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Cosmological solutions of $F(R,T)$ gravity model with k -essence

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Abstract. Now exist several alternative cosmological models that describe observable properties of our universe. In particular, it is such models as $F(R)$ and $F(T)$ gravity. We consider properties of their generalization as $F(R,T)$ model of gravity with kessence. We obtained some exact solutions of particular cass of scale factor a for general formof the $F(R,T)$ functions with scalar field. These solutions describe the accelerated/deccelerated periods of the universe.

1. Introduction

The discovery of the accelerated expansion of the universe requires the modernization of cosmology. As one of the way for the description, this cosmic acceleration is assumed to appear due to matter usually called Dark Energy with negative pressure (DE). But this DE has not yet been discovered. To explain this phenomenon, many theoretical models have been proposed, such as k-essence [1], [2], f-essence [3], [4], [5], [6], [7], g -essence [8], [9], [10] etc. In addition, Modified gravitational models are interesting like $F(R)$ gravity, $F(G)$ gravity, $F(T)$ gravity etc [11], [12],[13], [14], [15] [16]. For investigation of quantum and general gravity theories is interesting to investigate generalization of gravity as $F(R,T)$ gravity with k-essence, where R is the scalar of curvature and T is scalar torsion.

2. $F(R,T)$ gravity

The action of modified $F(R,T)$ gravity has the following form [11]:

$$S_{43} = \int \sqrt{-g} d^4x [F(R,T) + L_m], \tag{1}$$

where

$$\begin{aligned} R &= \epsilon_1 g^{\mu\nu} R_{\mu\nu} + u, \\ T &= \epsilon_2 S_\rho{}^{\mu\nu} T^\rho{}_{\mu\nu} + v. \end{aligned} \tag{2}$$

Here L_m is the matter Lagrangian, that for k-essence we can rewrite as $L_m = K = X - V(\varphi)$, $\epsilon_i = \pm 1$ is signature, R is the curvature scalar, T is the torsion scalar and $X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$, where φ- scalar field.

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3. $f(R,T,X,\varphi)$ gravity

For action of modified gravity in the most general form we consider here $F(R,T)$ as $f(R,T,X,\varphi) = F(R,T) + C_X X + C(\varphi)$ gravity within the framework of FriedmannRobertsonWalker (FRW) metric, with line element $ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)$ as

$$S = 2\pi^2 \int dt\, a^3 \left\{ f(R,T,X,\varphi) - \lambda_1 \left[R - u + 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)\right] - \lambda_2 \left[T - v + 6\left(\frac{\dot{a}^2}{a^2}\right)\right] - \lambda_3 \left[X - \frac{1}{2}\dot{\varphi}^2 \right] \right\}. \quad (3)$$

Here u, v -some function of $a, \dot{a}$.

For this signature we have

$$R = u - 6\left(\dot{H} + 2H^2\right), \quad (4)$$

$$T = v - 6H^2, \quad (5)$$

$$X = \frac{1}{2}\dot{\varphi}^2. \quad (6)$$

Hereafter denoted $f(R,T,X,\varphi)$ as F. By varying the action with respect to R , T and X, one obtains

$$\lambda_1 = F_R, \qquad \lambda_2 = F_T, \qquad \lambda_3 = F_X. \quad (7)$$

Here F_R is derivation of F function by R, F_T is derivation of F function by $T$and F_X is derivation of F function by X. After an integration by parts, the point-like Lagrangian have the following form

$$\begin{aligned} L &= a^3 \left[F - (R-u)F_R - (T-v)F_T\right] + 6\, a\dot{a}^2 \left[F_R - F_T\right] + \\ &+ 6a^2\dot{a}\left[\dot{R}F_{RR} + \dot{T}F_{RT} + \dot{X}F_{RX} + \dot{\varphi}F_{R\varphi}\right] - a^3 F_X \left[X - \frac{1}{2}\dot{\varphi}^2\right]. \end{aligned} \quad (8)$$

