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TACHYONIZATION COSMOLOGICAL MODEL IN THE FRAMEWORK OF LINEAR FORM-INVARIANCE TRANSFORMATIONS

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This work dedicated to the investigation cosmological models based on tachyon field. It was showed that Einstein's field equations for spatially flat, homogeneous and isotropic Friedman-Robertson-Walker space-time have form-invariance of symmetry, obtained by form-invariance of transformations. The form-invariance of transformations is specified in the general case by the linear function $\bar{\rho} = n^2\rho$, and in particular, we investigate for $n = -1$. The tachyonization of the cosmological model was made for the entire range of values of the barotropic index γ. For the power-law function of the scale factor, a method for obtaining the potential and the function of the tachyon field was shown. The resulting potential is equivalent to the potential used in string theory.

Keywords: tachyon field, form-invariance transformations, equation of state, barotropic index, state finder parameters, deceleration parameter.

Introduction

Our universe is currently undergoing an accelerated expansion phase. This is confirmed by various observational data [1-4]. Theorists speculate that there is a component of matter that currently dominates the energy density of the universe, which is why gravity is repulsive even under standard general relativity. Due to the lack of a complete understanding of the nature of this component, it is called dark energy. Many models of dark energy have been proposed in the literature [5-16]. In this article, we will consider a dark energy model based on a tachyon field using methods of form-invariance transformations.

Form-invariance transformations (FIT) preserves the form of the equations of motion, since it has form-invariance symmetry (FIS) [17]. It was shown in [18] that transformations affect the Hubble expansion rate, energy density, and pressure of the cosmic fluid. Such transformations belong to the Lie group. FIS defines a set of identical cosmological models, since each representation of the Lie group is associated with a certain cosmology, through certain fluids. From the quantum field theory the T-duality comes, which connects a theory compactified on a circle of radius R with another compactified theory on a circle of radius $1/R$ [19-20]. In cosmology, the duality of the scale factor is used [21], which reflects the invariance property of the equations of motion. For the spatially flat FRW metric, the radius R is replaced by the scale factor a, and the dual transformation $a \to a^{-1}$ connects the contracting cosmology with the expanding one [18]. In [22], a method for obtaining phantom k-essence cosmologies using FIS is shown, in which phantom symmetry affects the potential, which leads to an expanded super-accelerated tachyon field.

Due to the emergence of a large number of different theoretical models and the improvement in the reliability of observational data, there is a need for reliable statistics that could distinguish cosmological models of dark energy from each other and from the ΛCDM model. One of these statistics is the pair of the statefinder parameters$\{r, s\}$ [23]. In this paper, we will derive formulas for these parameters after applying FIT and find their values for the power scale factor. Then compare the results with the fixed point ΛCDM model.

1. Model

In the model we are investigating, we choose the action in the form

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\{\tfrac{1}{2}R + \mathcal{L}_m\}, \tag{1}$$

where R–is Ricci scalar, $\mathcal{L}_m$- density of matter Lagrangian.

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Friedman-Robertson-Walker metric(FRW) describes by the following expression

$$ds^2 = -dt^2 + a(t)^2(dx^2 + \mathrm{d}y^2 + dz^2), \tag{2}$$

where $a(t)$scalar factor of the universe.

Our aim is investigating internal symmetry Einstein equations jointly spatially flat, homogeneous, and isotropic universe FRW(2). Einstein equations conjointly FRW universe tend to Friedman equations

$$3H^2 = \rho, \tag{3}$$
$$3H^2 + 2\dot{H} = -p, \tag{4}$$

where$H = \frac{\dot{a}}{a}$is a Hubble parameter and "dot" denotes derivatives with respect to the cosmic time.

A consequence of the Friedman equations (3)-(4) is anenergy conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{5}$$

For another cosmological model filled with an ideal fluid with energy density $\bar{\rho}$ and pressure $\bar{p}$, one can obtain similar expressions [24-25]

$$3\bar{H}^2 = \bar{\rho}, \tag{6}$$
$$3\bar{H}^2 + 2\dot{\bar{H}} = -\bar{p}, \tag{7}$$
$$\dot{\bar{\rho}} + 3\bar{H}(\bar{\rho} + \bar{p}) = 0. \tag{8}$$

