

Analysis of ($^{11}\text{Li} + ^9\text{Be}$)-reaction in the framework of the time-dependent Schrodinger equation

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Neutron transfer and nucleus breakup cross sections in ($^{11}\text{Li} + ^9\text{Be}$)-reaction are calculated at energy range up to 32 MeV/nucleon. The evolution of probability density of external weakly bound neutrons of ^{11}Li and the probabilities of neutron transfer and nucleus breakup are determined based on a numerical solution of the time-dependent Schrodinger equation. Our calculation results are agree with the experiment.

Keywords: neutron transfer; nucleus breakup; time-dependent Schrodinger equation; shell model.

Introduction

The article studies processes of neutron transfer and nucleus breakup in reaction ($^{11}\text{Li}, ^9\text{Li}$) at ^9Be -target. Recently, in the review [1], we presented the latest experimental data on the total reaction cross sections and angular distributions of differential cross sections for elastic scattering of light weakly bound lithium nuclei ($^{6-9,11}\text{Li}$). Specific features of experimental excitation functions of the total reaction cross sections and dominant interaction mechanisms at near-barrier energies was

discussed. In [1], we also discussed the theoretical methods for describing the trend in behavior of the reaction cross section excitation functions. Study of the mechanisms of interaction of weakly bound light exotic nuclei arouse great interest. It is due to effects of a sharp increase of the total cross section in reactions with these nuclei, [2-5]. The ^{11}Li nucleus has a halo structure consisting of two external weakly bound neutrons. The neutron separation energy in the ^{11}Li nucleus is 0.4 MeV [6], which is significantly less than in stable lithium isotopes: ^6Li (5.7 MeV), ^7Li (7.3 MeV), ^9Li (4 MeV). For this reason, reactions of ^{11}Li nucleus with light nuclei are highly likely to involve neutron transfer and nucleus breakup.

Currently, one of the most widespread theoretical models for describing neutron transfer processes is Distorted Wave Born Approximation method [7]. In this method, particles rearrangement process in the reaction $A(a,b)B$ is divided into three stages: motion of projectile a in optical potential of target A , transfer of nucleon from projectile a to target A and motion of ejectile b in field of nucleus B . However, this division of interaction process into three stages does not allow analysis of the dynamics of taking place processes.

The use of the time-dependent Schrodinger equation to study rearrangement of neutrons during fusion reactions was proposed by [8,9]. Such approach makes it possible to obtain a microscopic description of dynamics of nuclear fusion [8,9], neutrons stripping and pick-up [10-14], and breakup of nuclei. This paper described neutrons transfer and breakup processes in a collision of ($^{11}\text{Li} + ^9\text{Be}$). As in [8,9], the spin-orbit interaction was not taken into account.

Theoretical calculations

Shell model without spin-orbit interaction was used for calculation of the radial parts of the neutron wave functions of the ^{11}Li nucleus. The radial Schrodinger equation for the central mean field $V(r)$ was solved by the difference method with matrix eigenvalues calculations [15]. Results for normalized radial part $R(r)$ of two wave functions for $1s$ and $1p$ levels neutrons were shown in Figure 1. Wave functions $\Psi(\vec{r})$ of the $1p$ state with projections of angular momentum $m_l = 0, \pm 1$ were used for initial conditions of the time-dependent Schrodinger equation. For used parameters of mean fields of nuclei $V_1(r), V_2(r)$, separation energies of neutron from target and projectile are equal to experimental values [6].

The time-dependent Schrodinger equation describing evolution of wave function of external neutron in field of colliding nuclei:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi + [V_1(|\vec{r} - \vec{r}_1(t)|) + V_2(|\vec{r} - \vec{r}_2(t)|)] \Psi. \quad (1)$$

The Eq. (1) was solved by the scheme used in [16]. Here $\vec{r}_1(t), \vec{r}_2(t)$ are the radii of the centers of the colliding nuclei which moved along line trajectories, since at high energies the motion of the centers of nuclei occurs along almost line trajectories.