4. The Noether Symmetries

For this Lagrangian Noether symmetry condition we will write as

$$XL = 0, \quad (9)$$

here

$$\begin{aligned} X &= \alpha\frac{\partial}{\partial a} + \beta\frac{\partial}{\partial R} + \gamma\frac{\partial}{\partial T} + \delta\frac{\partial}{\partial X} + \epsilon\frac{\partial}{\partial \varphi} + \\ &+ \dot{\alpha}\frac{\partial}{\partial \dot{a}} + \dot{\beta}\frac{\partial}{\partial \dot{R}} + \dot{\gamma}\frac{\partial}{\partial \dot{T}} + \dot{\delta}\frac{\partial}{\partial \dot{X}} + \dot{\epsilon}\frac{\partial}{\partial \dot{\varphi}}. \end{aligned} \quad (10)$$

The functions $\alpha, \beta, \gamma, \delta, \epsilon$ depend on the variables a, R, T, X, φ and then

$$\dot{\alpha} = \alpha_a\dot{a} + \alpha_R\dot{R} + \alpha_T\dot{T} + \alpha_X\dot{X} + \alpha_\varphi\dot{\varphi}\,, \quad (11)$$

$$\dot{\beta} = \beta_a\dot{a} + \beta_R\dot{R} + \beta_T\dot{T} + \beta_X\dot{X} + \beta_\varphi\dot{\varphi}\,, \quad (12)$$

$$\dot{\gamma} = \gamma_a\dot{a} + \gamma_R\dot{R} + \gamma_T\dot{T} + \gamma_X\dot{X} + \gamma_\varphi\dot{\varphi}\,, \quad (13)$$

$$\dot{\delta} = \delta_a\dot{a} + \delta_R\dot{R} + \delta_T\dot{T} + \delta_X\dot{X} + \delta_\varphi\dot{\varphi}\,, \quad (14)$$

$$\dot{\epsilon} = \epsilon_a\dot{a} + \epsilon_R\dot{R} + \epsilon_T\dot{T} + \epsilon_X\dot{X} + \epsilon_\varphi\dot{\varphi}. \quad (15)$$

Using this we can receive:

