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Cosmological Yang-Mills model with k-essence

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Abstract. We considered the $f(R)$ model of gravity with the Yang-Mills field and k-essence in four dimensions, together with the homogeneous, isotropic and flat Friedmann-Robertson-Walker universe. For this model we found equations of motion, a solution for a scale factor, a scalar field, a scalar potential is reconstructed, we studied the slow roll parameters. For the model under consideration, the of slow roll parameters satisfy the region of the inflationary stage. For later times was found the equation of state parameter ω, the deceleration parameter q the value of which corresponds to the accelerated expansion of the universe. Our model allows you to get an accelerated expansion of the universe in the inflationary period. Over time, the field decreases, rolls off slowly, viscosity has a lesser effect, and the universe leaves the inflationary regime, which shows the exponential dynamics of changes in the law of expansion of the universe.

1. Introduction

Over the past twenty years in astronomy, one of the most important discoveries has been the discovery of the fact that cosmic expansion accelerates with time. Moreover, this process began in a relatively recent era of cosmological history [1].

Cosmology is currently experiencing a large increase in activity. Observations of cosmic microwave radiation have confirmed inflation in the early universe and accelerated expansion of the late universe [2]-[4]. Also, as evidence of the accelerated expansion of the universe, observational data on type Ia supernovae [5],[6], large-scale structure of the universe [7], baryon acoustic oscillations and weak gravitational lensing were presented in [8]. Recently, a number of interesting reviews have appeared describing the cosmological evolution from inflation to the present [9]. Scalar-tensor and $F(R)$ theories were considered in [10]-[17]. Papers [18]-[25] studied the problems of dark energy, and [26], [27] considered $F(T)$ theory.

From the accelerated expanding universe, it follows that the fundamental theories are either incomplete or incorrect. Therefore, two ideas have appeared: up to 75% of the energy density of the universe exists in the form of dark energy with a large negative pressure responsible for accelerated expansion; the general theory of relativity needs to be revised on a cosmological scale [28].

The introduction of the Yang-Mills $F_{\mu\nu}^a$ field for $f(R)$ gravity allows us to find new approaches to solving problems of the observed accelerated expansion of the universe. Yang–Mills fields can interact with themselves and with each other, therefore they are nonlinear and the principle of superposition is not fulfilled for them. Their non-linearity complicates the search for solutions, so we will look for solutions in the mode of slow roll.

In this article, we consider an action of $f(R)$ gravity with a Yang-Mills field and k-essence. WE found the equations of motion. We construct a solution to the model under consideration. We determine whether such a model can describe the accelerated expansion of the universe.

2. $F(R)$ model of gravity with Yang-Mills field and k - essence

Consider the action of $f(R)$ gravity with a SU(2) Yang-Mills field and k-essence in four dimensions

$$S_{fYMk} = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\{f(R) - F^a_{\mu\nu}F^{\mu\nu a} + 2K(X,\phi)\}, \tag{1}$$

where SU(2) the Yang-Mills field A^a_μ has an internal symmetry index a and the field strength tensor is

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon^{abc} A^b_\mu A^c_\nu, \tag{2}$$

where ϵ^{abc} the Levi-Civita symbol and K is a function of its arguments, ϕ is a scalar function. Here

$$X = \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi \tag{3}$$

is the canonical kinetic term of a scalar field. ∇_μ covariant derivative.

The ansatz of the solution for SU(2) of the Yang-Mills field A^a_μ is a function of t. Then

$$F^1_{01} = -F^1_{10} = \dot{A}^1_1, \quad F^2_{02} = -F^2_{20} = \dot{A}^2_2, \quad F^3_{03} = -F_{30} = \dot{A}^3_3, \tag{4}$$
$$F^1_{23} = -F^1_{32} = A^2_2 A^3_3, \quad F^2_{13} = -F^2_{31} = A^1_1 A^3_3, \quad F^3_{12} = -F^3_{21} = A^1_1 A^2_2$$

dot means time derivative (all other components of $F^a_{\mu\nu}$ are equivalent to zero).

