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FRW COSMOLOGY OF NONCANONICAL FERMIONIC FIELD IN F(R) GRAVITY

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In cosmology fermion fields have been studied as possible sources of early and late time expansion without the need of a cosmological constant term or a scalar field. In most of the papers are considered fermions fields are minimally coupled to gravity. But, in recently appears works effects of fermionic fields non - minimally coupling to gravity [1-6]. The fermion fields has been investigated via several approaches, with results including exact solutions, numerical solutions, cyclic cosmologies and anisotropy-to-isotropy scenario, perturbations, dark spinors. The relation between general relativity and the equation for fermion fields is done via the tetrad formalism. The components of the tetrad play the role of the gravitational degrees of freedom

In this section we would like to present the derivation of the equations of motion for FRW metric in the f-essence.

Let us consider the following action of f-essence

$$S = \int d^4x\sqrt{-g}[F(R) + 2K(Y,\psi,\overline{\psi})], \tag{1}$$

$$\lambda = F_R \tag{2}$$

$$S = \int d^4x\sqrt{-g}\left[F(R) - \lambda\left(R - 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)\right) + 2K(Y,\psi,\overline{\psi})\right] \tag{3}$$

where R is the scalar curvature, Y is the kinetic term for the fermionic field ψ and K is some function of its arguments. In the case of the FRW metric

$$ds^2 = -dt^2 + a^2(dx^2 + dy^2 + dz^2), \tag{4}$$

R and Y have the form

$$R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right), \tag{5}$$

$$Y = 0.5i(\overline{\psi}\gamma^0\dot{\psi} - \dot{\overline{\psi}}\gamma^0\psi), \tag{6}$$

respectively. Substituting these expressions into (1) and integrating over the spatial dimensions, we are led to an effective Lagrangian in the mini-superspace $\{a,\psi,\bar{\psi}\}$

$$L = a^3F - a^3F_RR - 6\dot{a}^2aF_R - 6\dot{a}a^2F_{RR}\dot{R} + 2a^3K. \tag{7}$$

Variation of Lagrangian (7) with respect to a, R, yields the equation of motion of the scale factor

$$2a\ddot{a}F_R + \dot{a}^2F_R + \frac{a^2}{2}F - \frac{a^2}{2}F_RR + 2\dot{a}aF_{RR}\dot{R} + +a^2F_{RRR}\dot{R}^2 + a^2F_{RR}\ddot{R} + a^2K = 0. \tag{8}$$

$$a^3F_{RR}R - 6\ddot{a}a^2F_{RR} - 6\dot{a}^2aF_{RR} = 0. \tag{9}$$

The variation of Lagrangian (5) with respect to $\bar{\psi},\psi$ is the corresponding Euler-Lagrangian equations for the fermionic fields

$$K_Y\gamma^0\dot{\psi} + 1.5\frac{\dot{a}}{a}K_Y\gamma^0\psi + 0.5\dot{K}_Y\gamma^0\psi - iK_{\bar{\psi}} = 0, \tag{10}$$

$$K_Y\dot{\bar{\psi}}\gamma^0 + 1.5\frac{\dot{a}}{a}K_Y\bar{\psi}\gamma^0 + 0.5\dot{K}_Y\bar{\psi}\gamma^0 + iK_{\psi} = 0. \tag{11}$$

Another equivalence form is

$$3HK_{\dot{\psi}} + K_{\dot{\psi}\psi}\dot{\psi} + K_{\dot{\psi}\bar{\psi}}\dot{\bar{\psi}} + K_{\dot{\psi}\dot{\psi}}\ddot{\psi} + K_{\dot{\psi}\dot{\bar{\psi}}}\ddot{\bar{\psi}} + K_{\psi} = 0, \tag{12}$$

$$3HK_{\dot{\bar{\psi}}} + K_{\dot{\bar{\psi}}\psi}\dot{\psi} + K_{\dot{\bar{\psi}}\bar{\psi}}\dot{\bar{\psi}} + K_{\dot{\bar{\psi}}\dot{\psi}}\ddot{\psi} + K_{\dot{\bar{\psi}}\dot{\bar{\psi}}}\ddot{\bar{\psi}} + K_{\bar{\psi}} = 0. \tag{13}$$

Also the zero-energy condition is given by

$$L_{\dot{a}}\dot{a} + L_{\dot{R}}\dot{R} + L_{\dot{\psi}}\dot{\psi} + L_{\dot{\bar{\psi}}}\dot{\bar{\psi}} - L = 0, \tag{14}$$

which yields the constraint

$$-3a^{-2}\dot{a}^2F_(R) - 3\frac{\dot{a}}{a}F_{RR}\dot{R} - \frac{F}{2} + \frac{1}{2}F_RR + YK_Y - K = 0. \tag{15}$$