Figures 2 and 3 present results of a numerical solution of the time-dependent Schrodinger equation. As we see, the time-dependent Schrodinger equation

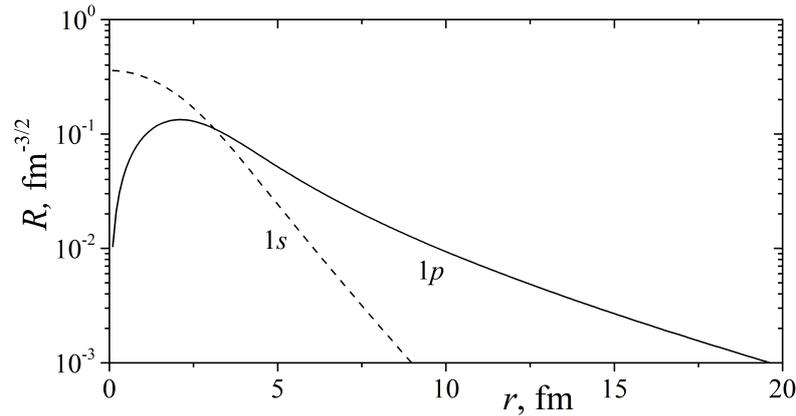


Figure 1. Radial part $R(r)$ of the wave functions for $1s$ and $1p$ levels neutrons in the ^{11}Li nucleus.

allowed us to assess visually the dynamics of taking place processes. The external neutrons of ^{11}Li nucleus may be removed during two processes: transfer to target nucleus and break up to unbound states of continuous energy spectrum. At a distance of about (15-16) fm between the centers of the nuclei, the probability density began to transfer to target (see Figure 2a,b and 3a,b). Transfer process was accompanied by breakup of ^{11}Li nucleus (see Figure 2c,d and 3c,d), due to low binding energy of external neutron of 0.4 MeV [6].

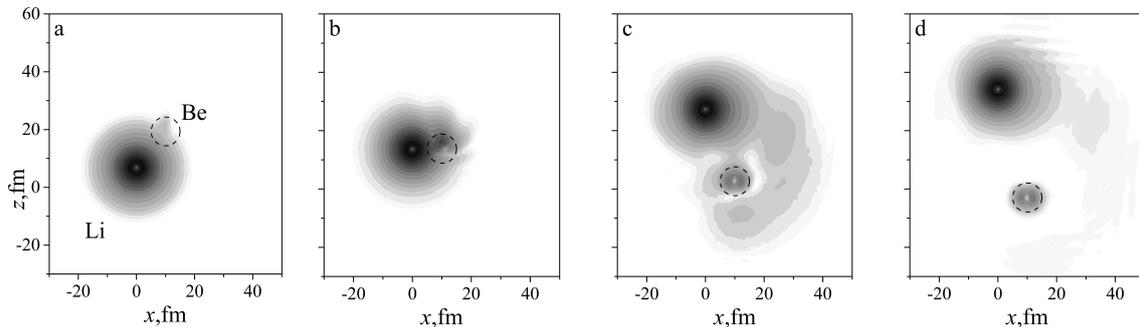


Figure 2. Evolution of probability densities of external neutron of ^{11}Li in the collision with ^9Be at 8 MeV/nucleon. Impact parameter $b = 10$ fm. Time scale from left to right. Dash circle is ^9Be nucleus.

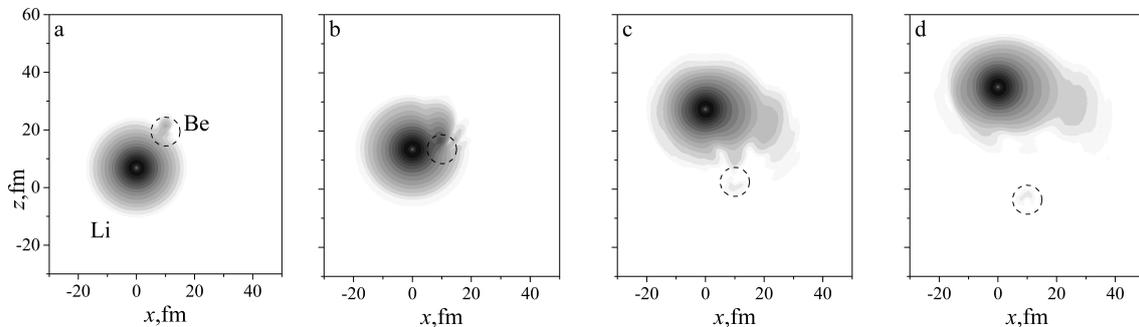


Figure 3. Evolution of probability densities of external neutron of ^{11}Li in the collision with ^9Be at 32.4 MeV/nucleon. Impact parameter $b = 10$ fm. Time scale from left to right. Dash circle is ^9Be nucleus.