Let as consider a homogeneous, isotropic and flat Friedmann-Robertson-Walker universe (FRW) filled with g-essence. In this case, the metric has the form

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2), \tag{5}$$

where $a(t)$ is a scale factor of the universe. For this metric, the scalar curvature is $R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)$ and a member of the Yang-Mills field will take the form

$$\begin{aligned}
F^a_{\mu\nu}F^{\mu\nu a} &= 2[g^{00}g^{11}(F^1_{01})^2 + g^{00}g^{22}(F^2_{02})^2 + g^{00}g^{33}(F^3_{03})^2 + g^{11}g^{22}(F^3_{12})^2 + \\
&+ g^{11}g^{33}(F^2_{13})^2 + g^{22}g^{33}(F^1_{23})^2] = \\
&= -\frac{2}{a^2}[(\dot{A}^1_1)^2 + (\dot{A}^2_2)^2 + (\dot{A}^3_3)^2] + \frac{2}{a^4}[A^1_1 A^2_2 + A^1_1 A^3_3 + A^2_2 A^3_3].
\end{aligned} \tag{6}$$

Then the action (1) is compatible with (5) and (6) can be written as

$$\begin{aligned}
S_{fYMk} = \frac{1}{8\pi G} \int d^4x \Big\{ &\frac{1}{2} a^3 f + a[(\dot{A}_1)^2 + (\dot{A}_2)^2 + (\dot{A}_3)^2] - \\
&- \frac{1}{a}[A^1_1 A^2_2 + A^1_1 A^3_3 + A^2_2 A^3_3] + a^3 K \Big\}.
\end{aligned} \tag{7}$$

In the case of the FRW metric (5), the equations of motion corresponding to the action (7) using the Euler-Lagrange equations and the zero-energy condition can be written as

$$3H^2 - \rho = 0, \tag{8}$$
$$2\dot{H} + 3H^2 + p = 0, \tag{9}$$
$$\ddot{A}^1_1 + H\dot{A}^1_1 + \frac{1}{2a^2}[A^2_2 + A^3_3] = 0, \tag{10}$$

$$\ddot{A}_2^2 + H\dot{A}_2^2 + \frac{1}{2a^2}[A_1^1 + A_3^3] = 0, \tag{11}$$

$$\ddot{A}_3^3 + H\dot{A}_3^3 + \frac{1}{2a^2}[A_1^1 + A_2^2] = 0, \tag{12}$$

$$K_X\ddot{\phi} + (\dot{K}_X + 3HK_X)\dot{\phi} - K_\phi = 0, \tag{13}$$

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{14}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter and the canonical kinetic term of the scalar field is $X = \frac{1}{2}\dot{\phi}^2$.

Energy density and pressure take the form

$$\rho = \frac{1}{f_R}\Big\{-3H\dot{R}f_{RR} + \frac{1}{2}Rf_R - \frac{1}{2}f + \frac{(\dot{A}_1^1)^2 + (\dot{A}_2^2)^2 + (\dot{A}_3^3)^2}{a^2} + \tag{15}$$
$$+\frac{A_1^1A_2^2 + A_1^1A_3^3 + A_2^2A_3^3}{a^4} + 2K_XX - K\Big\},$$

$$p = \frac{1}{f_R}\Big\{\dot{R}^2f_{RRR} + (2H\dot{R} + \ddot{R})f_{RR} - \frac{1}{2}Rf_R + \frac{1}{2}f + \frac{(\dot{A}_1^1)^2 + (\dot{A}_2^2)^2 + (\dot{A}_3^3)^2}{3a^2} + \tag{16}$$
$$+\frac{A_1^1A_2^2 + A_1^1A_3^3 + A_2^2A_3^3}{3a^4} + K\Big\}.$$

In this article, we will consider the action of the k-essence (7) with the Lagrangian

$$K = X - V_1(\phi) \tag{17}$$

and ansatz of the solution for the potential tensor equal to

$$A_\mu^a = (0, \phi(t)\delta_i^a). \tag{18}$$

Then the system of equations of motion (8)-(14) takes the form

$$3H^2 - \rho = 0, \tag{19}$$

$$2\dot{H} + 3H^2 + p = 0, \tag{20}$$

$$\ddot{\phi} + H\dot{\phi} + \frac{\phi}{a^2} = 0, \tag{21}$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{1\phi} = 0, \tag{22}$$