The investigated cosmological models are invariant with respect to each other and it is possible to introduce a relationship between the energy densities

$$\bar{\rho} = \bar{\rho}(\rho). \tag{9}$$

The Friedman equations have three unknown variables (H, p, ρ), and using connection Eq.(9) can find relation for that variables

$$\frac{\bar{\rho}}{\rho} = \frac{3\bar{H}^2}{3H^2}, \Rightarrow \quad \bar{H} = H\left(\frac{\bar{\rho}}{\rho}\right)^{\frac{1}{2}}, \tag{10}$$

$$\bar{\rho} + \bar{p} = \frac{\dot{\bar{\rho}}}{\dot{\rho}}\left(\frac{\rho}{\bar{\rho}}\right)^{\frac{1}{2}}(\rho + p) = \frac{d\bar{\rho}}{d\rho}\left(\frac{\rho}{\bar{\rho}}\right)^{\frac{1}{2}}(\rho + p), \tag{11}$$

$$\bar{p} = -\bar{\rho} + \frac{d\bar{\rho}}{d\rho}\left(\frac{\rho}{\bar{\rho}}\right)^{\frac{1}{2}}(\rho + p). \tag{12}$$

Each of investigating cosmological models is filled with perfect fluid with a barotropic equation of states accordingly$p = (\gamma - 1)\rho$ и $\bar{p} = (\bar{\gamma} - 1)\bar{\rho}$. Barotropic indices γ and $\bar{\gamma}$ have the next connection

$$\bar{\gamma} = \frac{\bar{\rho} + \bar{p}}{\bar{\rho}} = \frac{d\bar{\rho}}{d\rho}\left(\frac{\rho}{\bar{\rho}}\right)^{\frac{3}{2}}\gamma. \tag{13}$$

Form invariance transformation (10)-(12) generates Lie group [18]. The form-invariance of the symmetry is confirmed by the form-invariance of the transformations and shows the equivalence of the investigating models.

2. Linear FIT

FIT can be introduced by following linear function[18]

$$\bar{\rho} = n^2\rho, \tag{14}$$

where n is arbitrary constant. In that case equations (10)-(12)take the form

$$\bar{H} = nH, \tag{15}$$
$$\bar{\rho} + \bar{p} = n(\rho + p), \tag{16}$$
$$\bar{p} = n[p + (1 - n)\rho]. \tag{17}$$

Linear FIT induces linear expresses of variables(H, p, ρ). We obtain power law connection for scale factors by integrating Eq.(15)

$$\bar{a} = a^n \tag{18}$$

and from Eq.(13) transformation for barotropic index

$$\bar{\gamma} = \frac{\gamma}{n}. \tag{19}$$

We can relate the scale factor a of the original cosmological model to the scale factor $\bar{a} = a^n$ of another model due to the existence of the structure of the Lie group [18], [24].

3. Tachyon model

Let us investigate the behavior of the tachyon field and will show its transformation in accordance with the FIT (14)-(17). Density of matter Lagrangian tachyon field in a FRW metric becomes

$$\mathcal{L}_\phi = -V(\phi)\sqrt{1 - \dot{\phi}^2}, \tag{20}$$

here $V(\phi)$ is potential of tachyon field. We substitute Lagrangian (20) in action (1) using the Euler-Lagrange equation and obtain a dynamical system for the tachyon field as follows

$$3H^2 = \rho, \tag{21}$$
$$3H^2 + 2\dot{H} = -p, \tag{22}$$

where energy densityρ and pressure p are defined by expressions

$$\rho = \frac{V}{\sqrt{1-\dot{\phi}^2}}, \tag{23}$$
$$p = -V\sqrt{1 - \dot{\phi}^2} \tag{24}$$

and Klein-Gordon equation

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2} + 3H\dot{\phi} + \frac{V_\phi}{V} = 0. \tag{25}$$

We obtain an expression connecting the square of the derivative of the tachyon field $\dot{\phi}^2$ and the barotropic exponent γ , substituting into the equation of state $p = (\gamma - 1)\rho$ the value of the energy density ρEq.(23) and pressure pEq.(24) to analyze the stability of solutions. In this case it follows that

$$\gamma = \dot{\phi}^2, \tag{26}$$

where $0 < \gamma < 1$. Speed of sound $c_s^2 = 1 - \gamma > 0$or take into account(26)

$$c_s^2 = 1 - \dot{\phi}^2. \tag{27}$$

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The converted energy density and pressure of the tachyon field are equal

$$\bar{\rho} = \frac{\bar{V}}{\sqrt{1-\dot{\bar{\phi}}^2}} = \frac{n^2 V}{\sqrt{1-\dot{\phi}^2}}, \tag{28}$$

$$\bar{p} = -\bar{V}\sqrt{1-\dot{\bar{\phi}}^2} = -\left(1-\frac{\dot{\phi}^2}{n}\right)\frac{n^2 V}{\sqrt{1-\dot{\phi}^2}}, \tag{29}$$

