

# A method for extracting oil from a field with hard-to-recover reserves based on systems analysis methodology

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**Abstract.** The article examines the problems of solving the problem of optimizing oil production processes from a field with hard-to-recover oil reserves, characterized by shortages and fuzziness of the initial information. Equations for the continuity of the mass of matter in an oil reservoir have been derived for various cases. As a result of the study, by combining and modifying various optimality principles for fuzzy conditions, a formulation of the problem of multi-criteria optimization of oil production processes was obtained and an effective method for the resulting problem in a fuzzy environment was proposed. The effectiveness and novelty of the proposed heuristic algorithm lies in the fact that for the maximum use of the collected fuzzy information, it ensures high adequacy of solving fuzzy problems that arise in oil production practice. The sections of the studied multi-layer oil field Kenbai and the main properties of its layers are analyzed and described.

## 1 Introduction

Currently, more than 60% of Kazakhstan's oil reserves are confined to reservoirs with hard-to-recover oil reserves (HROR), and the share of fields with HROR tends to increase. Oil extraction from such oil fields using traditional methods does not provide high efficiency [1, 2]. In this regard, the development of new effective methods for increasing oil recovery, methods for optimizing technological operating modes of oil wells based on mathematical methods based on computer technology is currently an urgent task in petroleum science and the practice of oil and gas producing enterprises [3].

Currently, to increase the level of oil recovery from a reservoir, various technological, mathematical, physical and other methods are used; choosing the most effective of them

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requires a lot of research and considerable resources. One of the promising and effective methods for increasing oil recovery is the creation of automated systems based on mathematical models and algorithms for optimizing the technological regime of well operation [4, 5].

For the effective use of large oil reserves in the economy of Kazakhstan, as in other countries, it is necessary to have scientifically based methods of field development and oil production, based on modern achievements of mathematical methods, automation tools and information technology [6, 7]. These methods and tools, compared to physical and technological methods, are much more effective and efficient.

The development of an oil field and oil production is a set of measures aimed at ensuring the flow of oil from the deposit to the bottom of the wells, providing for this purpose a certain order of placement of wells in the area, the sequence of their drilling and commissioning, the establishment and maintenance of a certain mode of their operation. This requires scientifically based application of methods to improve the efficiency of oil production. In this case, quantitative formulation is very necessary, that is, the creation of mathematical models. Building a model of oil production processes is necessary to simulate and determine the optimal modes of the oil production process.

## 2 Materials and Methods

When conducting the study, the geological and physical properties of the oil field (Table 1) and the section diagram of the multi-layer Kenbai oil field (Figure 1), which is located in the Atyrau region of Kazakhstan, are used as materials. This deposit belongs to deposits with HROR [8]. The main research methods are mathematical methods: methods for modeling and optimizing oil production processes [9, 10]. Methods for mathematical modeling of oil production processes are based on the fundamental laws of nature: the law of conservation of matter and energy, as well as a number of physical, physicochemical laws and filtration laws. The law of conservation of matter in oil production models is written in the form of a differential equation of continuity of mass of matter or in the form of formulas expressing the material balance of substances. They are used to calculate data from oil production processes, and the corresponding calculation method is called the material balance method.

Let the mass  $\Delta M$  a substance with density  $\rho$  in a layer element with length  $\Delta x$ , thickness  $h$  and width  $b$ , measured in the direction perpendicular to the plane, with layer porosity  $m$  is determined by the formula [11]:  $\Delta M = \rho m h \Delta x$ . Suppose a substance enters a layer element

through its left edge at mass velocity  $\rho v_x + \frac{\partial \rho v_x}{\partial x} \Delta x$ , and its accumulated volume  $\delta \Delta M$

during the time  $\Delta t$ , then, taking into account the fact that more substance entered the element than came out of it, we can write:

$$\rho v_x b h \Delta x \Delta t - (\rho v_x + \frac{\partial \rho v_x}{\partial x}) b h \Delta x \Delta t = \delta \Delta M = \Delta(\rho m) b h \Delta x. \quad (1)$$

From equation (1) it follows  $\frac{\partial(\rho v_x)}{\partial x} + \frac{\Delta(\rho m)}{\Delta t} = 0$ . if  $\Delta t \rightarrow 0$ , then  $\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho m)}{\partial t} = 0$ .

