot\left(\tanh\left(-\frac{1}{8}(x-ct)\right)\right) - \frac{3}{8}\cdot\left(\tanh\left(-\frac{1}{8}(x-ct)\right)\right)^{-1}\right)^{-1}$.

In this paper, we studied the equations of the family of Camassa – Holm, particularly the Fornberg – Whitham equation. Using the tanh-coth method, we have constructed various exact wave solutions for this equation.

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Latest observation indicated that expansion of universe is accelerated. For explanation this late time accelerating, from scientists has been proposed two remarkable approaches. One is to assume contents of matter as scalar field in the right-hand side of the Einstein equation in the framework of general relativity, i.e. as scalar field may be consider phantom, quintessence, fermion, tachyon and etc. Another is to make modify the left-hand side of the Einstein equation [1-5], i.e. gravitational part.