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{23}$$

where

$$\rho = \frac{1}{f_R}\left[-3H\dot{R}f_{RR} + \frac{1}{2}Rf_R - \frac{1}{2}f + \left(\frac{1}{2} + \frac{3}{a^2}\right)\dot{\phi}^2 + 3\frac{\phi^2}{a^4} + V_1\right], \tag{24}$$

$$p = \frac{1}{f_R}\left[\dot{R}^2f_{RRR} + (2H\dot{R} + \ddot{R})f_{RR} - \frac{1}{2}Rf_R + \frac{1}{2}f + \left(\frac{1}{2} + \frac{1}{a^2}\right)\dot{\phi}^2 + \frac{\phi^2}{a^4} - V_1\right].$$

Equations (10)-(12) after substituting the potential tensor A_μ^a become identical and take the form of the equation (21).

Case $f = \frac{R^2}{6}$. Taking in to account that for the metric (5) the scalar curvature is $R = 6(\dot{H} + 2H^2)$ we rewrite the system of equations of motion (19)-(23) in terms of H

$$3H^2 - \rho = 0, \tag{25}$$

$$2\dot{H} + 3H^2 + p = 0, \tag{26}$$

$$\ddot{\phi} + H\dot{\phi} + \frac{\phi}{a^2} = 0, \tag{27}$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{1\phi} = 0, \tag{28}$$

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{29}$$

where

$$\rho = \frac{3[-2H\ddot{H} - 4H^2\dot{H} + \dot{H}^2 + 4H^4]}{2(\dot{H} + 2H^2)} + \frac{1}{2(\dot{H} + 2H^2)}\left[\left(\frac{1}{2} + \frac{3}{a^2}\right)\dot{\phi}^2 + 3\frac{\phi^2}{a^4} + V_1\right], \tag{30}$$

$$p = \frac{2H^{(3)} + 12H\ddot{H} + 5\dot{H}^2 + 4H^2\dot{H} - 12H^4}{2(\dot{H} + 2H^2)} + \frac{1}{2(\dot{H} + 2H^2)}\left[\left(\frac{1}{2} + \frac{1}{a^2}\right)\dot{\phi}^2 + \frac{\phi^2}{a^4} - V_1\right]. \tag{31}$$

From equations (30) and (31) we obtain the equation of state parameter

$$\omega = \frac{p}{\rho} = -1 + \frac{2H^{(3)} + 6H\ddot{H} + 8\dot{H}^2 - 8H^2\dot{H} + (1 + \frac{4}{a^2})\dot{\phi}^2 + 4\frac{\phi^2}{a^4}}{-6H\ddot{H} + 3\dot{H}^2 - 12H^2\dot{H} + 12H^4 + (\frac{1}{2} + \frac{3}{a^2})\dot{\phi}^2 + 3\frac{\phi^2}{a^4} + V_1}. \tag{32}$$

In a rapidly expanding universe, the scalar field rolls down very slowly, while the effective viscosity is proportional to the rate of expansion. In slow roll mode [28]

$$H\dot{\phi} >> \ddot{\phi}, \quad V_1(\phi) >> \dot{\phi}^2. \tag{33}$$

In this limit, the equations (27) and (28) take the form

$$H\dot{\phi} + \frac{\phi}{a^2} = 0, \tag{34}$$

$$3H\dot{\phi} + V_{1\phi} = 0. \tag{35}$$

Let us consider the law of expansion of the universe in the mode of slow rolling as

$$a = \exp(Ht). \tag{36}$$

Then from the equation (34) we find the scalar field function, the graph of which is shown in the figure 1

$$\phi = \exp\left(\frac{1}{2H^2 \exp(2Ht)}\right). \tag{37}$$

Knowing the function of the scalar field (37) and the law of expansion of the universe (36) from the equation (35) we find the potential of the scalar field

$$V_1 = 3H^2\phi^2(\ln\phi - \frac{1}{2}). \tag{38}$$

Due to the exponential increase in the scale factor, inflation describes flat space and, therefore, the parameter of equation of state is $\omega = -1$ and the deceleration is $q = -1$.
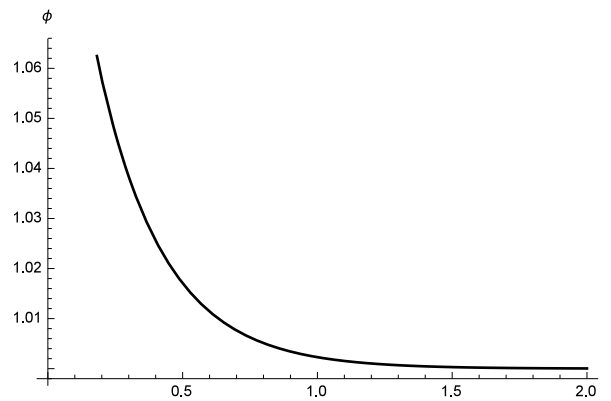
Figure 1. The dependence of the scalar field ϕ on time t