The resulting equation is the equation of continuity of the mass of substance in an oil reservoir with one-dimensional rectilinear movement of the substance saturating it. To obtain such an equation in the three-dimensional case, it is necessary to consider the mass balance in the volumetric element of the layer  $\Delta V = \Delta x \Delta y \Delta z$ . Considering the mass rates of entry of matter into the cube and displacement from it, as well as its accumulated volume in the cube, we can obtain:

$$\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z} + \frac{\partial(m\rho)}{\partial t} = 0. \tag{2}$$

The last equation (2) in general can be written in the following form:

$$\operatorname{div}(\rho v) + \frac{\partial(m\rho)}{\partial t} = 0. \tag{3}$$

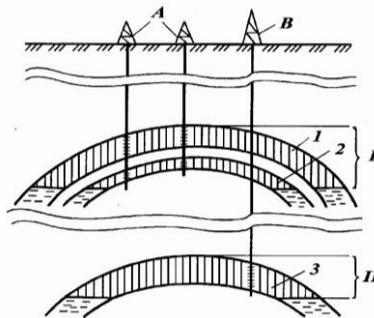
Then equations (2), (3) form a system of equations for the continuity of the mass of matter during its movement in three-dimensional measurement.

In addition to the complexity of the mathematical description of oil production processes, in practice there are problems of uncertainty arising from the shortage and fuzziness of the initial information necessary for developing models and optimizing these processes (12,13). Such problems are associated with the fact that complex processes, such as oil production, are difficult to describe quantitatively. In this case, modification and application of traditional deterministic, statistical approaches are impractical, since they do not provide significant results.

One of the promising approaches to overcome these problems, which significantly increases the efficiency of methods of mathematical modeling and optimization of complex, quantitatively difficult to describe oil production processes, is the reasonable use and formalization of a priori fuzzy information about the features of these processes. Effective formalization of such fuzzy information, which represents knowledge, judgments of the decision maker (DM), experts about the process under study, can be carried out on the basis of methods of expert assessments and the theory of fuzzy sets, the mathematical apparatus of which is described in the works [14–20].

### 3 Results

The main features of an oil production facility are the presence of industrial oil reserves in it and a certain group of wells inherent to this facility, with the help of which it is developed. For convenience, let us consider a cross-section of the studied Kenbai field shown in Figure 1.



**Figure 1.** Section of the multi-layered oil field Kenbai.

The field under study contains three layers that differ in thickness, areas of distribution of hydrocarbons saturating them and physical properties (Table 1).

**Table 1.** Basic properties of layers occurring within the field.

Geological and physical properties	Layer (see Figure 1)		
	1	2	3
Recoverable oil reserves, million tons	200.0	50.0	70.0
Thickness, m	10.0	5.0	15.0
Permeability, $10^{-3} \mu\text{m}^2$	100.0	150.0	500.0
Oil viscosity, $10^{-3} \text{Pa}\cdot\text{s}$	50	60	3

In this case, the bottom of the layer is located at a distance of 15 m from the roof of layer 2, and the bottom of layer 2 is vertically spaced from the roof of layer 3 by 1000 m. The table shows the main properties of layers 1, 2 and 3, located within the Kenbai field. In the field under consideration, it is advisable to identify two oil development and production objects, combining layers 1 and 2 into one development object (object I), and developing layer 3 as a separate object (object II). The inclusion of layers 1 and 2 in one object is due to the fact that they have similar values of permeability and viscosity of oil and are located at a short distance from each other vertically. In addition, the recoverable oil reserves in reservoir 2 are relatively small. Although reservoir 3 has smaller recoverable oil reserves compared to reservoir 1, it contains low-viscosity oil and is highly permeable. Consequently, wells that penetrate this layer will be highly productive. At the same time, it should be taken into account that, despite the significant difference in the parameters of layers 1, 2 and 3, the final decision on the allocation of development objects is made based on an analysis of the technological and technical-economic indicators of various options for combining layers into development objects.

An oil production system requires a set of interconnected engineering solutions that define objects; the sequence and pace of their drilling; methods of influencing layers in order to extract oil from them; optimization of oil production processes, etc. To effectively solve this problem, systems analysis methods should be applied (21). In this work, problems of multi-criteria optimization of the oil production process are studied and solved in order to increase oil recovery based on a systems approach.