where we used FIT (14) and pressure (17). From Eq.(19) and Eq.(26)

$$\dot{\bar{\phi}}^2 = \frac{\dot{\phi}^2}{n}. \tag{30}$$

We obtain the transformation expression for the tachyon field $\bar{\phi} = \frac{\phi}{\sqrt{n}}$, integrating the last expression. The transformation conditions for the potential of the tachyon field and the speed of sound are

$$\bar{V} = \bar{\rho}\sqrt{1-\dot{\bar{\phi}}^2} = n^2 V\sqrt{\frac{1-\frac{\dot{\phi}^2}{n}}{1-\dot{\phi}^2}}, \tag{31}$$

$$\bar{c}_s^2 = 1-\dot{\bar{\phi}}^2 = 1-\frac{\dot{\phi}^2}{n}. \tag{32}$$

The usual tachyon field corresponds to$0 < \gamma < 1$. Tachyonization of the model at $1 < \gamma$will be achieved by the complementarytachyon fieldϕ_c, and at $\gamma < 0$ - thephantom tachyon fieldϕ_{ph}. These two kinds of tachyon field can be introduced from the tachyon field analyzed above by applying the transformations [22, 26-27].

The complementary tachyon field $\phi_c$characterized by$1 < \gamma$or$1 < \dot{\phi}_c^2$and expressions for it can be obtained from the standard tachyon field by an internal transformation $1-\gamma \to -(1-\gamma)$, $1-\dot{\phi}^2 \to -(1-\dot{\phi}_c^2)$and $\sqrt{1-\dot{\phi}^2} \to \sqrt{-1}\sqrt{1-\dot{\phi}_c^2} = i\sqrt{1-\dot{\phi}_c^2}$. Having carried out a simultaneous replacement, we get

$$\bar{\rho}_c = \frac{|\bar{V}|}{\sqrt{\dot{\bar{\phi}}_c^2-1}}, \tag{33}$$

$$\bar{p}_c = |\bar{V}|\sqrt{\dot{\bar{\phi}}_c^2-1}. \tag{34}$$

The phantom tachyon fieldϕ_{ph}characterized by$\gamma < 0$, $\dot{\phi}_{ph}^2 = -\gamma$and expressions for it can also be obtained from the standard tachyon field by an internal transformation. In that case $\sqrt{\gamma} \to -i\sqrt{-\gamma}$, $\phi \to i\phi_{ph}$and$\dot{\phi}^2 \to i^2\dot{\phi}_{ph}^2 = -\dot{\phi}_{ph}^2$. Having carried out a simultaneous replacement, we get

$$\bar{\rho}_{ph} = \frac{\bar{V}}{\sqrt{1-\dot{\bar{\phi}}^2}} = \frac{n^2 V}{\sqrt{1-\dot{\phi}^2}}, \tag{35}$$

$$\bar{p} = -\bar{V}\sqrt{1-\dot{\bar{\phi}}^2} = -\left(1-\frac{\dot{\phi}^2}{n}\right)\frac{n^2 V}{\sqrt{1-\dot{\phi}^2}}. \tag{36}$$

Expressions for two new types of tachyon fields were found using simple internal symmetries. All of them are needed to describe the time evolution of the scale factor (18) for all values of γ and to carry out complete tachyonization of the flat FRW universe filled with an ideal fluid with barotropic equations of state $p = (\gamma-1)\rho$.

4. State finder parameters

The various properties of dark energy are highly dependent on the chosen model. Previously, specific evaluation criteria were invented in order to distinguish between different and competing cosmological models involving dark energy. In [23], [28], two parameters, called statefinders, were introduced, which make it possible to distinguish several models of dark energy. These parameters contain the scale factor $a(t)$ and its third derivative with respect to cosmic time t

$$r = \frac{\dddot{a}}{aH^3} = \frac{\ddot{H}}{H^3} - 3q - 2, \tag{37}$$
$$s = \frac{r-1}{3(q-1/2)}, \tag{38}$$

where q is deceleration parameter $q = -a\ddot{a}/\dot{a}^2 = -\ddot{a}/aH^2$. Using Eq.(15)andEq.(18)we get transformation condition for statefinder parameters and deceleration parameter

