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The proceedings are the papers of students, undergraduates, doctoral students and young researchers on topical issues of natural and technical sciences and humanities.

В сборник вошли доклады студентов, магистрантов, докторантов и молодых ученых по актуальным вопросам естественно-технических и гуманитарных наук.

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When $a_{ij} = 1, i \geq j \geq 1$ the inequality (1) was investigated in [1], [2] for various combinations of the parameters p, s and q.

Our main result reads as follows:

Theorem 1. Let $1 < p, s \leq q < +\infty$ and the elements of matrix (a_{ij}) satisfy condition (2). Then the inequality (1) holds if and only if $A = max\{A_1, A_2\} < \infty$, where

$$A_1 = \sup_{m\geq 1}\left(\sum_{i=1}^{m} u_i^q\right)^{\frac{1}{q}}\left(\sum_{j=m}^{\infty} a_{jm}^{p'} v_j^{-p'}\right)^{\frac{1}{p'}}\left(\sum_{i=m}^{\infty} w_i^{-s'}\right)^{\frac{1}{s'}}, \quad (3)$$

$$A_2 = \sup_{m\geq 1}\left(\sum_{i=1}^{m} a_{mi}^q u_i^q\right)^{\frac{1}{q}}\left(\sum_{j=m}^{\infty} v_j^{-p'}\right)^{\frac{1}{p'}}\left(\sum_{i=m}^{\infty} w_i^{-s'}\right)^{\frac{1}{s'}} \quad (4)$$

Moreover, where $A \approx C$(A is approximately equal to C) is best constant in (1).

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SEPARABILITY OF THE UNBOUNDED THIRD-ORDER DIFFERENCE OPERATOR

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We consider the following third-order difference equation

$$ly = -\Delta^{(3)}y_i + a_i\Delta y_i = a_i^{\alpha} f_i, \quad f_i \in l_p, \ \ i \in Z = \{0, \pm 1, \pm 2, \pm 3, \dots\}, \quad (1)$$

where

$$y = \{y_i\}_{i=-\infty}^{+\infty},$$

$$\Delta_- y = \{\Delta_- y\}_{i=-\infty}^{+\infty} = \{y_i - y_{i-1}\}_{i=-\infty}^{+\infty},$$

$$\Delta_+ y = \{\Delta_+ y\}_{i=-\infty}^{+\infty} = \{y_{i+1} - y_i\}_{i=-\infty}^{+\infty},$$

$$\Delta^{(3)}y_i = \Delta\left(\Delta^{(2)}y_i\right) = \Delta\{y_{i+1} - 2y_i + y_{i-1}\}_{i=-\infty}^{+\infty} = \{y_{i+1} - 3y_i + 3y_{i-1} - y_{i-2}\}_{i=-\infty}^{+\infty},$$

and $0 \prec \alpha \prec 1, \quad a_i \geq \varepsilon \succ 0$.

In this work we will set that equation has unique solution and for it the estimate

$$\left\| c_i^{\frac{1}{p}} \Delta y_i \right\|_{l_p} \le \left\| \frac{1}{c_i^{\frac{1}{p'}}} l y_i \right\|_{l_p}, \qquad \alpha + \frac{1}{p'} = 1, \qquad (1 < p < \infty) \tag{2}$$

holds.

The theory of differential operators finds various applications in all fields of modern science, in particular, in biology, economics, physics, and chemistry. Attention to difference operators and the equations derived from them originates from the use of difference operator devices in the study of the solvability of differential, integral and functional equations. The study of different types of difference equations is implemented in the works of many authors, including A.G. Baskakov, R. Bellman and K.L. Cook, I.C. Gohberg and I.A. Feldman, P.P. Zabreiko and Nguyen Van Minh, S.G. Crane, W.G. Kurbatov, B.M. Levitan and V.V. Zhikov, H.L. Masser and H.H. Schaeffer, W.M. Turin, D. Henry, M. Otelbaev, K.N. Ospanov.

Estimate of type (2) for the first order difference equation was obtained in [1] and for second order equation in the work [2, 3].

Our main result reads:

Theorem. Let $c_i \ge \varepsilon > 0$. Then equation (1) at $1 \ge \alpha \ge 0$ and for any right-hand side of $f = \{f_i\} \in l_{p'} (\alpha + \frac{1}{p'} = 1)$ has a solution satisfying inequality (2).