Let us formalize and present the formulation of the problem of optimizing oil production processes, which are characterized by multi-criteria and fuzziness, and propose a method for solving it.

Let  $f(\mathbf{x}) = f_1(\mathbf{x}), \dots, f_m(\mathbf{x})$  is a vector of criteria assessing economic efficiency, environmental safety and other indicators of oil production processes. Each of the  $m$  criteria depends on the vector of  $n$  parameters  $\mathbf{x} = (x_1, \dots, x_n)$ , for example: the method of influencing the bottomhole zone of wells (BZW), operating parameters of oil production, properties and characteristics of the oil deposit, etc. This dependence is described by models that describe oil production processes. In practice, there are always various restrictions (economic, technological, environmental), which can be described by certain functions of restrictions that may be fuzzy:  $\varphi(\mathbf{x}) \gtrsim b_q, q = \overline{1, L}$ . Input parameters also have their own change intervals:

$$x \in \Omega = [x_j^{\min}, x_j^{\max}], x_j^{\min}, x_j^{\max} - \text{lower and upper limits for changing parameter } x_j.$$

It is required to select an effective oil production option that provides an extreme value of the criterion vector while fulfilling the given constraints and takes into account the preferences of the DM. A formalized multi-criteria optimization problem with fuzzy constraints can be written in the following form:

$$\max_{\mathbf{x} \in X} f_i(\mathbf{x}), i = \overline{1, m}, \tag{4}$$

$$X = \{ \mathbf{x} \in X, \varphi_q(\mathbf{x}) \gtrsim b_q, q = \overline{1, L} \} \tag{5}$$

The solution to this multi-criteria optimization problem is the value of the parameter vector  $\mathbf{x}^* = (x_1^*, \dots, x_n^*)$ , providing such values of local criteria that satisfy the decision maker.

Known methods for solving such problems mainly consider single-criteria cases, there is no flexibility in taking into account the preferences of the decision maker, and the fuzzy problem at the stage of formulation based on the  $\alpha$ -level set is replaced by a set of clear problems, which leads to the loss of a significant part of the collected fuzzy information and reduces the adequacy of the solution. In practice, often fuzzy information (experience, knowledge, judgments of DMs, experts in natural language) are basic and familiar to humans. Converting a fuzzy description into a quantitative one is not always possible or is impractical. In this regard, this work uses an approach based on the development of a method for optimizing workable ones in a fuzzy environment, without converting the original fuzzy problem to a deterministic one, i.e., making maximum use of the available fuzzy information. For the convenience of setting and solving a fuzzy problem in a fuzzy environment, the formulation of the multi-criteria optimization problem (5)–(6) taking into account the fuzziness of the initial information can be written in the form:

Let  $\mu_0(\mathbf{x}) = (\mu_0^1(\mathbf{x}), \dots, \mu_0^m(\mathbf{x}))$  is a normalized vector of criteria  $f(\mathbf{x})$ , evaluating criteria for the efficiency of oil production. Let us assume that for each fuzzy constraint  $\varphi(\mathbf{x}) \gtrsim b_q, q = \overline{1, L}$  the membership function of its execution has been constructed  $\mu_q(\mathbf{x}), q = \overline{1, L}$ . There is either a known number of priorities for local criteria  $I_c = \{1, \dots, m\}$  and a weight vector reflecting the mutual importance of the constraints  $\boldsymbol{\beta} = (\beta_1, \dots, \beta_L)$ .

Then, by modifying the principles of the *main criterion* (MC) and *equality* (PR), the multicriteria optimization problem with fuzzy constraints can be written in the following form:

$$\max_{\mathbf{x} \in X} \mu_0^1(\mathbf{x}), \tag{6}$$

$$X = \left\{ \mathbf{x} \in \Omega, \wedge \arg(\max_{\mathbf{x} \in \Omega} \mu_0^i(\mathbf{x}) \geq \mu_R^i) \wedge \arg(\beta_1 \mu_1(\mathbf{x}) = \beta_2 \mu_2(\mathbf{x}) = \dots = \beta_L \mu_L(\mathbf{x})), i = \overline{2, m} \right\} \tag{7}$$

The scope of definition of variables  $\mathbf{x} = (x_1, \dots, x_n)$  and implementation of fuzzy constraints is determined based on the principle of equality. The following heuristic algorithm is proposed for solving the resulting multi-criteria fuzzy optimization problem (7)–(8):