Slope of potential $\epsilon(\phi)$ and curvature of potential $\eta(\phi)$, called slow roll parameters of the potential function and the scalar field defined as follows [29]

$$\epsilon(\phi) = \frac{1}{2}\left(\frac{V_\phi}{V}\right)^2, \quad \eta(\phi) = \frac{V_{\phi\phi}}{V}. \tag{39}$$

In our case, for the potential (38) we get

$$\epsilon(\phi) = \frac{2\ln^2\phi}{\phi^2(\ln\phi - \frac{1}{2})^2}, \quad \eta(\phi) = \frac{1 + \ln\phi}{\phi\ln\phi}. \tag{40}$$
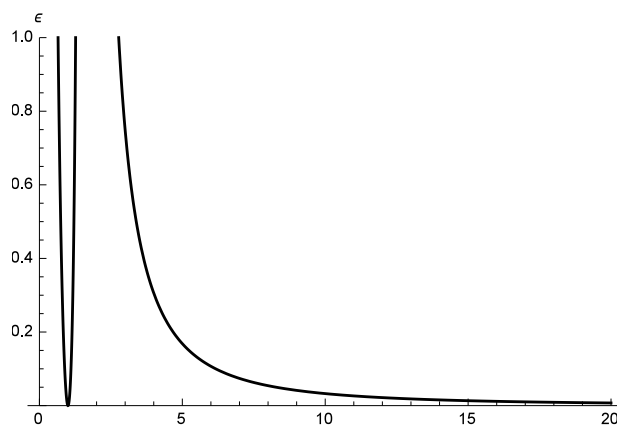

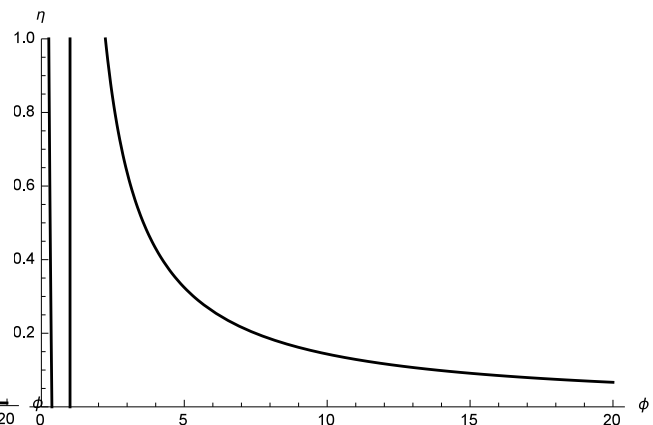**Figure 2.** Slope of potential $\epsilon(\phi)$.

Figure 3. Curvature of potential $\eta(\phi)$.

For inflation, it is necessary that $\epsilon(\phi) << 1$ and $\eta(\phi) << 1$. As can be seen from Figures 2 and 3 slow roll parameters of our models satisfy this condition. When the of slow roll parameters approach 1, the inflationary stage is exited.

3. Conclusion

We considered the $f(R)$ model of gravity with the Yang-Mills field and k-essence. For this model we found the equations of motion, the solution for the scale factor. The scalar field, the scalar potential were reconstructed, also we studied the slow roll parameters. For the model under

consideration, the parameters of slow rolling satisfy the region of the inflationary stage. For later times found the equation of state parameter ω, the deceleration parameter q the value of which corresponds to the accelerated expansion of the universe. Our model allows to obtain an accelerated expansion of the universe in the inflationary period. Over time, the field decreases, rolls off slowly, viscosity has a lesser effect, and the universe leaves the inflationary regime, which shows the exponential dynamics of changes in the law of expansion of the universe.

3.1. Acknowledgments

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