$$
\begin{aligned}
0 \;=\; & 6(\alpha\,[F_R - F_T] + \beta a\,[F_{RR} - F_{TR}] + \gamma a\,[F_{RT} - F_{TT}] + \delta a\,[F_{RX} - F_{TX}] + \\
&+\epsilon a\,[F_{R\varphi} - F_{T\varphi}] + \alpha_a 2a\,[F_R - F_T] + \beta_a a^2 F_{RR} + \gamma_a a^2 F_{TR} + \delta_a a^2 F_{RX} + \epsilon_a a^2 F_{R\varphi})\dot{a}^2 + \\
&+\dot{R}^2 6\alpha_R a^2 F_{RR} + \dot{T}^2 6\alpha_T a^2 F_{RT} + \dot{X}^2 6\alpha_X a^2 F_{RX} + \dot{\varphi}^2(\epsilon_\varphi a^3 F_X + 6\alpha_\varphi a^2 F_{R\varphi} + \\
&+\alpha\frac{3}{2}a^2 F_X + \beta\frac{1}{2}a^3 F_{XR} + \gamma\frac{1}{2}a^3 F_{XT} + \delta\frac{1}{2}a^3 F_{XX} + \epsilon\frac{1}{2}a^3 F_{X\varphi}) + \\
&+\dot{a}\dot{R}6a(\beta a F_{RRR} + \gamma a F_{RRT} + \delta a F_{RRX} + \epsilon a F_{RR\varphi} + 2\alpha_R\,[F_R - F_T] + \\
&+(\alpha_a a + \beta_R a + 2\alpha)F_{RR} + \gamma_R a F_{TR} + \delta_R a F_{RX} + \epsilon_R a F_{R\varphi}) + \\
&+\dot{a}\dot{T}6a(\beta a F_{TRR} + \gamma a F_{TRT} + \delta a F_{TRX} + \epsilon a F_{TR\varphi} + 2\alpha_T\,[F_R - F_T] + (\alpha_a a + 2\alpha + \gamma_T a)\,F_{TR} + \\
&+\beta_T a F_{RR} + \delta_T a F_{RX} + \epsilon_T a F_{R\varphi}) + \dot{a}\dot{X}6a(2\alpha_X\,[F_R - F_T] + a\beta_X F_{RR} + \\
&+a\gamma_X F_{TR} + \beta a F_{RXR} + \gamma a F_{RXT} + \delta a F_{RXX} + \epsilon a F_{RX\varphi} + (2\alpha + \alpha_a a + \delta_X a)\,F_{RX} + \\
&+\epsilon_X a F_{R\varphi}) + \dot{a}\dot{\varphi}6a(2\alpha_\varphi\,[F_R - F_T] + a\beta_\varphi F_{RR} + a\gamma_\varphi F_{TR} + \epsilon_a\frac{a^2}{6}F_X + \\
&+2\alpha F_{R\varphi} + \beta a F_{R\varphi R} + \gamma a F_{R\varphi T} + \delta a F_{R\varphi X} + \epsilon a F_{R\varphi\varphi} + \alpha_a a F_{R\varphi} + \\
&+\delta_\varphi a F_{RX} + \epsilon_\varphi a F_{R\varphi}) + \dot{R}\dot{T}6a^2(\alpha_R F_{TR} + \alpha_T F_{RR}) + \\
&+\dot{R}\dot{X}6a^2(\alpha_X F_{RR} + \alpha_R F_{RX}) + 6\dot{T}\dot{X}a^2(\alpha_X F_{TR} + \alpha_T F_{RX}) + \\
&+\dot{R}\dot{\varphi}a^2(6\alpha_\varphi F_{RR} + \epsilon_R a F_X + 6\alpha_R F_{R\varphi}) + \dot{T}\dot{\varphi}(6\alpha_\varphi a^2 F_{TR} + \\
&+\epsilon_T a^3 F_X + 6\alpha_T a^2 F_{R\varphi}) + \dot{X}\dot{\varphi}(\epsilon_X a^3 F_X + 6\alpha_X a^2 F_{R\varphi} + 6\alpha_\varphi a^2 F_{RX}) + \\
&+3a^2\alpha\left[F - (R-u)F_R - (T-v)F_T + \frac{1}{3}a\,(u_a F_R + v_a F_T)\right] + \\
&+\beta a^3\,[-(R-u)F_{RR} - (T-v)F_{TR}] + \gamma a^3\,[-(R-u)F_{RT} - (T-v)F_{TT}] + \\
&+\delta a^3\,[-(R-u)F_{RX} - (T-v)F_{TX}] + \epsilon a^3\,[F_\varphi - (R-u)F_{R\varphi} - (T-v)F_{T\varphi}] + \\
&+a^3\,[u_{\dot{a}} F_R + v_{\dot{a}} F_T]\,(\dot{a}\alpha_a + \dot{R}\alpha_R + \dot{T}\alpha_T + \dot{X}\alpha_X + \dot{\varphi}\alpha_\varphi) - \\
&-(\alpha 3F_X + \beta a F_{XR} + \gamma a F_{XT} + \delta a F_{XX} + \epsilon a F_{X\varphi})\,Xa^2. \qquad (16)
\end{aligned}
$$

From a Noether symmetry we have:

$$
\begin{aligned}
&\dot{a}^2: && (\alpha + 2a\alpha_a)\,[F_R - F_T] + a\,[\beta a F_{RR} + \gamma a F_{TR}]_a - \\
& && -\beta a F_{TR} - \gamma a F_{TT} = 0, && (17)\\
&\dot{R}^2: && 6\alpha_R a^2 F_{RR} = 0, && (18)\\
&\dot{T}^2: && 6\alpha_T a^2 F_{RT} = 0, && (19)\\
&\dot{X}^2: && 6\alpha_X a^2 F_{RX} = 0, && (20)\\
&\dot{\varphi}^2: && \epsilon_\varphi a + \alpha\frac{3}{2} = 0, && (21)\\
&\dot{a}\dot{R}: && 2\alpha F_{RR} + (\beta a F_{RR} + \gamma a F_{RT})_R + \alpha_a a F_{RR} = 0, && (22)\\
&\dot{a}\dot{T}: && 2\alpha F_{RT} + (\beta a F_{RR} + \gamma a F_{RT})_T + \alpha_a a F_{RT} = 0, && (23)\\
&\dot{a}\dot{X}: && (\beta a F_{RR} + \gamma a F_{RT})_X = 0, && (24)\\
&\dot{a}\dot{\varphi}: && 2\alpha_\varphi\,[F_R - F_T] + (a\beta F_{RR} + a\gamma F_{TR})_\varphi + \epsilon_a\frac{a^2}{6}F_X = 0, && (25)\\
&\dot{R}\dot{T}: && \alpha_R a^2 F_{TR} + \alpha_T a^2 F_{RR} = 0, && (26)\\
&\dot{R}\dot{X}: && \alpha_X a^2 F_{RR} + \alpha_R a^2 F_{RX} = 0, && (27)\\
&\dot{T}\dot{X}: && \alpha_X a^2 F_{TR} + \alpha_T a^2 F_{RX} = 0, && (28)
\end{aligned}
$$