Probabilities $p(b, E)$ of neutron transfer and breakup of nucleus, what depends on impact parameter b and energy E in the center of mass system, was calculated by integrating probability density over volume:

$$p_{\text{transfer}}(b, E) = \int_T |\Psi|^2 dV, \quad (2)$$

$$p_{\text{breakup}}(b, E) = C \int_B |\Psi|^2 dV. \quad (3)$$

Here C is a parameter that takes into account the uncertainty of the calculation of the breakup probability. It is due to slow separation of the wave function during breakup, see Figures 2d and 3d. For Eq. (2), T is region of integration within the beryllium nucleus volume that corresponds to transfer of neutron. In addition, for Eq. (3), B is region of integration over the volume outside lithium and beryllium that corresponds to the breakup of the ^{11}Li nucleus.

Calculations of time-dependencies of probabilities of collision presented on Figures 4 and 5 show the dynamics of neutron transfer and nucleus breakup. Due to slow separation of the wave function during the collision the end probability of breakup defines for a long time.

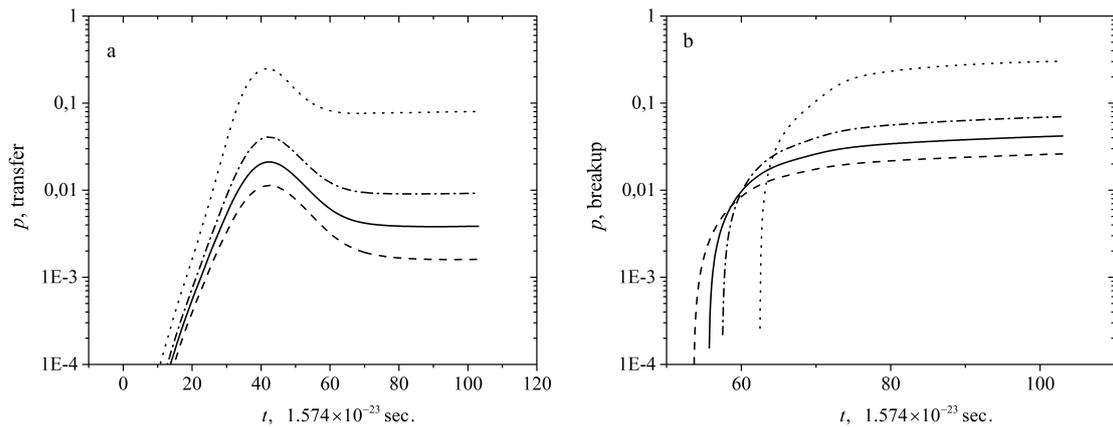


Figure 4. Time-dependencies of probabilities of transfer (a) and nucleus breakup (b) of collision at 8 MeV/nucleon. Dotted line – impact parameter of nuclei collision $b = 6$ fm. Dash-dotted line – $b = 9$. Solid line – $b = 10$ fm. Dashed line – $b = 11$ fm.

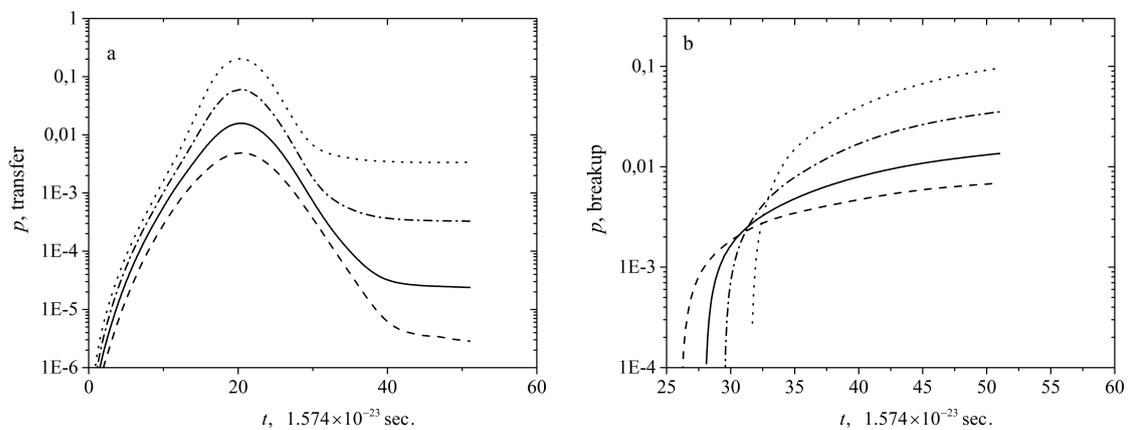


Figure 5. Time-dependencies of probabilities of transfer (a) and nucleus breakup (b) of collision at 32.4 MeV/nucleon. Dotted line – impact parameter of nuclei collision $b = 6$ fm. Dash-dotted line – $b = 9$. Solid line – $b = 10$ fm. Dashed line – $b = 11$ fm.