$$\bar{r} = \frac{\dddot{\bar{a}}}{\bar{a}\bar{H}^3} = \frac{a^2\dddot{a}}{n^2\dot{a}^3} + 3\frac{(n-1)a\ddot{a}}{n^2\dot{a}^2} + \frac{(n-1)(n-2)}{n^2}, \tag{39}$$
$$\bar{s} = \frac{\bar{r}-1}{3(\bar{q}-1/2)} = \frac{2}{3n}\left(1 - \frac{a^2\dddot{a}+(3n-1)a\dot{a}\ddot{a}}{2a\dot{a}\ddot{a}+(3n-2)\dot{a}^3}\right), \tag{40}$$
$$\bar{q} = -\frac{\bar{a}\ddot{\bar{a}}}{\dot{\bar{a}}^2} = -\frac{a\ddot{a}}{n\dot{a}^2} + \frac{1-n}{n}, \tag{41}$$

where to derive the transformation conditions, we used the derivatives

$$\dot{\bar{a}} = na^{n-1}\dot{a}, \tag{42}$$
$$\ddot{\bar{a}} = n(n-1)a^{n-2}\dot{a}^2 + na^{n-1}\ddot{a}, \tag{43}$$
$$\dddot{\bar{a}} = n(n-1)(n-2)a^{n-3}\dot{a}^3 + 3n(n-1)a^{n-2}\dot{a}\ddot{a} + na^{n-1}\dddot{a}. \tag{44}$$

5. Solution

Let consider case when in transformation (14) $n^2 = 1$. When our solution divided into two subcases$n = 1$ и $n = -1$

$$\bar{H} = H, \quad \bar{\rho} + \bar{p} = \rho + p, \quad \bar{a} = a, \tag{45}$$
$$\bar{H} = -H, \quad \bar{\rho} + \bar{p} = -(\rho + p), \quad \bar{a} = \frac{1}{a}. \tag{46}$$

The first subcase corresponds to the identical transformation, and the second case corresponds to the dual transformation, for which the energy density $\dot{\rho} = -3H(\rho + \mathrm{p}) \geq 0$ is an increasing function of time [18].

For $n = 1$, the equations for the statefinderparameters and the deceleration parameter (39) - (41) take the standard form (37) - (38), and for $n = -1$ we get

$$\bar{r} = \frac{a^2\dddot{a}}{\dot{a}^3} - \frac{6a\ddot{a}}{\dot{a}^2} + 6, \tag{47}$$
$$\bar{s} = -\frac{2}{3}\left(1 - \frac{a^2\dddot{a}-4a\dot{a}\ddot{a}}{2a\dot{a}\ddot{a}-5\dot{a}^3}\right), \tag{48}$$
$$\bar{q} = \frac{a\ddot{a}}{\dot{a}^2} - 2. \tag{49}$$

From the conservation equation (5)

$$\frac{\dot{\rho}}{\rho} = -3H(1+\omega), \tag{50}$$

where ω equation of state parameter depends on cosmic time accordingly $\omega = p/\rho$. We substitute equation (50) in Friedman equations(3)take the form

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$$1+\omega=-\frac{2\dot{H}}{3H^2}, \tag{51}$$

where we used the next relation $\frac{\dot{\rho}}{\rho}=\frac{2\dot{H}}{H}$. For tachyon field $\frac{p}{\rho}=\omega=\dot{\phi}^2-1$. When we can get expression $\dot{\phi}=\left(-\frac{2\dot{H}}{3H^2}\right)^{\frac{1}{2}}$, by integrating which we get

$$\phi(t)=\int\left(-\frac{2\dot{H}}{3H^2}\right)^{\frac{1}{2}}dt. \tag{52}$$

We multiply equations (23) and (24) and use the Friedman equation (3)

$$\rho p=-V^2,\qquad V=(-\rho p)^{\frac{1}{2}}=(-\omega)^{\frac{1}{2}}\rho=3H^2\left(1+\frac{2\dot{H}}{3H^2}\right)^{\frac{1}{2}}. \tag{53}$$

For any scale factor $a(t)$ find the time dependence of the potential $V(t)$ and the tachyon field $\phi(t)$, using equations (52) and (53) and hence the potential $V(\phi)$. Also from equation (52) we can conclude that for these models always $\dot{H}<0$. The tachyon potential, by analogy with the potential of a scalar field, can be used to control the expansion of the universe.