$$\dot{R}\dot{\varphi}: \quad 6\alpha_\varphi a^2 F_{RR} + \epsilon_R a^3 F_X = 0, \tag{29}$$

$$\dot{T}\dot{\varphi}: \quad 6\alpha_\varphi a^2 F_{TR} + \epsilon_T a^3 F_X = 0, \tag{30}$$

$$\dot{X}\dot{\varphi}: \quad \epsilon_X a^3 F_X = 0, \tag{31}$$

$$\begin{aligned} &3\alpha \left[F - (R-u)F_R - (T-v)F_T + \frac{1}{3}a\left(u_a F_R + v_a F_T\right)\right] + \\ &+\beta a\left[-(R-u)F_{RR} - (T-v)F_{TR}\right] + \gamma a\left[-(R-u)F_{RT} - (T-v)F_{TT}\right] + \\ &+\dot{a}\alpha_a a\left[u_{\dot{a}}F_R + v_{\dot{a}}F_T\right] + \dot{\varphi}\alpha_\varphi a\left[u_{\dot{a}}F_R + v_{\dot{a}}F_T\right] + \\ &+\dot{R}\alpha_R a\left[u_{\dot{a}}F_R + v_{\dot{a}}F_T\right] + \dot{T}\alpha_T a\left[u_{\dot{a}}F_R + v_{\dot{a}}F_T\right] + \\ &+\dot{X}\alpha_X a\left[u_{\dot{a}}F_R + v_{\dot{a}}F_T\right] + \epsilon a\left[F_\varphi\right] - \left(\alpha 3 F_X\right)X = 0. \end{aligned} \tag{32}$$

5. The Noether Symmetries Solution

First variant for solution we have find for $F_{RR} = F_{RT} = F_{RX} = 0$, and solution of this is a linear equation $F = s_1(\varphi)R + s_2(\varphi)T + s_3(\varphi)X + s_4(\varphi)$.

Second variant for solution we have find for $\alpha_R = \alpha_T = \alpha_X = 0$. In general the solution here will have this form:

$$f(R,T,X,\varphi) = F'\left(C_1(\varphi)R + C_2(\varphi)T\right) + C_X X + C(\varphi). \tag{33}$$

Than we can rewrite solution for action in most simple form as

$$S = \int \sqrt{-g}d^4x[\alpha R_s + \beta T_s + \alpha u + \beta v + L_m] = \int \sqrt{-g}d^4x[\alpha R_s + \overline{L_m}], \tag{34}$$

where $\overline{L_m} = \alpha u + \beta v + L_m$.
Here $L_m : \rho, p$ rewritten as $\overline{L_m} : \overline{\rho}, \overline{p}$.
Then the action we rewrite as

$$S = \int \sqrt{-g}d^4x[\alpha R + \beta T + L_m]. \tag{35}$$

For Friedman - Robertson - Walker (FRW) metric we can write density ρ and pressure p as

$$\begin{aligned} \rho &= 3H^2, \\ -p &= 2\dot{H} + 3H^2, \end{aligned} \tag{36}$$

The solution we look far as $a = a_0 t^n$, where n is constant. Than, since we solve the system for k-essences, we has

$$\begin{aligned} \overline{p} &= = \overline{K} = \frac{(2-3n)\,n}{t^2}, \\ \overline{\rho} &= = 2X\overline{K_X} - \overline{K} = \frac{3n^2}{t^2}. \end{aligned} \tag{37}$$

Solving these equations we obtain

$$\begin{aligned} \overline{K} &= \frac{(3n-2)\,n}{{C_1}^2 X^{\frac{3n-1}{2-3n}}}, \\ X &= = {C_1}^{\frac{2(2-3n)}{1-3n}} t^{\frac{2(2-3n)}{3n-1}}. \end{aligned} \tag{38}$$

6. Conclusions

We considered the generalization of $F[R, T]$ gravity with k-essence and found the exact analytical solution for this model.

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