Figure 6 shows linear regression of calculated neutron transfer and nucleus breakup probabilities at 8 and 32.4 MeV/nucleon. As the impact parameter increases, the probabilities decrease by several orders of magnitude.

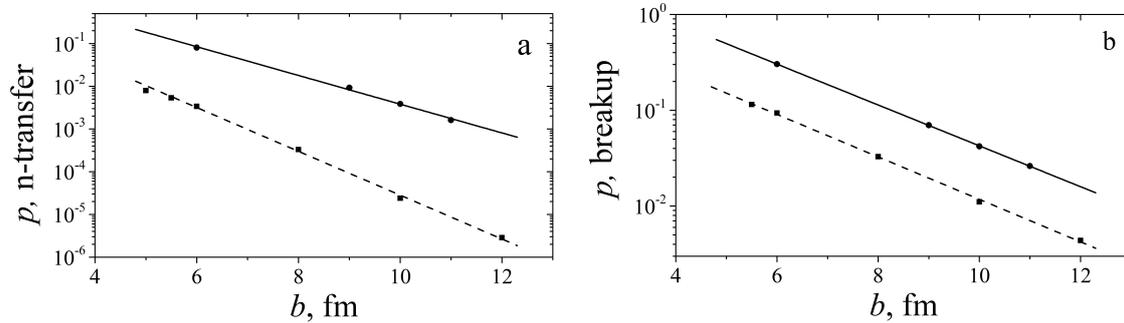


Figure 6. Dependencies of probabilities of transfer (a) and nucleus breakup (b) on impact parameter. $E = 8$ MeV/nucleon (solid line), 32.4 MeV/nucleon (dashed line).

The reaction cross sections (^{11}Li , ^9Li) were determined by integrating the sum of the probabilities of neutron transfer and nucleus breakup over impact parameter b :

$$\sigma_{-2n} = 2\pi \int (p_{\text{breakup}}(b, E) + p_{\text{transfer}}(b, E)) b db, \quad (4)$$

Figure 5 shows calculated cross sections for neutron transfer and nuclear breakup. For the reaction ($^{11}\text{Li} + ^9\text{Be}$) at 29.0 MeV/nucleon, experimental value of breakup two neutrons σ_{-2n} is about (470 ± 100) mb [17].

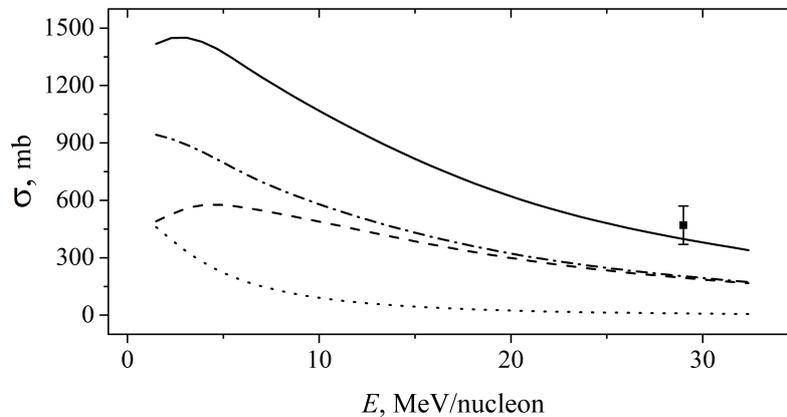


Figure 7. ■ – σ_{-2n} [17]. Dashed line – calculation for breakup of nucleus; Dotted line – calculation for neutron transfer; Dash-dot line – (^{11}Li , ^9Li) cross section ($C = 1$); Solid line – (^{11}Li , ^9Li) cross section ($C = 2$).

Conclusion

The cross sections of neutron transfer and nucleus breakup in ($^{11}\text{Li} + ^9\text{Be}$) was calculated based on the numerical solution of the time-dependent Schrodinger equation. The evolution of the probability density and the probability of transfer and breakup were determined based on a numerical solution of the time-dependent Schrodinger equation for external weakly bound neutrons of the ^{11}Li nucleus. The calculations results are agree with the experimental cross section of neutron

removal. Further theoretical and experimental improvements are needed by the more exactly description the processes of interaction of external neutrons in reactions with weakly bound nuclei.

Acknowledgments

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