Consider the case when the expansion of the universe obeys the power law

$$a=a_0t^{\alpha}, \tag{54}$$

where a_0 and α some positive constants, and for the accelerated expansion of the universe it is necessary$\alpha>1$. In this case, equations (52) and (53) have the following solutions

$$\phi(t)=\left(\frac{2}{3\alpha}\right)^{\frac{1}{2}}t+\phi_0,\quad V(t)=3\alpha^2\left(1-\frac{2}{3\alpha}\right)^{\frac{1}{2}}\frac{1}{t^2}, \tag{55}$$

where ϕ_0 integration constant. We find the potential $V(\phi)$ by replacing $t$from Eq.(55)

$$V=\frac{V_0}{(\phi-\phi_0)^2}, \tag{56}$$

where $V_0=2\alpha\left(1-\frac{2}{3\alpha}\right)^{\frac{1}{2}}$. This potential diverges at $\phi=\phi_0$ and corresponds to the typical potential of bosonic string theory. The converted scale factor is $\bar{a}=\bar{a}_0t^{\bar{\alpha}}$ с $\bar{a}_0=a_0^n$ и $\bar{\alpha}=n\alpha$. Expressions (55) correspond to the usual tachyon at $0<\gamma<1$. For the complementarytachyon field ϕ_c at $1<\gamma$, which is a stiff matter with a cosmology of deceleration, using FIT we obtain

$$\bar{\phi}_c=\left(\frac{2}{3\bar{\alpha}}\right)^{\frac{1}{2}}t+\phi_{c0},\quad \bar{\alpha}=\frac{1}{3}\left(1\pm\sqrt{1-\frac{9V_0^2}{4}}\right). \tag{57}$$

For phantom tachyon field ϕ_{ph} with $\gamma<0$

$$\bar{\phi}_{ph}=\left(-\frac{2}{3|\bar{\alpha}|}\right)^{\frac{1}{2}}t+\phi_{ph0},\quad \bar{\alpha}=\frac{1}{3}\left(1-\sqrt{1+\frac{9V_0^2}{4}}\right). \tag{58}$$

For the scale factor (54), the statefinderparameters (37) - (38) and the deceleration parameter take the form

$$r=1-\frac{3}{\alpha}+\frac{2}{\alpha^2},\quad s=\frac{2}{3\alpha},\quad q=-1+\frac{1}{\alpha} \tag{59}$$

and after FIT at $n = -1$ parameters(47)-(49)

$$\bar{r} = 1 + \frac{3}{\alpha} + \frac{2}{\alpha^2}, \quad \bar{s} = -\frac{2}{3\alpha}, \quad \bar{q} = -1 - \frac{1}{\alpha}. \tag{60}$$

We exclude the parameter α from equations (59) and (60)

$$r(s) = \frac{9}{2}s^2 - \frac{9}{2}s + 1, \quad r(q) = 2q^2 + q, \tag{61}$$
$$\bar{r}(\bar{s}) = \frac{9}{2}\bar{s}^2 - \frac{9}{2}\bar{s} + 1, \quad \bar{r}(\bar{q}) = 2\bar{q}^2 + \bar{q}. \tag{62}$$

Point $\{r, s\} = \{1,0\}$ is fixed point for ΛCDM-model [23]. It can be seen from equations (61) - (62) that the graphs of the functions $r(s)$ and $\bar{r}(\bar{s})$ pass through this point and are located to the right of it. Dependency graphs $\{r, q\}$ and $\{\bar{r}, \bar{q}\}$ pass in the past through the point $\{1, 0.5\}$ corresponding to the universe with a predominance of matter (SCDM) and the point in the future $\{1, -1\}$ corresponding to stable state (SS) - de Sitter extensions.

Conclusion

By researching our model, we have shown that form invariance transformations can be used to obtain new solutions to the Einstein equation. Moreover, FIT allows you to move from annon-stable cosmology to a stable one and vice-versa. A static universe containing an ideal fluid is always stable at the speed of sound $c_s^2 > 1/5$. If the initially investigated model has a barotropic index γ corresponding to an unstable solution, then after using the transformation rule (32) we can obtain a stable cosmological model.

As in [26], we proved the possibility of the existence of two types of extended tachyons. The complementary $(1 < \gamma)$ and the phantom $(\gamma < 0)$ tachyon fields were obtained from the standard tachyon field ($0 < \gamma < 1$). These fields were used to complete tachyonization of the FRW universe filled with an ideal fluid with barotropic equations of state $p = (\gamma - 1)\rho$ for all values γ and scale factor (54).

A method was shown for finding the time dependence of the potential $V(t)$ and the tachyon field $\phi(t)$ for any scale factor $a(t)$. We were convinced that the tachyon potential, by analogy with the potential of a scalar field, can be used to control the expansion of the universe.

Derived formulas of the statefinder and the deceleration parameter after applying FIT. From the performed study of our tachyon model using statefinder, it can be seen that the results obtained $\{r, s\} = \{1,0\}$ agree with the theory proposed in [23].

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