**Heuristic algorithm MC-PR:**

1. A priority number is set for local criteria  $I_c = \{1, \dots, m\}$ , where the main criterion has priority 1.

2. DM, experts appoint, in addition to the main one, boundary values of local criteria  $\mu_R^i, i = \overline{2, m}$ .

3. The value of the constraint vector weights is entered  $\boldsymbol{\beta} = (\beta_1, \dots, \beta_L)$ , where  $\sum_{q=1}^L \beta_q = 1, \beta_q \geq 0, q = \overline{1, L}$ , providing  $\beta_1 \mu_1(\mathbf{x}) = \beta_2 \mu_2(\mathbf{x}) = \dots = \beta_L \mu_L(\mathbf{x})$ .

4. If  $\mu_0^i(\mathbf{x}), i = \overline{1, m}$  are fuzzy, then a term set is defined for them and membership functions are constructed.

5. A term set is determined and membership functions for fulfilling fuzzy constraints are constructed  $\mu_q(\mathbf{x}), q = \overline{1, L}$ .

6. The problem of maximizing the main criterion  $\max_{\mathbf{x} \in X} \mu_0^1(\mathbf{x})$ , is solved on the set of feasible solutions  $X$  (8) and current solutions are determined:  $\mathbf{x}(\mu_R^i, \beta)$ ;  $\mu_0^1(\mathbf{x}(\mu_R^i, \beta)), \dots, \mu_0^m(\mathbf{x}(\mu_R^i, \beta))$  and  $\mu_1(\mathbf{x}(\mu_R^i, \beta)), \dots, \mu_L(\mathbf{x}(\mu_R^i, \beta))$ .

7. The resulting current solutions are presented to the DM. If the current results do not satisfy the DM, then new values  $\mu_R^i, i = \overline{2, m}$  and/or constraint vector weights  $\beta$  are assigned to them, then to improve the solution, return to point 4. If the current solutions satisfy the DM, then he selects the final, best solution based on his preference, taking into account the current situation in production and proceed to the next step 8.

8. The best solutions selected by DM are displayed: optimal vector values  $\mathbf{x}^*(\mu_R^i, \beta)$ ; satisfactory values of local criteria  $\mu_0^1(\mathbf{x}^*(\mu_R^i, \beta)), \dots, \mu_0^m(\mathbf{x}^*(\mu_R^i, \beta))$  and maximum degrees of fulfillment of fuzzy constraints  $\mu_1(\mathbf{x}^*(\mu_R^i, \beta)), \dots, \mu_L(\mathbf{x}^*(\mu_R^i, \beta))$ .

## 4 Discussion

The presented heuristic algorithm for solving the problem of multi-criteria optimization of oil production processes in a fuzzy environment is based on a modification of the principles of optimality of the main criterion and equality for working in a fuzzy environment. The algorithm operates in a dialog mode between the user, as a result, solutions are improved from iteration to iteration and this is repeated until the best solution is obtained that satisfies the DM.

The correctness and effectiveness of the developed heuristic method for solving multi-criteria fuzzy optimization problems for optimizing the oil production process is, first of all, determined by the unambiguity of the information requested from the user (DM) and his experience, knowledge and competence. In this regard, the professional language used for dialogue between the DM and the computer in the process of solving a problem should not contain synonyms and homonyms, which are a source of ambiguity in a strictly formalized language. Experienced, competent specialists who successfully make decisions in practice are selected as DMs.

## 5 Conclusion

The paper proposes a method for extracting oil from a field with hard-to-recover reserves based on the methodology of systems analysis, which makes it possible to effectively solve the multi-criteria problem of optimizing oil production processes in conditions of shortage and fuzziness of initial information.

The following main results and conclusions were obtained during the study:

- Equations for the continuity of the mass of matter in an oil reservoir were derived for the one-dimensional rectilinear movement of the substance saturating it and for the three-dimensional case;
- A cross-section of the multi-layer Kenbai oil field and the main properties of the layers occurring within this oil field have been studied and presented;
- Based on modification of the principles of optimality of the main criterion and equality, the formulation of the problem of multi-criteria optimization of oil production processes was formulated and a heuristic method for solving it was developed;
- The proposed fuzzy approach makes it possible to take maximum account of the initial fuzzy information, which ensures the efficiency and high adequacy of solving fuzzy problems that often arise in oil production practice.

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