To proof the theorem at first we denote $\Delta y_i = z_i$, so we have

$$lz = -\Delta^{(2)} z_i + c_i z_i = c_i^{\alpha} f_i . \tag{3}$$

Then multiply the both side of equation (3) by $z_i \left(z_i^2\right)^{\frac{\gamma}{2}}$ $(\gamma > -1)$ and sum over all by i we get:

$$\sum_{i=-\infty}^{+\infty} \left[-\Delta^{(2)} z_i \left(z_\alpha^2\right)^{\frac{\gamma}{2}} + c_i z_i \left(z_\alpha^2\right)^{\frac{\gamma}{2}} \right] = \sum_{i=-\infty}^{+\infty} c_i^{\alpha} f_i z_i \left(z_i^2\right)^{\frac{\gamma}{2}} \tag{4}$$

It is easy to check the following ratios:

$$\sum_{i=-\infty}^{+\infty} \left(\Delta^{(2)} z_i\right) z_i \left(z_i^2\right)^{\frac{\gamma}{2}} = \sum_{i=-\infty}^{+\infty} \left[(z_{i+1} - z_i) - (z_i - z_{i-1}) \right] z_i (z_i^2)^{\frac{\gamma}{2}} =$$

$$= \sum_{i=-\infty}^{+\infty} (z_{i+1} - z_i) z_i \left(z_i^2\right)^{\frac{\gamma}{2}} - \sum_{i=-\infty}^{+\infty} (z_{i+1} - z_i) z_{i+1} \left(z_i^2\right)^{\frac{\gamma}{2}} =$$

$$= \sum_{i=-\infty}^{+\infty} \left(z_{i+1} - z_i\right) \left[z_i \left(z_i^2\right)^{\frac{\gamma}{2}} - z_{i+1} \left(z_i^2\right)^{\frac{\gamma}{2}} \right].$$

Obviously, each summand of the sum on the right side of the last equality is not positive. Therefore, an estimate follows from the ratio (4):

$$\sum_{i=-\infty}^{+\infty} c_i |z_i|^{2+\gamma} \le \sum_{i=-\infty}^{+\infty} c_i^{\alpha} |z_i|^{1+\gamma} |f_i| \le$$

$$\leq \left(\sum_{i=-\infty}^{+\infty} c_i^{\alpha p} |z_i|^{(1+\gamma)p} \right)^{\frac{1}{p}} \left(\sum_{i=-\infty}^{+\infty} |f_i|^{p'} \right)^{\frac{1}{p'}}, \qquad \left(\frac{1}{p} + \frac{1}{p'} = 1,\ 1 \prec p \prec \infty \right).$$

Take p, α, γ satisfying conditions: $p\alpha = 1, \quad (1+\gamma)p = 2+\gamma, \quad p = \frac{1}{\alpha}, \quad \gamma = \frac{2-p}{p-1}$. Then the inequalities are eliminated: $1 < p < \infty, \gamma > -1$,

$$\sum_{i=-\infty}^{+\infty} c_i |z_i|^{2+\frac{2-p}{p-1}} \leq \left(\sum_{i=-\infty}^{+\infty} c_i |z_i|^{2+\frac{2-p}{p-1}} \right)^{\frac{1}{p}} \left(\sum_{i=-\infty}^{+\infty} |f_i|^{p'} \right)^{\frac{1}{p'}}.$$

Therefore, there is an assessment

$$\left(\sum_{i=-\infty}^{\infty} c_i |z_i|^{p'} \right)^{\frac{1}{p'}} \leq \left(\sum_{i=-\infty}^{+\infty} |f_i|^{p'} \right)^{\frac{1}{p'}} \qquad (\alpha + \frac{1}{p'} = 1).$$

Let $\{f_i\}$ be a finite sequence. Then equation (1), by virtue of condition $c_i \geq \varepsilon > 0$ in l_2 has a solution $\{y_i\}$. Using inequality (2), it is not difficult to make sure that this solution belongs to $l_{p'}$ at $\frac{1}{p'} + \alpha = 1$. Passing α to 0 or 1, we get that the above is true at $\alpha \in [0,1]$, and the inequality (2) holds.

If now $\{f_j\}$ belongs to $l_{p'}(1 \leq p' \leq \infty)$, but is not a finite sequence, then, approximating $\{f_j\}$ in $l_{p'}$ finite sequences, we get that equation (1) has a solution satisfying inequality (3).

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ONE CLASS OF PERTURBATION PROBLEMS FOR THE LAPLACE OPERATOR

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Perturbation theory is the most used techniques in quantum mechanics, atomic physics and especially in the study of dynamical systems. The classical method is for one perturbated problem obtaining complete system of eigenfunctions. (If system is complete, it is a basis of the space.) Main purpose of this work to defining does singular perturbated Laplace operators has a