Collecting all equations and rewriting using the Hubble parameter $H = (\ln a)_t$, we obtain a system of equations of f-essence (for the FRW metric case):

$$3H^2 - \rho = 0, \tag{14}$$

$$2\dot{H} + 3H^2 + p = 0, \tag{15}$$

$$a^3F_{RR}R - 6\ddot{a}a^2F_{RR} - 6\dot{a}^2aF_{RR} = 0, \tag{16}$$

$$3HK_{\dot{\psi}} + K_{\dot{\psi}\psi}\dot{\psi} + K_{\dot{\psi}\bar{\psi}}\dot{\bar{\psi}} + K_{\dot{\psi}\dot{\psi}}\ddot{\psi} + K_{\dot{\psi}\dot{\bar{\psi}}}\ddot{\bar{\psi}} + K_{\psi} = 0, \tag{17}$$

$$3HK_{\dot{\bar\psi}}+K_{\dot{\bar\psi}\psi}\dot\psi+K_{\dot{\bar\psi}\bar\psi}\dot{\bar\psi}+K_{\dot{\bar\psi}\dot\psi}\ddot\psi+K_{\dot{\bar\psi}\dot{\bar\psi}}\ddot{\bar\psi}+K_{\bar\psi}=0, \quad (18)$$

$$\dot\rho+3H(\rho+p)=0. \quad (19)$$

Here

$$\rho=\frac{-3\dot HF_R+3HF_{RR}\dot R+F/2-YK_Y+K}{F_R}, \quad (20)$$

$$p=\frac{F/2-F_RR/2+2HF_{RR}\dot R+F_{RRR}\dot R^2+F_{RR}\ddot R+K}{F_R}. \quad (21)$$

are the energy density and the pressure of f-essence. It is clear that these expressions for the energy density and the pressure represent the components of the energy-momentum tensor of f-essence as:

$$T_{00}=\frac{-3\dot HF_R+3HF_{RR}\dot R+F/2-YK_Y+K}{F_R}, \quad (22)$$

$$T_{11}=T_{22}=T_{33}=-\frac{F/2-F_RR/2+2HF_{RR}\dot R+F_{RRR}\dot R^2+F_{RR}\ddot R}{F_R}. \quad (23)$$

We introduce a useful model which is more applicable and more suitable for exact solutions. This model is described by

$$K(Y,\psi,\bar\psi)=Y-V(\bar\psi\psi). \quad (24)$$

To obtain the field equations, we substitute this form in Eqs. (13- 17). Another simple method is re-deriving these equations using the action directly, therefore in each of these equivalence methods we have the following equations of motion for Dirac fields:

$$3H(\frac{1}{2}i\bar\psi\gamma^0)+\frac{1}{2}i\gamma^0\dot{\bar\psi}-\frac{1}{2}i\dot{\bar\psi}\gamma^0-V_\psi=0, \quad (25)$$

$$3H(-\frac{1}{2}i\gamma^0\psi)=V_{\bar\psi}. \quad (26)$$

FRW equations in this case are

$$\left(2\frac{\ddot a}{a}+(\frac{\dot a}{a})^2\right)F_R+\frac{F}{2}-\frac{R}{2}F_R+2HF_{RR}\dot R+F_{RRR}\dot R^2+F_{RR}\ddot R+\frac{1}{2}i(\bar\psi\gamma^0\dot\psi-\dot{\bar\psi}\gamma^0\psi)-V(\bar\psi\psi)=0, \quad (26)$$

$$3H^2=\frac{-3\dot HF_R+3HF_{RR}\dot R+F/2-V(\bar\psi\psi)}{F_R}. \quad (27)$$

The general potential is $V(\bar\psi\psi)=2\bar\psi\psi$. For this special case, we have the next set of EOMs:

$$3H(\frac{1}{2}i\bar\psi\gamma^0)+\frac{1}{2}i\gamma^0\dot{\bar\psi}-\frac{1}{2}i\dot{\bar\psi}\gamma^0-2\bar\psi=0, \quad (28)$$

$$-\frac{3}{2}iH\gamma^0\psi = 2\psi, \quad (29)$$

$$2\dot{H} + \frac{1}{2}i(\overline{\psi}\gamma^0\dot{\psi} - \dot{\overline{\psi}}\gamma^0\psi) = 0. \quad (30)$$

Now we take the Dirac 2-spinor as $\overline{\psi} = (\psi_1, \psi_2)^\dagger \gamma^0$, the equation for spinor reads as

$$\dot{\psi} + \frac{3}{2}H\psi + 2i\gamma^0\psi = 0. \quad (31)$$

Thus we must solve the next system of ODEs:

$$\frac{d\log\psi_a}{dt} = \frac{3}{2}H\psi_a \pm 2, a = \{1,2\} = \{+,-\}, \quad (32)$$

which posses the following solution

$$\psi^T = (\psi_1(0)a(t)^{3/2}e^{2t}, \psi_2(0)a(t)^{3/2}e^{-2t}). \quad (33)$$

Using this form of the 2-spinor we can obtain the scale factor from the following equation

$$\ddot{y} + ie^{3y}\{|\beta|^2 e^{-4t} - |\alpha|^2 e^{4t}\}, \quad (34)$$

here $y \equiv \log(a(t)), \{\alpha, \beta\} \equiv \{\psi_1(0), \psi_2(0)\}$.

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NONLINEAR WAVES FOR KUDRYASHOV-SINELSHCHIKOV EQUATION

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In this paper, we consider the (3+1)-dimensional Kudryashov-Sinelshchikov (KS) equation in the following form [1]